

MICHAEL HALLETT

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Office: Leacock 913. Office Hours: To Be Announced.

Lectures: Tuesday, Thursday, 13.00–14.30, in room WONG 1030

Summary of Material. The aim of this course is to examine some parts of the geometry developed by Euclid in his remarkable work the *Elements* (c. 300 BCE), and in particular the role played in it by the famous ‘parallel postulate’. The *Elements* was based on 23 definitions, some so-called ‘Common Notions’ (‘CN’ for short), and 5 postulates, all except one of these apparently innocuous. This exception was the Fifth Postulate, the Parallel Postulate (‘EPP’ for short), and it is this postulate on which we will eventually focus. Basic geometry, in particular plane geometry, is then built up via *demonstrated* propositions in a careful order.

First, we will look at the nature of the system, in particular at what Euclid takes as primitives, then the things which he explicitly states as starting points, and (trickier!) what he implicitly assumes, and the way all this is made use of in the deductive arrangement of Book I, through the proofs.

In the course of this, we will consider a small selection of modern commentaries on Euclid’s system and its assumptions. After this, we will then look at the very important role the EPP plays in the development of Euclid’s system, why it is brought in, and what Euclid succeeds in proving with it.

We will then turn (as Euclid never does in any obvious way) to look at the status of the postulate. In particular, we will focus on Proclus’s claim (in his commentary on Book I of Euclid’s *Elements* (fifth century of the CE), that the EPP should *not* be listed among the postulates, but rather should be a *proved theorem*. If this is correct, *is* the EPP provable (using, say, just the other assumptions Euclid adopts), and can we actually prove it?

Proclus himself attempted a proof, and reported on another due to Ptolemy, both of which we will examine closely, since their unsatisfactoriness reveals how interconnected the EPP is with other assumptions. (There were various approaches made later, particularly by the geometers of the Muslim world, and we will look at some of these attempts, too, given time.)

Perhaps the most interesting attempt is one made much later by the (decidedly non-Ancient) English mathematician John Wallis in the 17th c. The reason for looking closely at this proof is largely for the light that it sheds on many of the implicit things in Euclid’s system which we will have stressed, and also for the way it reveals the intricacies of the EPP and the way it interconnects with other parts and assumptions of Euclid’s system.

Lastly, and we will sum this up quickly, the failure to provide convincing proofs of the EPP from the rest of Euclid’s system, particularly through attempts made by trying to prove it by *reductio* (assuming the negation of EPP, and then searching for a contradiction), eventually led to the consideration of what are now called *non-Euclidean geometries*. In particular, we will raise here the question of how this kind of enquiry led to an extension of Euclid’s project, particularly an extension to the fundamental question which occupied him of what rests on what.

The course will therefore have 5 distinct parts:

- Part I: Euclid, the Text of the *Elements*, and Euclid’s system, what is officially assumed, and what is implicitly assumed. (Important here will be the Proposition 4 of Book I of Euclid’s *Elements* justifying the Side-Angle-Side criterion of triangle congruence.)

- Part II: The role of the axioms/Postulates in Euclid's system. We will also consider here a limited number of modern commentaries on the *Elements*. *Major Question*: Is the *Elements* actually an axiomatisation of geometry as we understand this now?
- Part III: The deductive role of the EPP, in particular the build-up to the Pythagorean Theorem, Proposition 47 of Book I.
- Part IV: The status of the postulates, and in particular the EPP. Here most of the discussion will be sparked by Proclus's *Commentary*.
- Part V: Proofs of the EPP, particularly Ptolemy's, Proclus's, then much later (17th c.) Wallis's, and the structure of Saccheri's attempted proof. (Perhaps, too, some reading from Lambert and from Leibniz, depending on time.)
- Part VI: Very brief account of the birth of non-Euclidean geometries. The accommodation of proofs of unprovability. (For the sake of definiteness, the focus will be on the so-called 'Poincaré model', and we'll discuss the very modern idea of 'modelling' geometry which is opened up by this.)

The Purpose There are, roughly, three things:

1. Appreciation of what was achieved by science in the ancient world. The place of mathematics was intellectually *fundamental* to the Greeks (see the short extract from Shapiro), both for science (e.g., in astronomy, music) and philosophy (example of how we can know abstract truths, and general propositions).
2. Appreciation of how we can build up a science deductively from small, often apparently insignificant, beginnings.
3. Focus on the Euclidean Parallel Postulate, however, shows that in mathematics *ex nihilo nihil fit* ('nothing comes from nothing'), i.e., we cannot expect to get anything significant for nothing.
4. Appreciation of the later importance of proofs of unprovability, more generally of impossibility.

Course Material

1. The basis of the course will be the text of Euclid contained in Thomas Heath's translation of Euclid in *The Thirteen Books of Euclid's Elements*, in particular Volume 1, which contains Euclid's Book I. Heath's historical notes and commentaries are also very useful.
 2. Also very important to us will be the text of Proclus's *Commentary on the First Book of Euclid's Elements*, in the translation by Glenn Morrow.
 3. In addition, there will be a number of important articles designed to throw light on (or perhaps just raise questions about) different aspects or details of the things we will consider.
 4. All of this material will be made available on *myCourses*.
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Delivery of Lecture Material

The course will be in person; there will be three lecture-hours per week, on Tuesdays and Thursdays, together with some Office Hours, as yet unscheduled. These, too, will be in person.

- I will prepare a series of Handouts, which will often be in note or question form, and which will be posted regularly on *myCourses*. My intention then is to elaborate on these in the lectures.
- I will invite questions to be posted on *myCourses*, and whenever there are enough of these, they will be discussed in lectures in the week following.
- I may make use of the video format (not live, but posted) for discussion of extra matters, which are germane, but perhaps tangential to the core subject matter.
- I will also occasionally make use of the same (posted-video) format in the case of (I hope rare) enforced absence from the classroom. This will be managed/announced on an *ad hoc* basis.

myCourses

As will be clear from the above, McGill's *myCourses* system will be crucial for the course. This is where the reading material, the Handouts, a Discussion Board, etc. are to be found, and very importantly it's where Announcements will appear noting forthcoming deadlines, postings, changes etc.

But please note that the *myCourses* system uses *only* your official McGill e-mail address, so this (as well as NB *myCourses*) should be checked regularly and routinely.

Marking and Assessment: The final mark is composed of marks assigned to three reading assignments (in total 20%), a short sketch paper due around the middle of the term (30%), and a final take-home exam (worth 50%) due in the exam period with deadline as set by the university's rules on these. All work will be on questions *assigned* and posted on *myCourses*, and must be *submitted* there. The reading journals will require short answers (≤ 100 words) to specific questions about the reading material assigned, and might focus on readings not discussed at length in lectures. The short/sketch paper is to be ≤ 500 words in length on one of the topics to be assigned. (More details on this when the topics are assigned.) The take-home exam is expected to be ≤ 2000 words in length.

More detailed instructions for each assignment will be given later. However, you can expect that the Sketch will be due at roughly the half-way point of the course (Week 7), and the take-home exam some time after the end of the teaching Semester during the official examination period (after Week 14). The reading assignments will be at roughly the 1/4 (Week 4), 3/4 (Week 10) and 4/4 (Week 13) points in the 13 week teaching semester.

Submission of Work. All work is to be submitted *electronically*, to *myCourses* as PDF documents. (WORD files are NOT acceptable: PDFs can be created very simply from any word processor files.) The titles of the files submitted are to be of the form 'Bloggs-G-350-X', where 'Bloggs' is here a placeholder for *your surname only as it appears on the course registration*, 'G' is a placeholder for your first given name as it appears on the registration sheet, 'X' is a placeholder either for (as appropriate) 'Sketch', 'Final' or 'Assignment-*n*', where '*n*' will be either '1', '2', '3', again as appropriate.

Policy for Late Work: Extensions to deadlines set will be granted only in **exceptional** circumstances, usually only for medical reasons or other, similar emergencies (which of course include COVID-related difficulties), and with a medical note or other appropriate documentation wherever appropriate. Late work will be penalised at the rate of 5 percentage points per day overdue, so half a grade-scale per day.

Students who wish to make arrangements with the Office for Students With Disabilities are encouraged to make these arrangements as soon as possible, and to inform me about the nature of the Accommodation agreed. NB

McGill Policies

1. *McGill University values academic integrity. Therefore all students must understand the meaning and consequences of cheating, plagiarism and other academic offences under the Code of Student Conduct and Disciplinary Procedures. (See www.mcgill.ca/integrity for more information.)* NB
2. *In the event of extraordinary circumstances beyond the University's control, the content and/or evaluation scheme in this course is subject to change.*
3. *In accord with McGill University's Charter of Students' Rights, students in this course have the right, without seeking permission, to submit in English or in French any written work that is to be graded.*
4. *As instructors of this course, the Lecturers and TAs endeavor to provide an inclusive learning environment. If you experience barriers to learning in this course, do not hesitate to discuss them with me/us or with Student Affairs or the Office for Students with Disabilities, <https://www.mcgill.ca/osd>, 514-398-6009.*
5. *McGill University is on land which is the traditional and unceded territory of the Kanien'keha:ka (Mohawk), a place which has long served as a site of meeting and exchange amongst nations.*