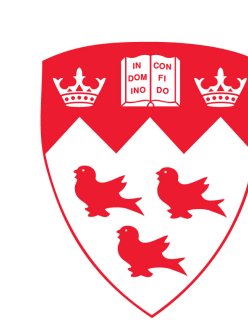


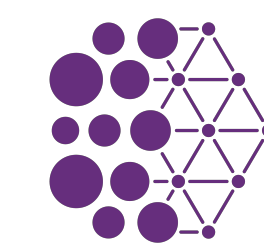
Simultaneous detection and mapping in the olfactory sensing



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Chen Jiang^{1,2*}, **Matthew Y. He^{1*}**, Cengiz Pehlevan^{4,5,6}, Venkatesh N. Murthy^{4,5,7}, Jacob A. Zavatone-Veth^{4,8†}, Paul Masset^{1,3†}

1. Department of Psychology, McGill University, Canada
2. Ph.D. Program in Quantitative Life Science, McGill University, Canada
3. Mila - Quebec Artificial Intelligence Institute, Canada
4. Center for Brain Science, Harvard University, USA

5. Kempner Institute for the Study of Natural & Artificial Intelligence, Harvard University, USA
6. John A. Paulson School of Engineering and Applied Sciences, Harvard University, USA
7. Department of Molecular and Cellular Biology, Harvard University, USA
8. Society of Fellows, Harvard University, USA

Summary:

- Explain anatomical structure: mitral cells and tufted cells play different roles in odor inference.
- Provide a framework for solving constraint optimization problems through recurrent neural networks (RNNs)
- Provide a framework for mapping computations performed by RNN onto biologically plausible circuits.

Keywords:

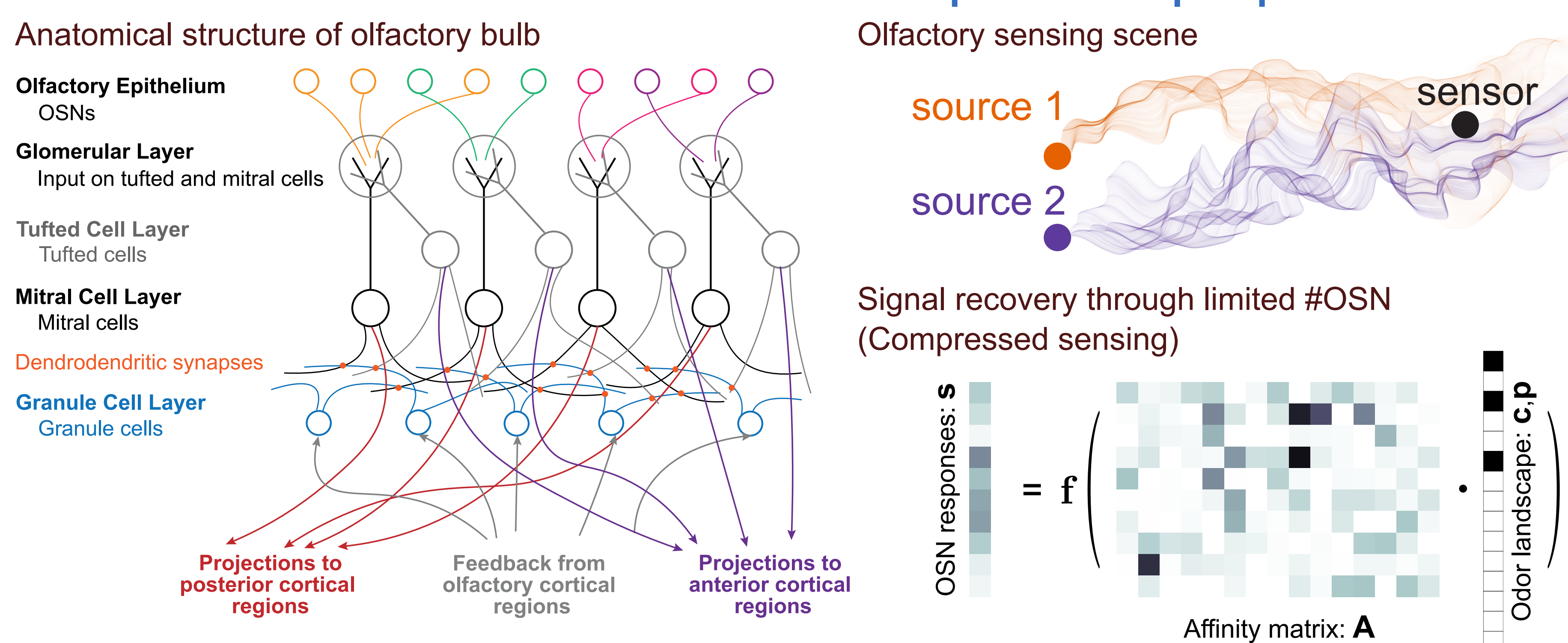
• **Theoretical/Computational Neuroscience**
• **Applied Mathematics, Sampling**
Constraint Optimization, # Diffusion Processes
Mirror Descent, # Langevin Dynamics
Bayes Inference, # Recurrent Neural Network (RNN)

Preprint:

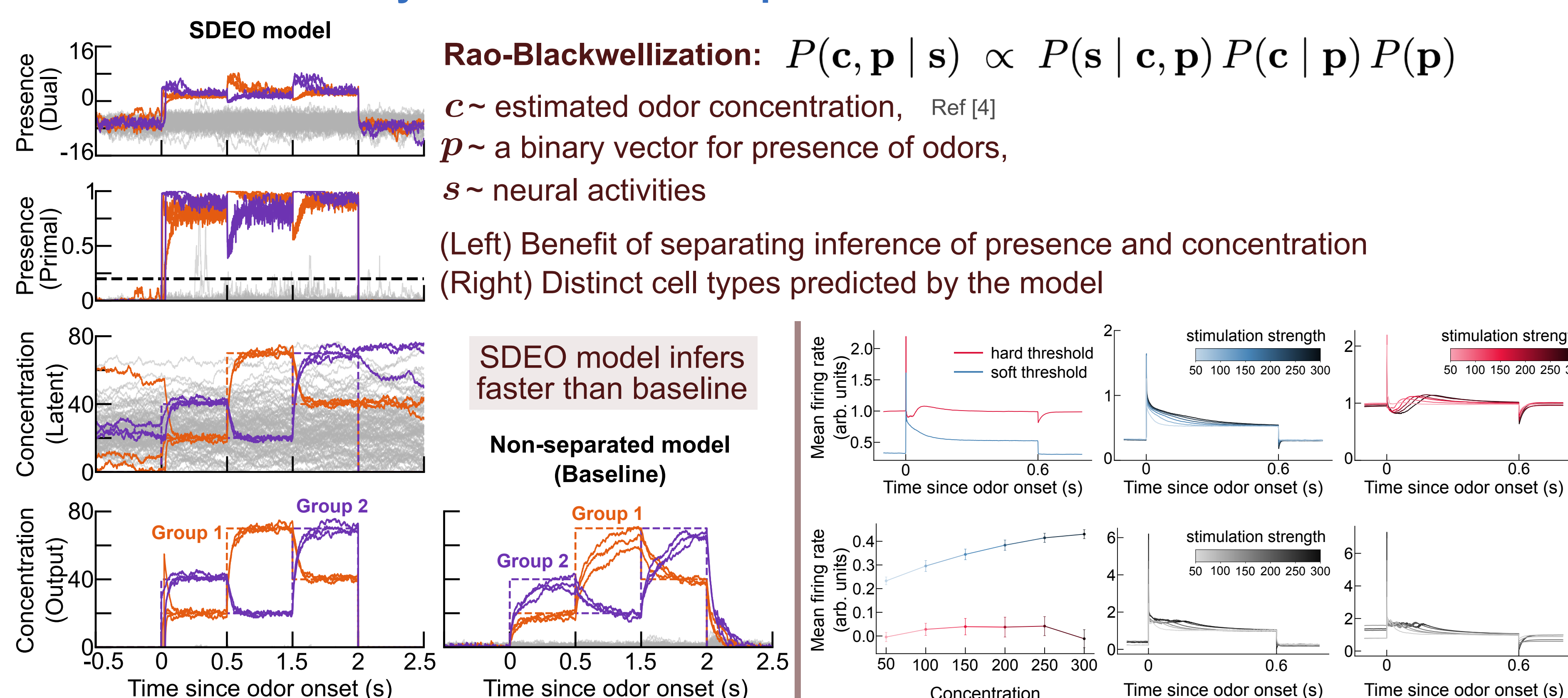


Olfactory Sensing

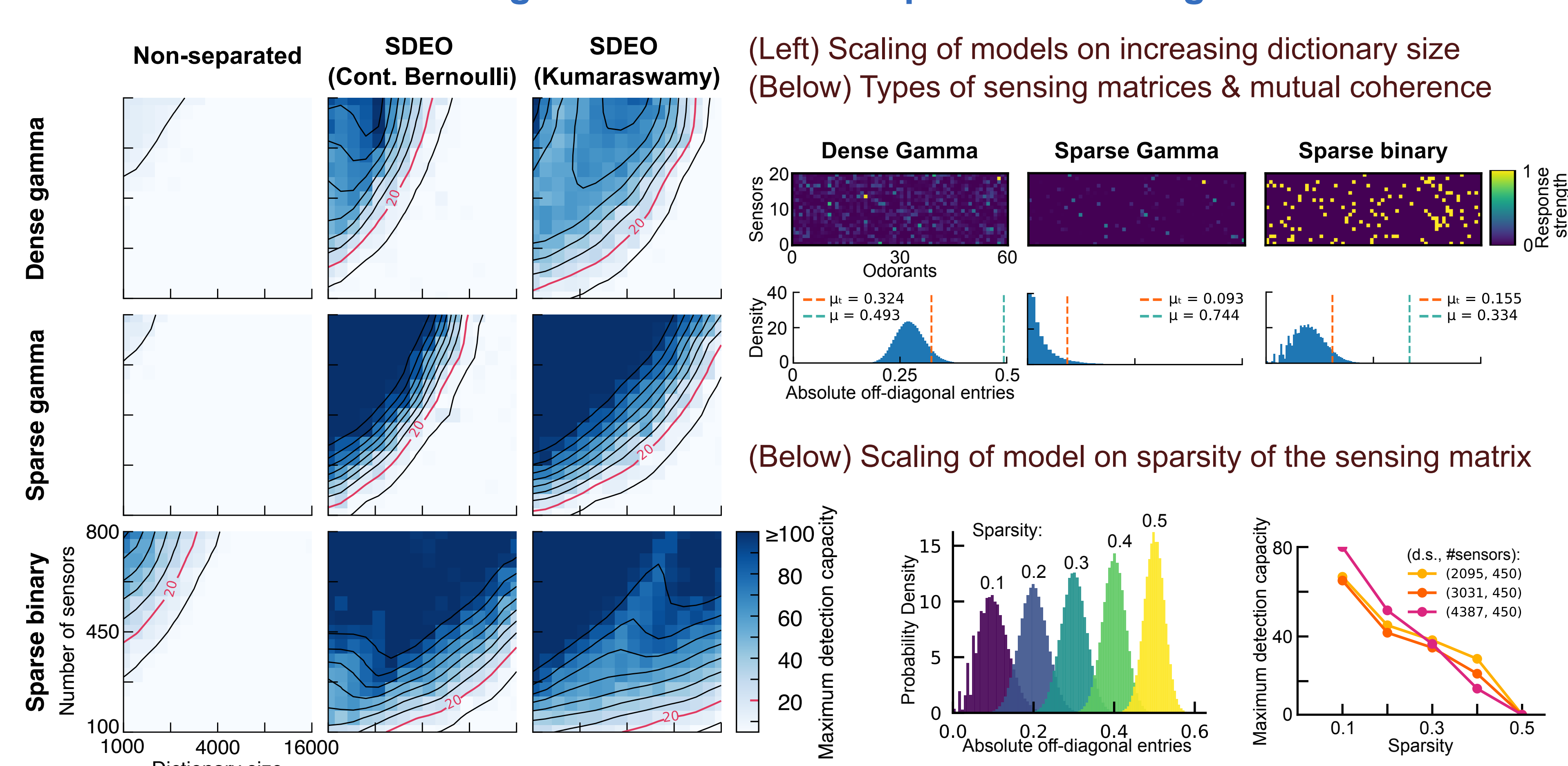
Anatomical structure & Constraint optimization perspectives



Bayesian inference of presence and concentration



Scaling of the model & Compressed sensing



Langevin Dynamics

Standard & Mirrored Langevin Dynamics

Exponential family probability distribution
 $P(\theta) \propto e^{-V(\theta)}$ Energy function
Langevin Dynamics
 $d\theta_t = -\nabla V(\theta_t) dt + \sqrt{2} dB_t$
Empirical distribution converges to $P(\theta)$ as $t \rightarrow \infty$ and $dt \rightarrow 0$
Mirrored Langevin Dynamics: with ϕ convex, transform constrained variable $\mathbf{x}$ into unconstrained dual-space variable $\mathbf{y}$ via the mirror map:
Ref [7]

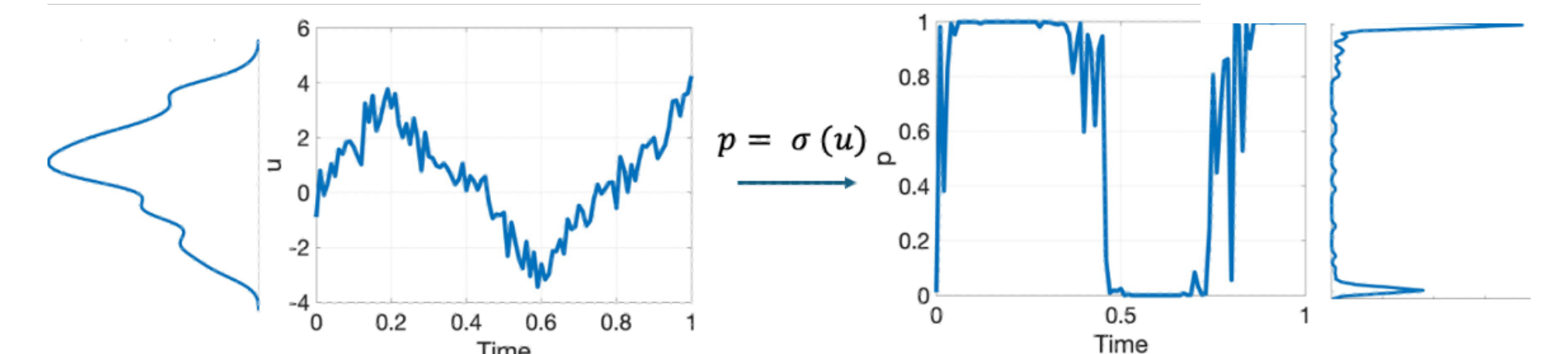
$\mathbf{y} = \nabla_{\mathbf{x}} \phi(\mathbf{x})$
With Fenchel conjugate $\phi^*(\mathbf{y}) = \sup_{\mathbf{x}} \{\langle \mathbf{y}, \mathbf{x} \rangle - \phi(\mathbf{x})\}$
 $\mathbf{x} = \nabla_{\mathbf{y}} \phi^*(\mathbf{y})$
Langevin dynamics in the dual space:
 $d\mathbf{y}(t) = \nabla_{\mathbf{y}} \log Q(\mathbf{y}) dt + \sqrt{2} d\mathbf{B}_u(t)$
For $\mathbf{x}(\mathbf{y})$ to have the desired distribution, we need the density:
 $Q(\mathbf{y}) = \det(\nabla_{\mathbf{x}}^2 \phi(\mathbf{x}))^{-1} P(\mathbf{x}(\mathbf{y}))$
As we have: $\nabla_{\mathbf{y}} \mathbf{x}(\mathbf{y}) = \nabla_{\mathbf{y}}^2 \phi^*(\mathbf{y}) = [\nabla_{\mathbf{x}}^2 \phi(\mathbf{x})]^{-1} \equiv \mathbf{H}^{-1}$, by chain rule we have:
 $\nabla_{\mathbf{y}} \log Q(\mathbf{y}) = \mathbf{H}^{-1} [\nabla_{\mathbf{x}} \log P(\mathbf{x}) - \nabla_{\mathbf{x}} \log \det \mathbf{H}]$ Ref [8,9,10]

Langevin Dynamics for concentration

Defining the energy function $\nabla V(\mathbf{p}, \mathbf{c}) = -\log P(\mathbf{p}, \mathbf{c})$
The gradient in our model is: $-\nabla_{\mathbf{c}} V = \mathbf{p} \odot \mathbf{A}^T (\mathbf{h} - \mathbf{1}) + (\alpha - 1) \mathbf{1} \odot \mathbf{c} - \beta \mathbf{1}$
where we denote neuron population: $\mathbf{h} = \mathbf{s} \odot (\mathbf{r}_0 + \mathbf{A}(\mathbf{c} \odot \mathbf{p}))$
Thus, we have the dynamics: $d\mathbf{c}(t) = -\nabla_{\mathbf{c}} V dt + \sqrt{2} d\mathbf{B}_{\mathbf{c}}(t)$

Mirrored Langevin Dynamics for presence

Express $\mathbf{p} \in [0, 1]^{n_{\text{odor}}}$ in terms of an unconstrained variable $\mathbf{u} \in \mathbb{R}^{n_{\text{odor}}}$
a continuous relaxation $\mathbf{p} = \sigma_{\gamma}(\mathbf{u})$ $\sigma_{\gamma}(\mathbf{x}) = \frac{1}{1 + \exp(-\gamma \mathbf{x})}$



The gradient is: $-\nabla_{\mathbf{p}} V = \mathbf{c} \odot [\mathbf{A}^T (\mathbf{h} - \mathbf{1})] + \log \left(\frac{\alpha}{1 - \alpha} \right) \mathbf{1}$
Construct a mirror map $h_{\gamma} : [0, 1]^d \rightarrow \mathbb{R}^d$ with Fenchel conjugate satisfying: $\frac{\partial h_{\gamma}^*}{\partial u_i} = \sigma_{\gamma}(u_i)$

After computation we obtain: $h_{\gamma}(\mathbf{p}) = \frac{1}{\gamma} \sum_j [p_j \log p_j + (1 - p_j) \log(1 - p_j)]$

Thus the mirrored Langevin dynamics is:
 $d\mathbf{u}(t) = -\gamma [\mathbf{p} \odot (\mathbf{1} - \mathbf{p}) \odot \nabla_{\mathbf{p}} V + 2\mathbf{p} - \mathbf{1}] dt + \sqrt{2} d\mathbf{B}_u(t)$

Circuit Implementation

Introducing hard threshold presence: $\tilde{\mathbf{p}} = \arg \max_{k \in \{0,1\}} [k\mathbf{p} + (1 - k)\mathbf{p}_{th}]$
Replace the $\mathbf{p}$ in concentration dynamics with $\tilde{\mathbf{p}}$ and we have:
 $d\mathbf{c}(t) = [\tilde{\mathbf{p}} \odot [\mathbf{A}^T (\tilde{\mathbf{h}} - \mathbf{1}) + (\alpha - 1) \mathbf{1} \odot \mathbf{c} - \beta \mathbf{1}]] dt + \sqrt{2} d\mathbf{B}_{\mathbf{c}}(t)$ with neurons: $\tilde{\mathbf{h}} = \mathbf{s} \odot (\mathbf{r}_0 + \mathbf{A}(\mathbf{c} \odot \tilde{\mathbf{p}}))$
We can derive the update rules for the two distinct neuron population and consider the model as an RNN driven by these neurons (with additional types not shown here):
 $\tau_h d\mathbf{h}(t) = \mathbf{s} - \mathbf{h} \odot (\mathbf{r}_0 + \mathbf{A}(\mathbf{c} \odot \mathbf{p}))$, $\tau_{\tilde{h}} d\tilde{\mathbf{h}}(t) = \mathbf{s} - \tilde{\mathbf{h}} \odot (\mathbf{r}_0 + \mathbf{A}(\mathbf{c} \odot \tilde{\mathbf{p}}))$

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Acknowledgements

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