PHIL 311: Philosophy of Mathematics — Fall 2024

General information

Course schedule: Tuesdays and Thursdays, 4:05pm–5:25pm
Course location: Education Building 211
First day of classes: August 29, 2024
Last day of classes: December 3, 2024
Course prerequisites: Introduction to Deductive Logic (Phil 210) or permission by the instructor.

Instructor information

Instructor: Dirk Schlimm
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Office hours: tba on myCourses

Course content

Mathematics has exerted a particular attraction to philosophers throughout history. For example, tradition has it that the phrase "Let no one ignorant of geometry enter" marked the entrance to Plato's Academy, Kant famously argued that "5+7=12" is a synthetic proposition that is knowable a priori, and Frege worried how we can determine whether Julius Caesar is a number or not. And even after more than 2000 years of philosophical reflections on the nature of mathematical truths, the status of mathematical objects, the sources of mathematical knowledge, the applicability of mathematics in science, and the methodology of mathematical practice, these topics still continue to puzzle philosophers.
This course provides an historically informed introduction to philosophy of mathematics. It is intended to present an overview of prominent issues and arguments, to motivate the students to appreciate this fascinating subject matter, and enable them to discuss contemporary research in philosophy of mathematics. To this end, philosophical reflections on mathematics and particular episodes in the history of mathematics will be presented and discussed side by side. Simple examples from mathematical practice (mainly geometry, arithmetic, and algebra) serve as illustrations for the subject matter the philosophical reflections are about, and, at the same time, they serve as proving ground for the adequateness of the philosophical claims about mathematics.

Required course materials

All readings will be made available on myCourses.

Assessment

Students are expected to attend and participate in class, do the assigned readings, complete weekly homework assignments, take three in-class quizzes, write a critical summary of a research article, and write a term paper. The final grade depends on:

Submission of weekly homework assignments	10%
Three in-class quizzes (worth 5%, 10% and 10%)	25%
Meeting milestones for critical summary	5%
Critical summary of a research article	20%
Meeting milestones for term paper	5%
Term paper	35%

Late days. Every student can take up to two "late days" for meeting milestones and handing in the homework assignments or papers during the semester.

Assignments. Weekly assignments consist of readings and reflection questions. Answering these will help being prepared for the lectures. Assignments will not be graded, but will receive a participation grade if a meaningful submission is made.

Quizzes. The in-class quizzes will each consist of three simple questions that are based on the homework assignments and material discussed in class. They are graded for correctness, clarity, and the appropriate use of terminology.

Milestones. Milestones for the critical summary and term paper consist of an announcement of the topic of interest, posting a reading list and an outline, giving and receiving peer reviews. The dates are listed in the schedule, below. Meeting a milestone will receive a participation grade.

Grading criteria for papers. The critical summary and term paper are graded by the instructor with a letter grade. To do excellent work in this course, you need to be able to do more than just reiterate what various authors have written or what was said in class. An 'A' indicates that you not only understand and comprehend the material, but that you have thought critically about it, fully fleshing out its subtleties and implications so that you can creatively apply the material at many levels. A 'B' reflects an above-average understanding of the material without any major errors; however, 'B' work doesn't capture the complexity of the issues and tends toward accurate summary rather than independent analysis. A 'C' suggests a struggle with the material that manages an average, basic comprehension of it but is flawed by some significant misunderstandings or errors. A 'D' indicates only a rudimentary comprehension of part of the material with most of the material being misunderstood; 'F' indicates no understanding of the material.

Grading scale. The critical summary and term paper are graded with letter grades, while the homework submissions, quizzes, and milestones are graded on a scale from 0 to 10. For the calculation of the final grade, letter grades are converted to the 0-10 scale as follows:

F	D	C	C+	B-	B	B+	A-	A
4.00	5.25	5.75	6.25	6.75	7.25	7.75	8.25	8.75

The conversion from the 0-10 scale to letter grades is according to the standard McGill scale:

F	D	C	C+	B-	B	B+	A-	A
0–4.99	5.0–5.49	5.5–5.99	6.0–6.49	6.5–6.99	7.0–7.49	7.5–7.99	8.0–8.49	8.5–10

Communication

For any communication about the course (that does not concern personal matters), please use the myCourses discussion board. Any submissions should be in PDF format (no Word documents, please).

Technology use

Use of Artificial Intelligence and LLM technology, such as ChatGPT, is not permitted in this class.

McGill policy statements

Academic integrity. McGill University values academic integrity. Therefore all students must understand the meaning and consequences of cheating, plagiarism and other academic offences under the Code of Student Conduct and Disciplinary Procedures (see http://www.mcgill.ca/students/srr/honest).

Copyright and intellectual property. Instructor generated course materials (e. g., handouts, notes, summaries, exam questions, etc.) are protected by law and may not be copied or distributed in any form or in any medium without explicit permission of the instructor. Note that infringements of copyright can be subject to follow up by the University under the Code of Student Conduct and Disciplinary Procedures.

Language of assessment. In accord with McGill University's Charter of Students' Rights, students in this course have the right to submit in English or in French any written work that is to be graded.

Extraordinary circumstances. In the event of extraordinary circumstances beyond the control of McGill University, assessment tasks in a course are subject to change, provided students are sent adequate and timely communications regarding the change.

Schedule

	Date	Hw	Topic	Content	
			Part I – **Introduction**		
1.	Th, Aug 29		Introduction to the course	What is mathematics? Basic terminology (objects, concepts)	
2.	Tu, Sep 3	1	Approaches, questions, and positions	General structure of philosophical inquiries. Main questions and tensions. Biases	(Wk 1)
			Part II – **Historical positions**		
3.	Th, Sep 5		○ Euclid's geometry	Definitions. Deductive structure of a theory. Primitive terms and axioms.	
4.	Tu, Sep 10	2	Plato	Truth. Forms. Recollection (Add/Drop deadline)	(Wk 2)
5.	Th, Sep 12		Aristotle	Abstraction	
6.	Tu, Sep 17	3	Ancient math and phil.	Discussion	(Wk 3)
				Critical summary: Announcement of topic of interest	
7.	Th, Sep 19		Kant	Space and time as forms of intuition. Pure intuition	
			Part III – **19th Century developments (Geometry)**		
8.	Tu, Sep 24	4	○ Analytic geometry and projective geometry	What is geometry? Duality of projective geometry. Meanings and language diverge	(Wk 4)
			Quiz 1	Critical summary: Reading list	
9.	Th, Sep 26		○ Non-Euclidean geometry	Geometry and the nature of space	
10.	Tu, Oct 1	5	Pasch and Empiricism	Deductive rigor.	(Wk 5)
				Critical summary: Outline	
11.	Th, Oct 3		Hilbert and Frege on geometry	Truth and consistency.	
			Part IV – **Foundational positions (Arithmetic)**		
12.	Tu, Oct 8	6	Frege's *Foundations of arithmetic*	Criticism of abstractionism. Logicism. Sense and reference.	(Wk 6)
				Critical summary: Full draft to peers	
13.	Th, Oct 10		Frege and Russell	Julius Caesar problem. Paradoxes. Theory of types.	
				Critical summary: Peer review	
	Tu, Oct 15		No class	*Reading Break*	
	Th, Oct 17		No class	*Reading Break*	
14.	Tu, Oct 22	7	Hilbert's Programme	Consistency proofs, finitary arithmetic.	(Wk 8)
				Critical summary due	

	Date	Hw	Topic	Content	
15.	Th, Oct 24		Formalism and Incompleteness	Gödel's incompleteness theorems. Limitations of formal systems.	
16.	Tu, Oct 29		Intuitionism	Mathematics as mental construction.	(Wk 9)
				Term paper: Topic announcement	

Part V – **Later 20th century positions**

17.	Th, Oct 31	8	Carnap and Quine	Linguistic frameworks, the analytic/synthetic distinction. Naturalism, the web of belief.	
18.	Tu, Nov 5		Set-theoretic realism: From Cantor to Gödel	Infinite sets. Intuition and analogies to science. Maddy.	(Wk 10)
			Quiz 2	Term paper: Outline and abstract	
19.	Th, Nov 7	9	○ Group theory	Abstract algebra.	
20.	Tu, Nov 12	10	Structuralism	Methodological structuralism (Benacerraf, Shapiro)	(Wk 11)

Part VI – **Philosophy of mathematical practice**

21.	Th, Nov 14		Lakatos: *Proofs and Refutations*		
22.	Tu, Nov 19	11	Learning arithmetic	Mathematical cognition.	(Wk 12)
				Term paper: Full draft to peers for review	
23.	Th, Nov 21		Mathematical notation: Numerals	Informational and computational equivalence. Roman numerals.	
24.	Tu, Nov 26	12	Mathematical notation: Logical notations		(Wk 13)
			Quiz 3	Term paper: Peer review to author	
25.	Th, Nov 28		tba		
26.	Tu, Dec 3		Review		(Wk 14)
				Term paper due	

Note that this schedule is *tentative* and may be changed in the course of the semester! (v2, Sep. 4, 2024)