Laws of Nature and Branching Spacetime

What is a law of nature? An elegant answer to this question is to be found in Armstrong (1983), in which laws are identified with non-logical contingent relations of "nomic necessitation" between universals. For example, the law that all protons have rest-mass *m* takes the form N(F,G), where F and G are the universals "being a proton" and "having rest-mass *m*" respectively, and N is the relation of contingent necessitation that connects them. It is not *logically* necessary that being a proton should imply having rest-mass *m*, but it is contingently and *physically* necessary. Armstrong distinguishes between laws and law-statements, which are attempts to put laws of nature into verbal or mathematical form:- on this see also Lehoux (2006).[1] Law-statements can be true or false. When they are true, what makes them true is a law.

In this paper a different but related analysis of laws is put forward. Laws are based not on universals but on patterns of instantiation of event-types in branching spacetime. The end-result resembles Armstrong's in providing an objective, realist basis for laws of nature, but the branching universe that underlies them is more complex and elaborate than the world of universals. As in David Lewis' theory of Humean supervenience, laws supervene on how events are distributed in space and time. Humean supervenience is "the doctrine that all there is to the world is a vast mosaic of local matters of particular fact, just one little thing and then another" (Lewis (1986), p. ix). In Lewis' picture, laws emerge from the mosaic of events in spacetime, which in Lewis' metaphysics is unbranched. Moving to a more complex branching structure that permits both actual and possible events to belong to a single spacetime history, the account of laws put forward here provides for both (i) the contingency, and (ii) the factual necessity, of laws of nature.

Branching spacetime has been described and analyzed in numerous publications by Belnap, McCall, and co-workers: Belnap (1992), Belnap, Perloff and Xu (2001), McCall (1994), McCall (2000), McCall (2009). What branches in branching spacetime are *histories*, i.e. complete 4-dimensional differential manifolds that extend back to the beginning of the universe and forward to the end of time (if there is one). The mode of branching between any two 4D histories is along a 3D spacelike hypersurface which serves as the "branch point" between them. For pictures of such branching see McCall (1994), p. 87; (2009), pp. 421-24; and figure 1 below, reproduced from McCabe (2005), p. 666. This diagram is derived by McCabe from Earman (1986), p. 225.

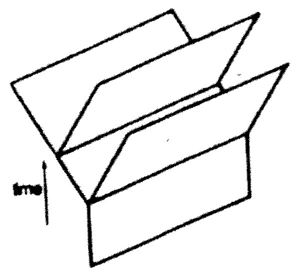


Figure 1.

The overall structure of the branched universe takes the form of a tree, with the past consisting of a unique 4D trunk, the future of a set of physically possible branching histories, and the present being at the first branch point (figure 2):

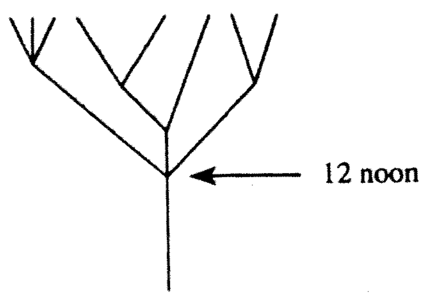


Figure 2.

As time progresses, a single one of the possible futures at the first branch point gets randomly selected as the *actual* branch, becoming part of the past, while the others vanish. If figure 2 is a model of the universe at time t, figure 3 represents it a moment later:

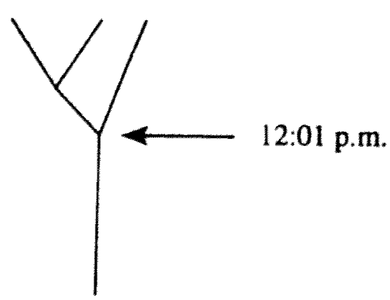


Figure 3.

This progressive branch loss or attrition represents the flow of time. The model is a dynamic one, in which the first branch point moves steadily up the tree, and sets of possible future histories are successively reduced to a single present and past. The future branches are *real*, but they are not *actual*. There is no danger of running out of branches: for all we know, they extend infinitely far into the future. The universe is a tree that "grows" (or more precisely "shrinks") by losing branches, and progressive branch loss constitutes the flow of time. Far from being a subjective illusion, depending on the existence of conscious beings, time flow is an objective feature of the world. A very different picture of spacetime branching, with no branch loss or attrition, is found in Penrose (1979), p. 592. In this model the branched structure remains fixed and invariant, while the observer moves up the tree.

1. Natural law supervenience.

Moving now to laws of nature, if throughout the universe-tree at any given period, some set of initial conditions A occurs at a branch point, and if all the branches above A contain B, then during that period B is *physically necessitated* by A:

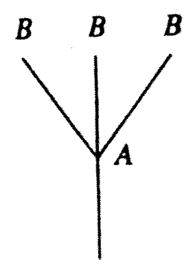


Figure 4

If this pattern of A's and B's is repeated everywhere throughout the branching structure, then "All A is B" states a law of nature. What it states is an objective fact about the universe: the law of nature supervenes upon the distribution of events in the world. Armstrong, as stated above, distinguishes sharply between *laws* and *law-statements*. Law-statements are human creations, whereas laws are repetitive patterns of events in space and time.

An interesting feature of this theory is that natural laws can *come into being* with the passage of time. Imagine a situation in which, at the time of the big bang, the pattern of A's and B's was not always as in figure 4, but in some instances followed figure 5:

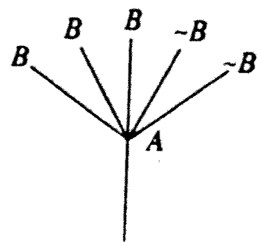


Figure 5

Then "All A is B" would not, from the beginning of time, state a law of nature. But suppose further that after a few billion years of branch attrition, no instances of figure 5's pattern remained, leaving only cases in which all branches above A initial conditions were B-branches (figure 4). In that case we could say that "All A is B" had *become* a law. Because of branch attrition, laws of nature constitute a monotonic increasing set. No laws are ever lost, and new ones come into being as the universe passes through different epochs.

2. Probabilistic laws.

Not all laws are exceptionless; many are probabilistic. The laws of particle decay fall into this category. To say that free neutrons have a half-life of 886 seconds means that if a free neutron is located at a first branch point, exactly half the branches above that point show the event of the neutron decaying into a proton, an electron, and an electron anti-neutrino within 886 seconds. Then subsequently, half the branches above each non-decay branch show the neutron decaying within another 886 seconds, etc. (figure 6):

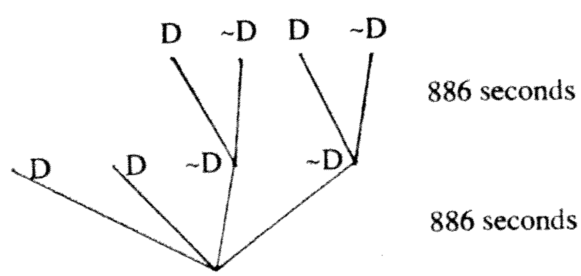


Figure 6

A nice feature of the branched theory of laws is that the time interval of 886 seconds, which is an essential component of the law of free neutron decay, is built directly into spacetime structure. This holds also for exceptionless laws. If a copper wire is moved through a magnetic field, an electric current is induced to flow in the wire, not instantly but after a short time lag. The length of the time lag can be found in the branched structure.

3. Non-rational probability values.

A different example of a probabilistic law is based on the Stern-Gerlach experiment in quantum mechanics. If a spin-1/2 particle such as an electron in the quantum state |z-spin up> passes through a Stern Gerlach magnet slanted at 30° to the vertical z-axis, then the electron has a probability of $\sin^2 30° = 3/4$ of exiting in the spin-up channel of the apparatus, and a probability of $\cos^2 30°$ of exiting in a spin-down channel (figure 7):

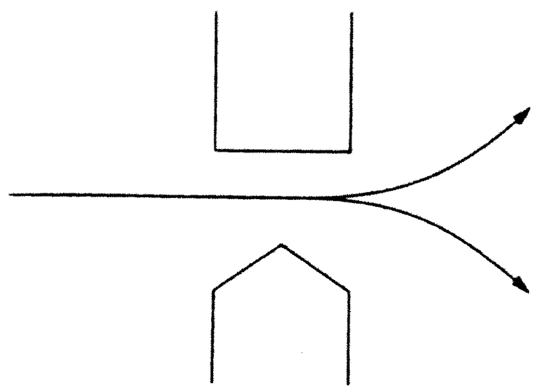


Figure 7

In branching space-time, these probabilities derive from the fact that 3/4 of the branches above the experiment's initial conditions show the electron exiting spin-up, and 1/4 of the branches show it exiting spin-down. In branching space-time, probability = relative proportionality of sets of branches. This works easily for rational probability values, but what if the apparatus is oriented not at 30° but at 20° to the z-axis? In that case the probability of the electron being measured spin-up is $\sin^2 20°$, an irrational number. How can irrational-valued proportionalities between sets of branches exist in branching spacetime?

This question is unanswerable if we restrict ourselves to finite sets. All proportions between finite sets take rational values only. If infinite sets are considered, on the other hand, there is a difficulty, since Cantor showed in the 19th century that there exist no well-defined proportionalities among infinite sets. One might think, for example, that the proportion of even to odd numbers among the integers was 1/2, but if the integers are ordered differently:

<u>**1**</u> 2 4 <u>**3**</u> 6 8 <u>**5**</u> 10 12 <u>**7**</u> ...

then the proportion of even to odd numbers seems to be 2/3, not 1/2. In Cantorian set theory, there exist no fixed proportions among infinite sets. How is it possible, in branching spacetime, to admit probabilities of future events that take irrational values?

A solution of this problem can be found that does not contradict Cantor's position. If the set of branches above the initial conditions of an electron entering a Stern-Gerlach apparatus possesses a particular kind of structure described as a "decenary tree", then any probability at all of the electron emerging spin-up, whether rational or irrational, can be represented by branch proportionality. A decenary tree is a tree, each branch of which branches in 10 at every one of a denumerably infinite set of branching levels (figure 8, where the time axis extends to the right):

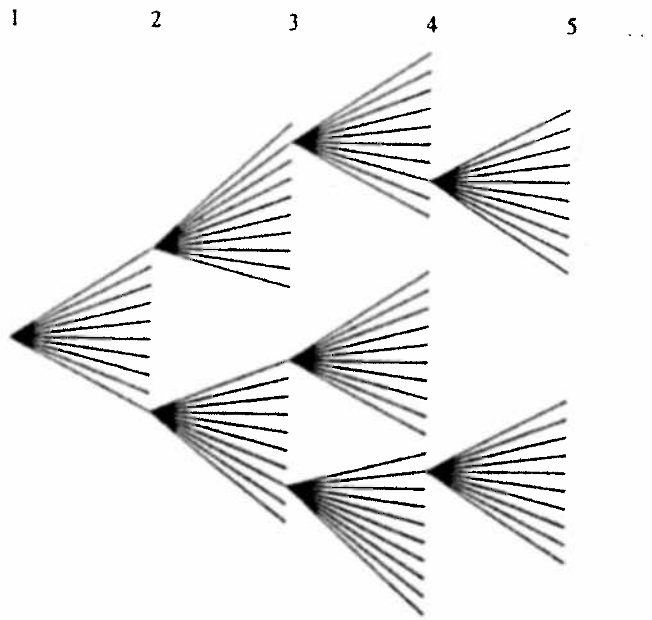


Figure 8

A decenary tree contains a non-denumerable infinity of branches, 10 to the power aleph-null.[2] But despite its many branches, a decenary tree need not extend far along the time axis. It may be compressed down into an arbitrarily short but non-zero interval of time, as shown in figure 9:

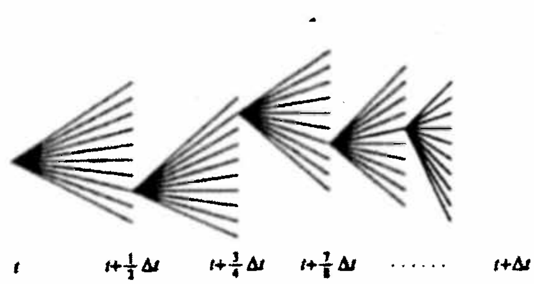


Figure 9

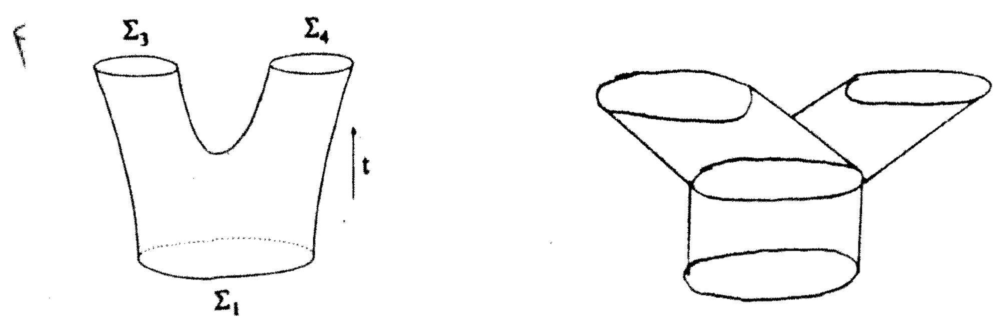
Let the first branching level be at time t, resulting in 10 branches, the second at t + 1/2 Δt, yielding 100 branches, the third at t + 3/4 Δt, yielding 1000, etc. By t + Δt there are infinitely many. A decenary tree provides probability values for outcomes equal to any real number between zero and one, whether rational or irrational, as follows.

An electron with vertical spin has a chance of $\sin^2 20° = 0.884023\ldots$ of exiting in the apparatus' spin-up channel when the magnet is tilted at 20° to the vertical, and a probability of

$\cos^2 20° = 0.115976...$ of exiting in the spin-down channel. Using "u" to denote "spin-up" outcomes and "d" to denote "spin-down", let 8 of the first 10 branches at the base of the decenary tree be u-branches, let 1 branch be a d-branch, and let 1 branch be "open", i.e. be a branch in which the electron is not yet in either channel. Once a branch becomes either u or d, it and all its daughter branches remain u or d for the rest of the decenary tree. At the second branching level, where the "open" branch splits in 10, again let 8 of its descendants be u-branches, 1 a d-branch, and let 1 remain open. At the third level, let 4 of the open branch's descendants be u-branches, let 5 be d-branches, with one left open. Continuing this process, the decimal numbers 0.884023... and 0.115976... are simply reproduced on the branches of the tree, with the result that when time $t + \Delta t$ arrives, exactly $\sin^2 20°$ of its branches are spin-up, and $\cos^2 20°$ are spin-down.[3] The decenary tree consequently provides all experimental outcomes with precise probability values, whether rational or irrational.

4. Branching along hyperplanes and non-locality.

In the spacetime structure envisaged here, histories split along 3D spacelike hyperplanes, as in figure 1 above. Borde et al. (1999), p. 3464, McCabe (2005), p. 673, and Earman (2008), p. 194, picture a very different kind of structure called a "trousers world":



(i) (ii)

Figure 10

In the trousers world (i) there is no well-defined borderline between trunk and branches, whereas in (ii) the branches split along a surface. As will be seen below, splitting along a surface is essential in understanding how branch attrition can throw light on the non-local distant correlations of the EPR experiment. The spacelike hyperplanes along which histories divide are relative to frames of reference. Each one is a constant-time hyperplane in a frame, the events that lie along it being simultaneous with one another in the frame in question. If a model branches along hyperplanes, its branching structure will be different not only at different times (because of branch attrition), but at different *frame-times*. At any given time, the shape of the structure will depend upon the frame of reference or coordinate system used to describe it. The way in which the shape of the universe tree changes from one frame of reference to another will be a "perspectival" change, analogous to the different aspects presented by a 3D object when viewed from different directions. In one frame, the universe model branches along one parallel family of hyperplanes; in another frame, along a different parallel family. These differences can be "transformed away" by changing to a new coordinate system. Time flow, modeled by branch-attrition, will always be time flow *within a frame of reference*. This fact provides part of the basis for getting clear about the twins paradox (McCall and Lowe (2003), McCall (2006)).

Proceeding now to the Bell-EPR experiment and the puzzling non-local correlations that accompany it, it will be shown that branching along hyperplanes can elucidate matters. Suppose that a pair of photons, in an entangled quantum state and with anti-correlated polarizations, is emitted in opposite directions from a source S by positronium decay. If the angle of plane polarization of the photons is measured by two similarly-aligned polarization analyzers, the two outcomes will always disagree: if the angle of polarization of one photon is found to be "vertical", that of the other will always be "horizontal", and vice versa (Ballentine (1990), p. 448). Now let the analyzers be mis-aligned, say one vertical and the other at an angle φ to the vertical, as in figure 11:

Figure 11

The paths of the two photons in branching space-time are shown in figure 12, in which A is the measurement event of the left photon and B is that of the right photon:

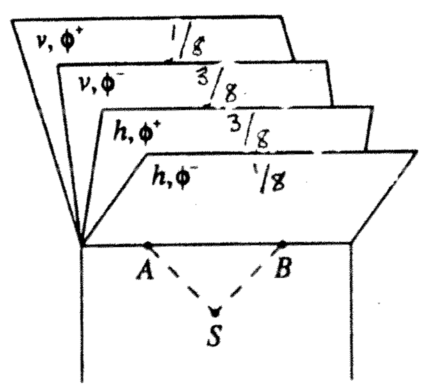


Figure 12

Where the possible measurement outcomes are *v* (vertical) and *h* (horizontal) on the left, and φ+ and φ- on the right, the probabilities of the joint outcomes (*v*, φ+), (*v*, φ-), (*h*, φ+) and (*h*, φ-) on left and right are respectively $1/2\sin^2\varphi$, $1/2\cos^2\varphi$, $1/2\cos^2\varphi$ and $1/2\sin^2\varphi$. In figure 12, the figures shown on the four branches represent the *relative proportions* of sets of branches containing the different joint outcomes. When $\varphi = 30^\circ$, these values equal 1/8, 3/8, 3/8, and 1/8, and consequently the *probabilities* that the actual branch will be selected randomly from one or another of these sets are also 1/8, 3/8, 3/8 and 1/8.

A particular significance attaches to these four joint probability values: they show that the measurement outcomes on left and right are not probabilistically independent. A necessary and sufficient condition for the outcome v on the left to be independent of the outcome φ+ on the right is that the joint probability p(*v*, φ+) should be equal to the product of the two individual probabilities p(*v*) and p(φ+). (If the probability of Smith being left-handed is 1/10, and the probability of Jones being left-handed is also 1/10, then the probability of them both being left-handed, barring some mysterious linkage, should be 1/10 x 1/10 = 1/100.) In figure 12, p(*v*) = p(*v*, φ+) + p(*v*, φ-) = 1/8 + 3/8 =1/2, and p(φ+) = p(*v*, φ+) + p(*h*, φ+) = 1/8 + 3/8 = 1/2. The product of p(*v*) and p(φ+) is 1/2 x 1/2 = 1/4, but the joint probability p(*v*, φ+) equals 1/8, not 1/4, showing that the probability of the left photon being measured *v* is not independent of the right photon being measured φ+. What conceivable mechanism links the two outcomes that could account for the statistical dependence of one upon the other?

The problem of providing a physical linkage that explains the distant correlations of measurement outcomes in the EPR experiment seems impossible, given that the two photons emitted from S are retreating from each other at the speed of light. If the left and right analyzers are both aligned HV, horizontal-vertical, and if the left photon is measured *v*, how does the right photon "know" that it must pass *h*? If the left photon is measured *v*, and if the right analyzer is inclined at 30° to the vertical, how does the right photon know that its probability of passing 30+ should be 1/8? If the left photon had been measured *h* instead of *v*, its twin's probability of passing 30+ would have been 3/8 rather than 1/8. How does the information concerning the left outcome reach it in time?

A possible explanation, namely that the photons might be accompanied by instruction sets, or "local hidden variables", telling them how to be measured if they encounter measuring devices set at different angles, is ruled out by the experimental violation of Bell's inequality in any sufficiently long run of trials, when the left polarizer can be set at either HV or 30°, and the right polarizer at either 30° or 60° (see McCall (1994), pp. 98-99). In addition, faster-than-light signaling is a non-starter in modern physics. Failing local hidden variables and faster-than-light communication, how can the observed distant correlations between the left and right measurement outcomes be accounted for?

There remains one possibility: that the distant correlations are the result of branch-attrition in branching spacetime, where the branches divide along spacelike hyperplanes. In this process a single "actual" future is selected instantaneously along a 3-dimensional dividing surface that extends across the entire universe. If both the left and the right EPR measuring events are located on that hyperplane, and if the two polarization analyzers are both aligned HV, then in the branching structure every branch above the hyperplane contains a *v*-outcome on the left if and only if it contains an *h*-outcome on the right. In this way, no matter how far apart the analyzers are positioned, the two outcomes will always disagree.

If on the other hand the two analyzers are mis-aligned, the left one being HV and the right inclined at an angle φ to the vertical, and if (an additional complication) the exact setting of the right one is not made until the photons have already left the source, the branching structure that explains the observed results is that of figure 13:

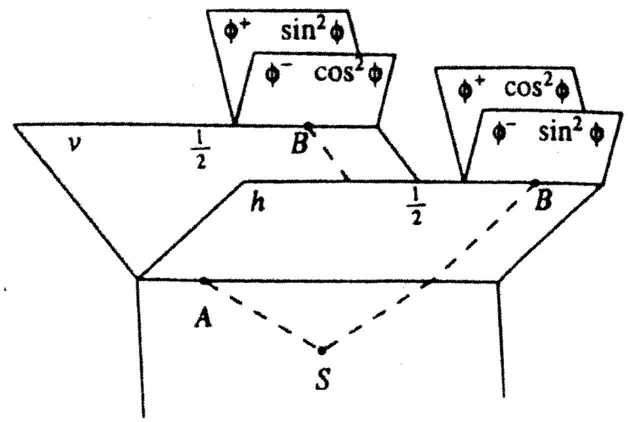


Figure 13

Since the photons are moving in opposite directions at the speed of light, the two measurement events are spacelike separated: neither lies within the light-cone of the other. Therefore it is always possible to find a frame of reference *f* in which the left measurement at point A occurs before the right measurement at B, as in figure 13. In frame *f*, since at A no measurement of the right photon has yet taken place, the branches above the constant-time hyperplane that passes through A are equally divided into *v*-branches and *h*-branches. Meanwhile the track of the right photon continues through all these branches until point B, where the right measurement occurs and the structure again divides, this time into φ+ and φ- branches. How does the right photon learn what its probability is of being measured φ+ or φ-?

Well, it has no choice. Since in frame *f* the left measurement has already taken place, because of branch-attrition the right photon finds itself either on an actual *v*-branch (if that was the outcome of the left measurement), or on an actual *h*-branch. If the former, its probability of passing φ+ at B is $\sin^2\varphi$, and if the latter, its probability is $\cos^2\varphi$. These probabilities are built into the model as seen through the perspective of frame *f*. If on the other hand we were to ask how the left photon knew what *its* probability of passing either *v* or *h* was, given that the right measurement had already taken place, the answer would be found in the perspectival branching structure based on a frame in which B occurs before A (figure 14):

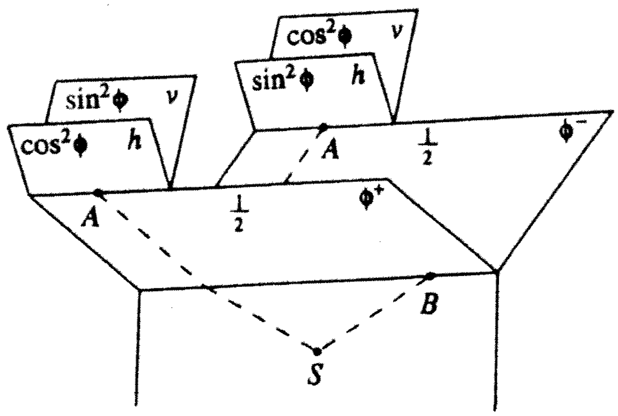


Figure 14

In either case, branch-attrition along spacelike hyperplanes accounts for the apparent "action at a distance" exhibited by measurements performed on two-particle entangled systems.

5. Branching spacetime and the Hausdorff property.

A topological space is called a Hausdorff space if for each pair x, y of distinct points of the space, there exist disjoint neighborhoods Nx and Ny of x and y. If two spacetime histories H_1 and H_2 branch, and if their mode of branching is what is known as "lower cut", then their union is a Hausdorff space. But if the mode of branching is "upper cut", their union is not Hausdorff. The names lower and upper cut were originally used by Belnap, and the difference between them is as follows (McCall (1994), pp. 289 ff.). In a Y-shaped space, the division between the two 1-dimensional arms of the "Y" is *lower cut* if the least upper bound of the trunk of the "Y" coincides with the greatest lower bound of each of the branches. But if the trunk has no least upper bound, i.e. is an open set of points, while each of the branches possesses its own distinct greatest lower bound, then the branching is *upper cut*. See figure 15, where (iii) is from Visser (1996), p. 252, and McCabe (2005), p. 668:

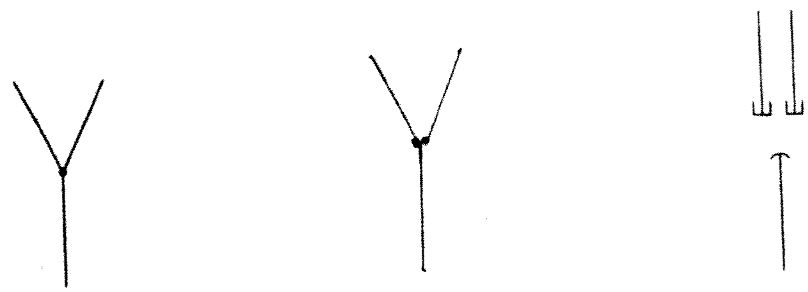


Figure 15

As figure 15 shows, lower cut branching is Hausdorff and upper cut is non-Hausdorff. Around any two distinct points *x* and *y* belonging to (i), there exist disjoint open sets Nx and Ny that do not overlap. This is not true of the two points *x* and *y* in (ii) and (iii) that are the greatest lower bounds of the left branch and the right branch respectively. Any neighborhood of *x* includes points belonging to the trunk, and the same is true of any neighborhood of *y*. So any such pair of neighborhoods overlap, and the two points *x* and *y* are not Hausdorff separated.

As far as real, physical spacetime is concerned, e.g. as dealt with in general relativity, how seriously should a failure of the Hausdorff property be taken? Very seriously, in the opinion of many experts. We find in Earman (2008), pp. 199-200, the following:

"The assumption of Hausdorffness is explicitly invoked only sporadically in textbooks on general relativity. But it is implicitly assumed in so many standard results in GTR that dropping it would require a major rewriting of textbooks. Here are two examples of widely used results that depend on Hausdorffness. (i) A compact set of a topological space is closed – if the space is Hausdorff. (ii) If a sequence of points of a topological space converges the limit point is unique – if the space is Hausdorff. The situation is best summed up by a dictum of Robert Wald ... 'Asking what relativistic physics would be like without Hausdorffness is like asking what the earth would be like without its atmosphere'. "

Given the importance of the Hausdorff property, it would seem either that the idea of spacetime branching should be abandoned entirely, or, failing that, that only lower-cut branching should be recognized. Earman would favour the first alternative. But in connection with the second, an equally important consideration is that lower-cut branching spaces are not locally Euclidean. A space is "locally Euclidean" if it is homeomorphic to a Euclidean space for all sufficiently small neighborhoods around any point. Around a lower-cut branch point, however, a space cannot be Euclidean for any neighborhood no matter how small (see figure 15). In a Y-shape, no neighborhood around the branch point can be part of a 1-dimensional Euclidean line, and a lower-cut branching space is not locally Euclidean. McCabe lists what must be given up if one works with a branching non-locally-Euclidean space:

"There is no tangent vector space at any of the branch points; one cannot take the sum of the vectors which are tangent to different branches emanating from the same point. There is therefore no tensor algebra at these points either. The Einstein field equations, the energy conditions of general relativity, and the expression for the local conservation of energy-momentum, cannot hold at the branching points because these expressions and equations are tensorial." (McCabe (2005), p. 670)

Supporters of branching spacetime are confronted with a dilemma. If the branching is upper-cut, the Hausdorff property is violated, whereas if it is lower-cut, the space is not locally Euclidean. Which to choose? I would suggest choosing upper-cut branching, for the reason that every separate complete history in upper-cut space is a differentiable, locally Euclidean, Hausdorff manifold. Physics can be done on such manifolds. By contrast, individual histories in lower-cut branching spacetime are not locally Euclidean at branch points, and at these points the Einstein field equations do not hold. In opting for upper-cut spaces that are everywhere locally Euclidean, Hausdorffness is sacrificed only at the level of the entire branching space. Each individual history, to which the Einstein equations apply, is Hausdorff.

6. Summing up: the pros and cons of invoking branching.

The time has come to weigh the advantages of spacetime branching against the disadvantages, and to reach a conclusion. It has been argued that a dynamic branching structure, with branch attrition, can provide an ontological basis for laws of nature; i.e. what makes true law-statements true. Such a structure exhibits a graphic difference between past and future, and provides an objective model of time flow. It allows for the possibility that natural laws may evolve through time, with new laws coming into being in the future. In addition, the particular way in which future histories branch in decenary trees creates numerical values for relative proportionalities of sets of branches, and provides every future event, and every probabilistic law, with an exact, non-subjective probability value. To the author's knowledge, nowhere else can we find an example of a precise numerical value being encoded ontologically, in the physical world. The theoretical values of the probabilities of outcomes of quantum experiments may be calculated using vectors in Hilbert space. But what is it about the real world that makes experimental outcomes conform to their theoretical Hilbert-space probabilities? Where is the real – as opposed to the theoretical – probability value to be found? I suggest that it can be found in real proportionalities of real sets of future spacetime branches.

Finally, branching along hyperplanes can explain the long-distance correlations between outcomes of measurements performed on pairs of particles in entangled quantum states. Branch attrition along such hyperplanes furnishes what amounts to instantaneous communication of information across arbitrarily large distances, without anything "travelling" at superluminal speeds. If spacetime were not to branch, none of these things would be possible. The conclusion of the paper is strongly to suggest the incorporation of branching into the vocabulary of theoretical physics.

Footnotes

[1] Lehoux raises the question of when it has become common to describe regularities in the natural world as operating according to strict mathematical laws, and says that this practice dates back only to the 16th and 17th centuries. The matter is discussed in Giere, *Science without Laws* (1999). A mathematical law or equation would be one variety of what Armstrong calls a "law-statement".

[2] Human beings, who have 10 fingers and use decimal number systems, find decenary trees easy to grasp. Binary trees, or ternary trees, would do as well, but are harder to work with.

[3] On top of every one of the branches in a decenary tree, including the "open" branch, there sits at t + delta-t the base of another decenary tree. However, the base branch point of that next tree is not "open", but (in the case of the Stern-Gerlach experiment) is either determinately spin-up or determinately spin-down. Although each branch of the lower tree has no "last" point, every tree at the next level has a "first" point. The universe is, in effect, a huge stack of decenary trees.

References

Armstrong, D. (1983), *What is a Law of Nature?*

Ballentine, L. (1990), *Quantum Mechanics.*

Belnap, N. (1992), "Branching space-time", *Synthese* 92, pp. 385-434.

Belnap, N., Perloff, M. and Ming Xu (2001), *Facing the Future.*

Borde, A. et al. (1999), "Causal continuity in degenerate spacetimes", *Classical and Quantum Gravity* 16, pp. 3457-3481.

Earman, J. (1986), *A Primer on Determinism.*

------------ (2008), "Pruning some branches from 'Branching Spacetimes' ", in D. Dieks (ed.) *The Ontology of Spacetime* II, pp. 187-205.

Giere, R.N. (1999), *Science without Laws.*

Lehoux, D. (2006), "Laws of nature and natural laws", *Studies in History and Philosophy of Science* 37, pp. 527-49.

Lewis, D. (1986), *Philosophical Papers,* vol. II

McCabe, G. (2005), "The topology of branching universes", *Foundations of Physics Letters* 18, pp. pp. 665-76.

McCall, S. (1994), *A Model of the Universe*.

------------ (2000), "QM and STR: The combining of quantum mechanics and relativity theory", *Philosophy of Science* 67 (Proceedings), pp. S535-S548.

------------ (2006), "Philosophical consequences of the twins paradox", in D. Dieks, (ed.) *The Ontology of Spacetime*, pp. 191-204.

------------ (2009), "Objective quantum probabilities", in Greenberger, Hentschel and Weinert (eds) *Compendium of Quantum Physics*, pp. 420-425.

McCall, S. and Lowe, E.J. (2003), "3D/4D equivalence, the twins paradox and absolute time", *Analysis* 63, pp. 114-123.

Penrose, R. (1979), "Singularities and time-asymmetry", in S.W. Hawking and W. Israel (eds) *General Relativity: An Einstein Centenary Survey*, pp. 581-638.

Visser, M. (1995), *Lorentzian Wormholes: From Einstein to Hawking*.