# MECHANICAL LABORATORY 

## MECH 362 A,B

# Department of Mechanical Engineering McGill University <br> Montreal, Quebec 

FALL 2007

## Contents

1 Introduction to Mechanical Laboratory ..... 7
1.1 Introduction ..... 7
1.2 Evaluation ..... 8
1.3 Laboratory Preparation ..... 8
1.4 Laboratory Performance ..... 10
1.5 Experimental Results ..... 10
1.6 Discussion of Results and Conclusions ..... 10
1.7 Collecting and Presenting Experimental Data ..... 12
1.8 References ..... 14
I Fluid Mechanics ..... 15
2 F1-Airflow Measurement ..... 17
2.1 Objectives ..... 17
2.2 Apparatus ..... 17
2.3 Apparatus Setup ..... 18
2.4 Instrumentation ..... 18
2.4.1 Manual Data Acquisition ..... 18
2.4.2 Electronic Data Acquisition ..... 18
2.5 Procedure ..... 19
2.5.1 Preliminary Check ..... 19
2.5.2 Preliminary Measurements ..... 19
2.5.3 Experimental Procedure ..... 19
2.6 Results ..... 20
2.7 Theory ..... 21
2.7.1 Actual Mass Flow Rate ..... 21
2.7.2 Theoretical Mass Flow Rate from Obstruction Meters ..... 22
2.7.3 Head Loss Coefficient ..... 24
3. F2- Laminar and Turbulent Flow Through Pipes ..... 25
3.1 Objectives ..... 25
3.2 Apparatus ..... 25
3.3 Apparatus Setup ..... 26
3.4 Instrumentation ..... 26
3.5 Procedure ..... 26
3.5.1 Preliminary Measurements and Checks ..... 26
3.5.2 Experimental Procedure ..... 26
3.6 Theory ..... 27
3.6.1 Pressure Loss Coefficient
(or Coefficient of Friction) ..... 28
3.6.2 Theoretical Results from Literature ..... 29
4 F3 - Wind Tunnel Lift and Drag ..... 30
4.1 Objectives ..... 30
4.2 Apparatus ..... 30
4.3 Appartatus Setup ..... 31
4.4 Instrumentation ..... 31
4.5 Procedure ..... 31
4.5.1 Preliminary Measurements ..... 31
4.5.2 Experimental Procedure ..... 31
4.6 Theory ..... 32
II Heat Transfer ..... 36
5 H1 - Double-Pipe Heat Exchanger ..... 38
5.1 Objectives ..... 38
5.2 Apparatus ..... 38
5.3 Instrumentation ..... 40
5.4 Procedure ..... 40
5.4.1 Preliminary Measurements ..... 40
5.4.2 Operation for Hot \& Cold Water Flows ..... 40
5.4.3 Operation for Steam 7 Cold Water Flows ..... 41
5.5 Results ..... 42
5.6 Theory ..... 42
6. H2 - Transient Convective Heat Transfer ..... 46
6.1 Objectives ..... 46
6.2 Apparatus ..... 46
6.3 Apparatus Setup ..... 47
6.4 Instrumentation ..... 47
6.5 Procedure ..... 48
6.5.1 Preliminary Checks \& Measurements ..... 48
6.5.2 Experimental Procedure ..... 48
6.5.3 Instructions for Using the Computer "RUN" and "CORREL". ..... 49
6.6 Results ..... 50
6.7 Theory ..... 50
7. H3-Turbulent Internal Forced Convection ..... 54
7.1 Objectives ..... 54
7.2 Apparatus ..... 54
7.3 Instrumentation ..... 55
7.4 Procedure ..... 55
7.4.1 Preliminary Checks ..... 55
7.4.3 Preliminary Measurements ..... 56
7.4.3 Main Experiment ..... 57
7.5 Results ..... 57
7.6 Theory ..... 59
7.7 References ..... 59
III Thermodynamics ..... 61
8. T1 - Ruston-Hornsby Diesel Engine ..... 63
8.1 Objective ..... 63
8.2 Apparatus ..... 63
8.3 Instrumentation ..... 64
8.4 Procedure ..... 64
8.5 Results ..... 66
8.6 Theory ..... 67
8.6.1 Brake Power ..... 67
8.6.2 Indicated Power ..... 67
8.6.3 Air to Fuel Ratio (/F) ..... 68
8.6.4 Specific Fuel Consumption (SFC) ..... 69
8.6.5 Mechanical, Overall, and Ideal Efficiency ..... 69
8.6.6 Heat Balance ..... 70
9. T2 - Variable Compression Ratio Engine ..... 72
9.1 Objective ..... 72
9.2 Apparatus ..... 72
9.3 Instrumentation ..... 73
9.4 Procedure ..... 73
9.4.1 General ..... 73
9.4.2 Recorded Measurements ..... 74
9.5 Results ..... 75
9.6 Theory ..... 75
9.6.1 Indicated Power ..... 75
9.6.2 Air Standard Efficiency ..... 75
10. T3 - Small Gas Turbine Demonstration Unit ..... 77
10.1 Introduction ..... 77
10.2 System Description ..... 78
10.2.1 gas Generator ..... 79
10.2.2 Jet Pipe and Thrust Nozzle ..... 80
10.2.3 Fuel System ..... 80
10.2.4 Lubricating System ..... 80
10.2.5 Starting and Ignition System ..... 80
10.3 Physical Layout ..... 81
10.4 Theory ..... 82
10.4.1 The Open Gas Turbine Process ..... 82
10.5 Results ..... 86
10.6 Controls ..... 87
11 T4 - Air Conditioning Plant ..... 88
11.1 Objectives ..... 88
11.2 Apparatus ..... 88
11.3 Apparatus Setup ..... 89
11.4 Instrumentation ..... 90
11.5 Procedure ..... 90
11.5.1 Experimental Procedure ..... 90
11.5.2 Shutdown Procedur ..... 91
11.6 Results ..... 91
11.7 Theory ..... 93
11.7.1 Introduction ..... 93
11.7.2 The Psychrometric Chart ..... 93
IV Vibrations ..... 100
12 V1 - Rotating Unbalance ..... 102
12.1 Objectives ..... 102
12.2 Apparatus ..... 102
12.3 Apparatus Setup ..... 103
12.4 Instrumentation ..... 103
12.5 Procedure ..... 104
12.5.1 Preliminary Calculations ..... 104
12.5.2 Preliminary Measurements ..... 105
12.5.3 Experimental Procedure ..... 105
12.6 Results ..... 106
12.7 Theory ..... 107
12.7.1 Critical Damping ..... 109
12.7.2 Damping Ratio ..... 109
12.7.3 Oscillatory Motion ..... 109
12.7.4 Non-Oscillatory Motion ..... 109
12.7.5 Critically Damped Motion ..... 110
12.7.6 Logarithmic Decrement ..... 110
12.7.7 Theoretical Envelope ..... 110
12.7.8 Forced Vibration ..... 110
12.7.9 Rotating Unblance ..... 112
12.7.10 Vibration Absorber ..... 112
12.7.11 Support Motion ..... 113
13 V2 - Vibrations of Continuous Systems ..... 118
13.1 Objectives ..... 118
13.2 Apparatus ..... 118
13.3 Apparatus Setup ..... 119
13.4 Instrumentation ..... 119
13.5 Procedure ..... 119
13.5.1 Preliminary Checks ..... 119
13.5.2 Preliminary Measurements ..... 120
13.5.3 Experimental Procedure ..... 120
13.6 Results ..... 120
13.7 Theory ..... 121
14 V3 - Single Degree of Freedom Systems ..... 126
14.1 Objectives ..... 126
14.2 Apparatus ..... 126
14.3 Apparatus Steup ..... 126
14.4 Instrumentation ..... 127
14.5 Procedure ..... 127
14.5.1 Preliminary Checks ..... 127
14.5.2 Experimental Procedure ..... 128
14.6 Results ..... 130
14.7 Theory ..... 131

## Chapter 1

## Introduction to Mechanical Laboratory

### 1.1 Introduction

The purpose of Mech. Lab I is to expose you to experimental equipment, data collection, and reporting which would support the theoretical background obtained in the following courses:

- Thermodynamics I: 305-240A,B
- Fluid Mechanics I: 305-331A,B
- Heat Transfer: 305-346A,B
- Dyn. of Vibrations: 305-315B

Measurement Lab. 305-261B is a pre-requisite.
It is highly recommended that you sign up to perform two experiments in the appropriate area(s) the term following completion of one or more of the above courses. You should register for course 305-362A, B only for that term in which you will complete all eight experiments in the four areas. At the end of the final term you will write an exam covering the eight experiments that you performed.

The undergraduate laboratory has undergone considerable renovation in the past few years. Several new experiments have been obtained, data acquisition systems have been installed, and access to PCs in the laboratory are now available for preparation of spreadsheets, processing data, and writing reports. EXCEL and WORD for WINDOWS are the two sofware programs available for spreadsheets, graphs; and word processing. All of this has been done with the help of student funds to upgrade and modernize the laboratory.

### 1.2 Evaluation

In this spirit of change, it is proposed to alter the evaluation system. The preparation, performance of the experiment, collection of the data, and analysis of the results will be emphasized, and the writing of a formal report eliminated. A report will be handed in at the end of the 3 hour laboratory session. Every effort will be made to ensure that data collection for all experiments can easily be completed in $1 \frac{1}{2}$ to 2 hours, leaving $1 \frac{1}{2}$ to 1 hour for discussion of results and conclusions. The following is the point allocation used to determine the marks for the reports:

| Lab Preparation | 25 |
| :--- | :--- |
| Lab Performance | 25 |
| Experimental Results | 25 |
| Discussion of Results | 25 |
|  | 100 |

The following is the allocation of points to determine the final grade:

| Group Mark $100 \times 8$ experiments | 800 |
| :--- | :--- |
| Individual Lab. Exam | 200 |
|  | 1000 |

### 1.3 Laboratory Preparation

Advance preparation before running the experiment is most important to enable you to perform the experiment and complete your report within the
allotted 3 hour laboratory session. Before doing the experiment you should have prepared the following:

1. a clear statement of the objective, i.e. what you want to achieve from the experiment;
2. a schematic diagram of the apparatus showing the location of all the instrumentation;
3. list all instrumentation including accuracy and calibration curves if required (see experimental results);
4. data sheets for recording primary data;
5. data sheets for calculating data to be plotted;
6. appropriate graph paper required for plotting the data;
7. appropriate constants or graphs from the literature to compare with the experimental results.

The following are available to assist you in preparing for the laboratory experiments:

1. appropriate course notes;
2. laboratory notes on individual experiments as contained in this lab manual; these will contain sections on:

- objective
- apparatus
- apparatus setup
- instrumentation
- experimental procedure
- results
- theory
- references

3. previous laboratory student reports are NOT permitted during the performance of the labs.

### 1.4 Laboratory Performance

To a large extent, your performance in the laboratory will depend on how well prepared you are as a team. This will improve your efficiency and accuracy in collecting data, calculating errors and final results, and submitting the final report writing the three hour time limit.

### 1.5 Experimental Results

Neatness, accuracy, and completeness of results and curves will be considered.

### 1.6 Discussion of Results and Conclusions

Correct interpretation of your results, and comparison with results in the literature will be considered.

The teaching assistants will evaluate your performance in these categories according to the following evaluation form.

# Mechanical Laboratory 305-362 REPORT EVALUATION 



AVERAGE TOTAL: $/ 10$

### 1.7 Collecting and Presenting Experimental Data

The correct interpretation of data collected from measurement devices, and the proper presentation of the calculated results are important factors in experimental work. This section deals briefly with what is expected in all laboratory reports concerning the presentation of data and results, and points out some of the common errors made in the past by students.

First consider the raw data obtained from reading the scale on an instrument of measurement. How accurately can you read the scale on a common mercury-in-glass thermometer? How close is the reading on the scale to the true temperature of the fluid that is to be measured? The first question is a matter of judgment and ability to observe the scale. The second question depends on the accuracy of the measurement device or the closeness of the observation to the true value of the fluid temperature. The true value is never known so the accuracy or error, the difference between the true value and observed value, cannot be obtained directly. The error is dependant on the calibration of the scale, either by the manufacturer, or a calibration of the measurement device in the laboratory. A simple rule of thumb is to estimate the error as between 0.5 and 1.0 percent of the magnitude of the scale.

No measurement can be repeated exactly. The precision, or closeness of a grouping of a number of measurements, may be determined by taking a number of readings at one test condition. The precision of the observations may depend on a number of factors; i.e. a change in the observer, a change in ambient conditions affecting the instrumentation, a change in the substance being measured, etc. The precision of the data will affect the error of the measurement if it is greater than the previously estimated error. This commonly occurs in experiments where it is difficult to maintain steady state conditions.

Now that the raw data and estimated error have been obtained, it is used to calculate various parameters of interest to be plotted in graphical form. The absolute or percentage error of the calculated parameters must be determined. This may be done using the percentage error or the absolute error for the raw data. To illustrate how the calculated error can vary considerably
for the raw data error, consider the following example:

$$
\begin{aligned}
T_{H} & =100^{\circ} \mathrm{C}(\text { error }=1 \%) \\
T_{C} & =95^{\circ} \mathrm{C}(\text { error }=1 \%)
\end{aligned}
$$

now

$$
X=\frac{T_{H}-T_{C}}{T_{H}}
$$

and

$$
\begin{aligned}
& X_{\max }=\frac{101-94}{99}=0.0707 \\
& X_{\min }=\frac{99-96}{101}=0.0297
\end{aligned}
$$

so that

$$
X=0.05 \pm 0.02(\text { error }=40 \%)
$$

When plotting the results of an experiment it is good practice to indicate the error in the parameters on the graph. This may be effectively done with error bars drawn directly on the graph. Scales for the graphs should be carefully chosen so that all the data can be plotted including the absolute error bars. It now remains to draw a curve through the different data points. Great care should be exercised in doing this so that the results can be interpreted correctly. A common error is to join all the data points through the centre of the error bars irrespective of the shape of the curve. If the resulting curve shows points of inflection or discontinuities and there is no physical explanation for them, then it is most probable that another type of error has been made at one of the test conditions. Such error could be due to a temporary malfunction of the equipment or the instrumentation, an error in reading the scale, or an error in calculation. In almost all the experiments a best fit smooth continuous curve, or line, should be drawn through the test points.

### 1.8 References

1. Measurement Laboratory Course Notes
2. Bragg, G.M., Principles of Experimentation and Measurements
3. Hall, C.W., Errors in Experimentation

## Part I

## Fluid Mechanics

Fluid flow phenomena are so prevalent, that perhaps one is not aware of their all-encompassing presence. Being such, fluid mechanics is not only the providence of the mechanical engineer, but is truly a multi-disciplinary field attracting researchers in chemical engineering, materials science, petroleum engineering, civil engineering, environmental science, meteorology, geology, and astronomy. However, eventhough we are surrounded by fluid flow phenomena, to isolate and measure fluid flow properties is a difficult endeavor, even for the simplest cases.

To gain insight into some of the most basic tools available in experimental fluid mechanics, three labs will be performed. These labs serve as an elegant introduction to the myriad measurement techniques that have been developed over the course of the last 150 years. The complexity of fluid flow measurement should be quite evident after the completion of these labs.

## Chapter 2

## F1- Airflow Measurement

### 2.1 Objectives

1. To calibrate and asses a number of different obstruction type metering devices for measuring the volume flow rate of air in a pipe at low Mach numbers.
2. To compare the results of this experiment with those from the published literature.

### 2.2 Apparatus

Transparent acrylic tube:

$$
\begin{aligned}
& \text { length }=15.24 \mathrm{~m} \\
& \mathrm{ID}=\quad 7.62 \mathrm{~cm}
\end{aligned}
$$

Thin plate orifice:
Thick plate orifice:
Bell nozzle:
Venturi meter:

Dimensions are given on obstruction measurement devices

### 2.3 Apparatus Setup

A transparent acrylic tube is horizontally supported along its length. Obstruction devices are installed in series along the length of the tube, each being connected to a main manometer board from which the static pressure readings can be observed. A constant speed centrifugal compressor forces air through the tube. The flow rate is controlled by a gate valve immediately downstream of the compressor. The air exhausts to atmosphere at the other end of the tube.

### 2.4 Instrumentation

### 2.4.1 Manual Data Acquisition

- Glass Tube Manometer Board

This instrument is used for the static pressure readings taken from various locations on each obstruction device. The locations are indicated by numbers on the manometer board and each device. The corresponding number on the device and the manometer board will give the pressure drop for that location. The liquid used in the board is coloured water.

- Pitot Traverse

The pitot traverse is located at the tube exit. The velocity profiles obtained are used to get an accurate measure of the air mass flow rate.

- Inclined Manometer
- Mercury Thermometer
- Mercury Barometer
- Sling Psychrometer-for the measurement of wet and dry bulb temperatures.


### 2.4.2 Electronic Data Acquisition

- Pressure Transducers
- Scani Valve


### 2.5 Procedure

### 2.5.1 Preliminary Check

1. The gate valve should be closed before the compressor is turned on.

### 2.5.2 Preliminary Measurements

1. Check the barometric pressure and the wet and dry bulb temperatures, and obtain the humidity ratio from a psychrometric chart.
2. Turn on the compressor.
3. Attain the maximum fiow rate by fully opening the gate valve.
4. Estimate the flow velocity using the appropriate selection in the data acquisition program.
5. By closing the gate valve, obtain a reasonable minimum velocity and quantify it by the same procedure as in step 4.
6. Calculate two other equally spaced flow rates in between to end up with four velocities. Note: the velocity is proportional to (dynamic pressure) ${ }^{1 / 2}$.

### 2.5.3 Experimental Procedure

1. Set the flow rate to the minimum value. Note: The reading for steps 2,3 , and 4 that follow are obtained by the data acquisition program.
2. Measure the average total and static pressures from the pitot tubes located at $3 / 4$ of the tube radius.
3. Record the appropriate pressure readings from the manometer boards.
4. Determine the total pressure profile at the end of the tube. Take total pressure readings at the following radii positions which approximately represent the mean velocity for five concentric equal areas:

$$
\text { - } R_{1}=1.63 \mathrm{~mm}
$$

- $R_{2}=5.12 \mathrm{~mm}$
- $R_{3}=9.03 \mathrm{~mm}$
- $R_{4}=13.55 \mathrm{~mm}$
- $R_{5}=19.14 \mathrm{~mm}$
- $R_{6}=27.73 \mathrm{~mm}$

At each radius, three readings should be taken (one in each Pitot traverse orientations of $0^{\circ}, 90^{\circ}$, and $180^{\circ}$ ). As well, take a total pressure reading at the centerline of the tube.
5. Repeat steps 1 to 4 for the next 3 flow rates.

### 2.6 Results

1. Plot the velocity profiles from the pitot traverse results.
2. Determine the actual mass flow rate.
3. Determine the theoretical mass flow rates from the static pressure plots for each obstructive flow device.
4. Determine the discharge and pressure loss coefficients for each device.
5. Plot coefficients as a function of Reynolds number.
6. Determine the percentage error for the $3 / 4$ radius pitot tube, and check the results with a theoretical analysis assuming a velocity profile $U / U_{c}=(Y / R)^{1 / 7}$.
7. Explain physically the variation of discharge coefficients with Reynolds number, and from one obstruction device to another.
8. Discuss the relative merits and drawbacks of each device.

### 2.7 Theory

The measurement of mass flow rate is an important aspect in many engineering systems. Four obstruction devices are to be examined in this experiment. These include a

1. venturi meter
2. bell nozzle
3. thin plate
4. thick plate

By comparing the "actual" mass flow rate to the "theoretical" mass flow rate, the discharge coefficient can be determined for each device at a given flow rate.

### 2.7.1 Actual Mass Flow Rate

The most accurate determination of the mass flow rate of air through the tube is obtained from the pitot traverse average dynamic head results. A schematic of the traverse is shown in Figure 2.1 where:


Fig. 2.1

- $p_{\mathrm{S}}=$ static pressure $\left(\mathrm{kN} / \mathrm{m}^{2}\right)=(\mathrm{kPa})$
- $p_{\mathrm{T}}=$ total pressure
- $\delta p=p_{\mathrm{T}}-p_{\mathrm{S}}=$ dynamic pressure
- $p_{\mathrm{a}}=$ atmospheric pressure
- $\delta h=$ manometer reading
- $\overline{\delta \hbar}=$ average manometer readings for equal areas
- $\theta=$ slope of the manometer
- $\rho_{f}=$ density of manometer fluid (water $=1000 \mathrm{~kg} / \mathrm{m}^{3}$ )
- $\mathrm{T}_{\mathrm{S}}=$ static temperature of air
- $\rho_{\mathrm{a}}=\frac{p_{\mathrm{s}}}{\mathrm{RT} \mathrm{T}_{\mathrm{s}}}$ (assume ideal gas: $\mathrm{R}=287 \mathrm{~J} / \mathrm{kg}-\mathrm{K}$ )
- ID $=$ inside diameter of tube

$$
\begin{equation*}
\delta p=\rho_{f}(\delta h \sin \theta) \frac{g}{g_{c}} \tag{2.1}
\end{equation*}
$$

where

$$
\begin{aligned}
& g=9.8 \mathrm{~m} / \mathrm{s}^{2} \\
& g_{c}=1 \mathrm{~kg} \mathrm{~m} / \mathrm{N}-\mathrm{s}^{2}
\end{aligned}
$$

Then from Bernoull's equation for steady, inviscid, incompressible flow

$$
\begin{gather*}
p_{\mathrm{T}}-p_{\mathrm{S}}=\frac{1}{2} \rho_{\mathrm{a}} u^{2}  \tag{2.2}\\
\bar{u}=\sqrt{2 \delta p g_{\mathrm{c}} / \rho_{\mathrm{a}}}  \tag{2.3}\\
\dot{m}_{\mathrm{act}}=\rho_{\mathrm{a}} \bar{u} \pi \mathrm{D}^{2} / 4 \tag{2.4}
\end{gather*}
$$

### 2.7.2 Theoretical Mass Flow Rate from Obstruction Meters

All of the obstruction meters are based on the measurement of the static pressure difference between the position upstream of the restriction (at position 1 in Figure 2.2) and at the restriction (position 2). In Figure 2.2:


Fig. 2.2

- $\mathrm{A}_{1}=$ cross sectional area of tube $-\pi \mathrm{D}_{1}^{2} / 4$
- $\mathrm{A}_{2}=$ cross sectional area of restriction $-\pi \mathrm{D}_{2}^{2} / 4$

Now, Bernoulli's equation is

$$
\begin{equation*}
p_{s_{1}}+\frac{u_{1}^{2}}{2 g_{c}}=p_{s_{2}}+\frac{u_{2}^{2}}{2 g_{c}} \tag{2.5}
\end{equation*}
$$

Conservation of mass yields

$$
\begin{equation*}
\rho_{1} \mathrm{~A}_{1} u_{1}=\rho_{2} \mathrm{~A}_{2} u_{2}=\dot{m} \tag{2.6}
\end{equation*}
$$

Also, for an incompressible fluid

$$
\begin{equation*}
u_{1}=\frac{\mathrm{A}_{2}}{\mathrm{~A}_{1}} u_{2} \tag{2.7}
\end{equation*}
$$

Combining equations $2.5-2.7$ yields

$$
\begin{equation*}
\dot{m}_{\text {theoretical }}=\rho_{1} \mathrm{~A}_{1}\left[\frac{2 g_{c}\left(p_{s_{1}}-p_{s_{2}}\right)}{\left(\frac{A_{1}}{\mathrm{~A}_{2}}\right)^{2}-1}\right]^{1 / 2} \tag{2.8}
\end{equation*}
$$

$\dot{m}_{\text {theoretical }}$ is not accurate due to the difficulty in recording the true static pressure at the restriction and the presence of a vena-contracta after the orifice exit which tends to reduce the area of the restriction. To allow for the above restriction, meters are calibrated for a discharge coefficient, $\mathrm{C}_{\mathrm{d}}$, which may vary with the flow velocity through the restriction.

$$
\begin{equation*}
\mathrm{C}_{\mathrm{d}}=\frac{\dot{m}_{\text {actual }}}{\dot{m}_{\text {theoretical }}} \tag{2.9}
\end{equation*}
$$

### 2.7.3 Head Loss Coefficient

There is a total pressure loss across the obstruction devices $\left(p_{0_{1}}-p_{o_{3}}\right)$. The total pressure loss is equal to the static pressure loss assuming the flow velocity is the same upstream and downstream of the obstruction. The pressure loss is non-dimensionalized to give the head loss coefficient as:

$$
\begin{equation*}
\beta \equiv \frac{p_{\text {tot }_{1}}-p_{\text {tot }_{2}}}{\frac{1}{2} \rho u_{1}^{2}} \tag{2.10}
\end{equation*}
$$

which may be approximated as:

$$
\begin{equation*}
\beta=\frac{p_{\mathrm{S}_{1}}-p_{\mathrm{S}_{3}}}{p_{\mathrm{S}_{1}}-p_{\mathrm{S}_{2}}} \tag{2.11}
\end{equation*}
$$

To avoid excessive power losses this value should be maintained at the lowest possible value. As restriction area increases, the losses decrease, as does the sensitivity of the device.

## Chapter 3

## F2- Laminar and Turbulent Flow Through Pipes

### 3.1 Objectives

1. To determine the friction coefficient for air and water flow through smooth pipes at different flow velocities.
2. To show the experimental results fall on a single curve when plotted non-dimensionally.
3. To compare the results of this experiment with those from the published literature.
4. To establish the precision and accuracy of the instruments.

### 3.2 Apparatus

Stainless Steel Tubing (with static pressure taps):
(1) Water tube : $\mathrm{ID}=8.08 \mathrm{~mm}$
(2) Air tube :
$\mathrm{ID}=7.82 \mathrm{~mm}$

Variable Air and Water Flow Supply

### 3.3 Apparatus Setup

The steel tubing for both the air and water experiments are fixed to a table and connected to their respective flow supplies. Bleed valves and motor speed control the air flow. Tap valves control the water flow. The tubes have several pressure taps located along their lengths so that the pressure measurement instruments may be attached.

### 3.4 Instrumentation

- Water Filled Manometers - used in the water flow experiment.
- Inclinable Lambrecht Alcohol Manometers - used in the air flow experiment.
- Mercury Barometer
- Thermometer
- Pitot Tube


### 3.5 Procedure

### 3.5.1 Preliminary Measurements and Checks

1. Record the barometric pressure before and after the experiment.
2. Record the wet and dry bulb temperatures.
3. Check the level and zero settings of the Lambrecht manometers.

### 3.5.2 Experimental Procedure

Water Flow Measurements

1. Take 3 manometer readings, 2 static and 1 dynamic, for approximately 10 equally spaced fiow velocities over the dynamic pressure range available on the manometer. Note that the velocity is proportional to (dynamic pressure $)^{1 / 2}$.
2. Measure the water temperature at the pipe exit at least three times during the experiment.
3. To check for precision of measurements and possible hysteresis, return to at least one flow condition and repeat the manometer readings.

## Air Flow Measurements

The same as water flow measurements.
Note!! Be careful not to blow the Lambrecht manometers. This will occur when the manometers are inclined and the flow velocity is increased, or the outlet is partially blocked when taking the air temperature. At low flow rates the air bleed valves should be open, as the air supply becomes unstable at the low motor speeds.

### 3.6 Theory

The determination of pressure from manometer readings and velocity from dynamic pressure is reviewed in the theory section for the previous experiment F1, i.e.

$$
\begin{align*}
\delta p_{\mathrm{dyn}} & =\rho_{f}(\delta h \sin \theta) \frac{g}{g_{c}}  \tag{3.1}\\
u_{\mathrm{center}}^{\prime} & =\sqrt{2 \delta p_{\mathrm{dyn}} g_{c} / \rho_{\mathrm{a} / \mathrm{w}}} \tag{3.2}
\end{align*}
$$

- $\delta p=p_{\mathrm{T}}-p_{\mathrm{S}}=$ dynamic pressure
- $\delta h=$ manometer reading
- $\theta=$ slope of the manometer
- $\rho_{f}=$ density of manometer fluid
- $\mathrm{T}_{\mathrm{S}}=$ static temperature of air
- $\rho_{\mathrm{a} / \mathrm{w}}=$ density of air or water, depending on the case
- $\rho_{\mathrm{a}}=\frac{\mathrm{Ps}}{R T_{\mathrm{s}}}$ (assume ideal gas: $\mathrm{R}=287 \mathrm{~J} / \mathrm{kg}-\mathrm{K}$ )
- $\rho_{\mathrm{w}}=$ density of water
- $g=$ acceleration of gravity $\left(9.807 \mathrm{~m} / \mathrm{s}^{2}\right)$
- $g_{c}=$ conversion constant ( $1 \mathrm{~kg} \mathrm{~m} / \mathrm{N}-\mathrm{s}^{2}$ )
$u^{\prime}=$ centre line velocity

$$
\begin{equation*}
\mathrm{Re}^{\prime}=\frac{\rho_{\mathrm{a}} u^{\prime} \mathrm{D}}{\mu}=\frac{u^{\prime} \mathrm{D}}{\nu} \tag{3.3}
\end{equation*}
$$

Where for air and water, $\mu$ and $\nu$ are obtained from tables as functions of temperature.

### 3.6.1 Pressure Loss Coefficient (or Coefficient of Friction)

The static pressure loss of a fluid flowing in a tube is due to the surface skin friction. Figure 3.1 shows the forces acting on a finite control volume of fluid.


Fig. 3.1
Note: for steady-state condition

$$
\begin{aligned}
& \text { pressureforce }=\text { shearforceatthewall } \\
& \qquad \pi r^{2} \mathrm{~d} p=2 \pi \mathrm{rd} x \tau_{w}
\end{aligned}
$$

and, the shear stress at the wall

$$
\tau_{w}=\frac{1}{2} \frac{\mathrm{dp}}{\mathrm{~d} \frac{\mathrm{x}}{\mathrm{r}}}=\frac{1}{4} \frac{\delta p}{\ell / \mathrm{D}}
$$

The shear stress or pressure drop ( $\delta \mathrm{p}$ ) are non-dimensionalized with $\frac{1}{2} \rho u^{2} / g_{c}$ to give

$$
C_{f}^{\prime}=\frac{\tau_{w}}{\frac{1}{2} \rho u^{\prime 2} / g_{c}}
$$

and

$$
f^{\prime}=\frac{\delta p}{\frac{\frac{1}{2} \rho u^{\prime 2}}{g_{c}}\left(\frac{\ell}{\mathrm{D}}\right)}=4 C_{f}^{\prime}
$$

Here we determine $\delta$ p and measure $\ell$ and D directly, so it is most direct to plot $\log f^{\prime}$ vs. $\log \mathrm{Re}^{\prime}$. The primes indicate $f^{\prime}$ and $\mathrm{Re}^{\prime}$ are based on the centre line velocity rather than the average velocity.

### 3.6.2 Theoretical Results from Literature

For laminar tube flow Poiseille derived the following relationship:

$$
f=\frac{64}{\operatorname{Re}}
$$

or

$$
f^{\prime}=\frac{32}{\mathrm{Re}^{\prime}}
$$

i.e. $u_{\text {centre }}=2 u_{\text {ave }}$.

For turbulent flow in smooth tubes, Blasius derived the following relationship:

$$
C_{f}^{\prime}=\frac{0.0555}{\mathrm{Re}^{\prime 1 / 4}}
$$

or

$$
f^{\prime}=\frac{0.222}{\operatorname{Re}^{\prime 1 / 4}}
$$

## Chapter 4

## F3- Wind Tunnel Lift and Drag

### 4.1 Objectives

1. To investigate the relationship between the Reynolds number and the drag coefficient for different body shapes.
2. To investigate the relationship between angle of attack and lift and drag coefficients for an aerofoil with and without flap.
3. To compare the results of this experiment with those from the published literature.

### 4.2 Apparatus

Subsonic Wind Tunnel - test section: $\quad 30 \mathrm{~cm}$ octogonal $\times 45 \mathrm{~cm}$ long max. speed: $\quad 26 \mathrm{~m} / \mathrm{s}$

Models -

1. sphere, concave and convex hemispheres, circular disc and streamline body for drag.
2. aerofoil with adjustable flap for lift and drag.

### 4.3 Apparatus Setup

The stem support onto which the different models are mounted is connected to the balance system, which is calibrated to measure lift and drag forces directly. The stem drag alone must be accounted for. The air velocity is adjusted by varying the far motor speed.

### 4.4 Instumentation

- Inclined Manometer

Measures atmospheric pressure relative to static pressure at the beginning of the test section. It is calibrated to give the air flow velocity in the test section.

- Two Component Balance System

The lift and drag forces are measured directly from calibrated scales connected to the stem support.

### 4.5 Procedure

### 4.5.1 Preliminary Measurements

1. Carry out a dimensional analysis relating lift and drag to the parameters defining the physical situation (see theory).

### 4.5.2 Experimental Procedure

1. Obtain the stem drag data and plot the stem drag versus velocity.
2. Obtain the drag data for a sphere, a flat plate, a convex and concave hemisphere and for a streamline body.
3. Obtain the lift and drag for a two dimensional aerofoil at various angles of attack, with and without the flap.
4. Plot the drag coefficient versus Reynolds number for 5 drag models.
5. Plot the lift and drag coefficients for the aerofoil with and without flap versus angle of attack at a constant Reynolds number.

### 4.6 Theory

The fluid forces acting on a body can be divided into two parts, drag and lift.

Drag force is defined as the force exerted on a body by a fluid moving in the direction of the free stream flow. Drag is dependant basically on two factors:

1. Properties of the body itself, which include shape and surface conditions.
2. Properties of the flow.

Drag is comprised of two components, namely pressure drag and skin friction drag.

Pressure drag is the result of flow reversal due to an adverse pressure gradient (Refer to Figure 4.1). As a result, separation of the flow from the body occurs, which generates a wake behind the body. The larger the wake, the greater the pressure drag exerted on the body.

Skin friction drag, on the other hand, is due to shear stresses in the boundary layer on the body surface.

Drag is an undesireable quantity which impedes the movement of an object through a fluid. In engineering applications, pressure drag has a greater effect on a body than skin friction. Thus, in an effort to avoid separation as long as possible, the body of an object is made to be streamlined.

Lift forces are caused primarily as the result of a pressure gradient between the top and bottom portions of a body. When separation of the flow occurs, the result is stall: a loss of lift and an increase in drag.

To examine the effects of different parameters on lift and drag, a relationship between the drag coefficient and/or the lift coefficient versus the Reynolds number should be used. Drag (lift) on a fully submerged body in a moving fluid is dependent on

1. characteristic dimensions of the body
2. fluid density
3. fluid viscosity
4. fluid (or body) velocity

Criteria of similarity is a method by which to determine the number of non-dimensional groups that can be formed from the independant parameters affecting the flow around a specific body. To determine the criteria of similarity, the independant parameters are subtracted from the basic units of those parameters (such as time, length and mass). In the case of this experiment, one criteria of similarity is available:

$$
\begin{equation*}
\frac{D}{1 / 2 \rho v^{2} A}=f\left(\frac{\rho v d}{\mu}\right) \tag{4.1}
\end{equation*}
$$

where D-drag
$\rho$ - fluid density
$v$ - fluid or body velocity
A - projected area of body facing flow
d - diameter of body
$\mu$.- fluid viscosity

Eqn. (4.1) defines the drag coefficient $C_{D}$. If D is replaced by L in Eqn. (4.1), where L is the lift force, then this equation defines the lift coefficient $C_{L}$.

The lift and drag forces on an aerofoil, as examined in this experiment, are dependant on the Reynolds number as well as the angle of attack (refer to Figure 4.2). From Figure 4.2 it is clear that

$$
\begin{equation*}
D=f_{1}(\rho, \mu, v, \alpha, c) \tag{4.2}
\end{equation*}
$$

$$
\begin{equation*}
L=f_{2}(\rho, \mu, v, \alpha, c) \tag{4.3}
\end{equation*}
$$

Through dimensional analysis, both the lift and drag on an aerofoil can be formulated as

$$
\begin{align*}
& C_{D}=f_{3}(R e, \alpha)  \tag{4.4}\\
&=\frac{D}{1 / 2 \rho v^{2} A_{p}}  \tag{4.5}\\
& C_{L}=f_{4}(R e, \alpha)=\frac{L}{1 / 2 \rho v^{2} A_{p}}
\end{align*}
$$

where $A_{p}$ - planform area of the aerofoil
To represent lift and drag graphically, one can plot $C_{D}$ and $C_{L}$ for various angles of attack (refer to Figure 4.3).


Fig. 4.1: Onset of separation.


Fig. 4.2: Life and drag forces on an aerofoil.


Fig. 4.3: Life and drag coefficients vs. angle of attack.

Part II

## Heat Transfer

Heat Transfer plays a key role in the development of almost every emerging technology. For instance, it is well established that one of the main factors inhibiting the development of even faster microchips than exist today is the effective removal of the heat generated within the chips. The heat generation increases as the chip speed increases or the size decreases. One of the many reasons that the use of high temperature superconductors is currently limited is because of the inability to maintain a constant enough temperature for them to operate; the temperature band in which these materials become superconducting is very narrow. Also the thermal efficiency of turbines increases as the operating temperature increases; however, the inability to effectively cool the turbine blades so that they do not melt or sustain damage has prevented their commercial introduction. These situations and many more rely on adroit application of heat transfer principles.

The scenarios above primarily involve heat transfer through two modes: conduction and convection. A thorough understanding of these modes will aid in attacking heat transfer issues in many diverse situations. To gain this understanding, it is important to have a feel for heat transfer in several basic situations like those presented in the following three chapters. Insight regarding the link between theory and experiment will be gained. A double-pipe heat exchanger will be tested and analyzed; this heat exchanger is representative of a large class of widely exploited heat exchangers. Transient external forced convective heat transfer is examined, and the variation of the Nusselt number with the Reynolds number is determined. Finally turbulent internal forced convection will be examined in detail.

From these three situations, an appreciation for the inherent link between theory and experiment in the field of heat transfer should be obtained. Because of the complex nature of the involved phenomena, correlations based on experiment are often the only recourse when approaching a heat transfer problem. Performing these labs should give the student a taste for the difficulty in obtaining these correlations.

## Chapter 5

## H1- Double-Pipe Heat Exchanger

### 5.1 Objectives

1. To determine the heat transfer rate between a flow of hot water and/or steam and a flow of cold water as they pass through the 2,4 , or 6 pass double-pipe heat exchanger in a parallel and/or counter flow manner. At least 2 different arrangements from the above possibilities should be tested.
2. The overall heat transfer coefficients and the heat exchanger effectiveness should be determined from the experimental data for the doublepipe heat exchanger configurations chosen and compared to what you would expect theoretically.

### 5.2 Apparatus

A schematic layout of the Lab Sciences 6 pass heat exchanger is shown in Figure 5.1. The dimensions and material of the six pass tubes are given in Table 5.1.

Cool water enters the exchanger at $C$, passes through a flow meter, then goes through the outer annuli of all the passes in series. The cold water
ter control; valves (1), (2), (3), and (4) may be adjusted so that the cool water enters at pass 1 and exits at pass 6, i.e. valves (1) and (2) open and (3) and (4) closed, or enters at pass 6 and exits at pass 1, i.e. valves (1) and (2) closed and (3) and (4) open. In either case, the cool water exits at $D$.

For all tests, cooling water flows in the annuli of all passes. The cooling water is made to flow by connecting a water pump, located in an outside container, with a plastic hose to the inlet a $C$. The flow rate is regulated with a valve on the flowmeter. The flowmeter is not accurate and thus the actual mass flow rate must be determined by collecting the cool water leaving the exchanger at $D$ in a calibrated container.

The boiler and super heater are used to heat water to supply either superheated steam of hot water.

For operation with superheated steam, valve (10) is open and valve (11) is closed so that water from the boiler supply enters the boiler/superheater combination directly by gravity. Caution: when operating in this mode extreme care must be taken to ensure that the boiler supply water tank does not run dry. The steam from the super heater enters the inside tubes of pass 1 and 2 only and exits at $B$ under atmospheric pressure. The condensate is collected at the outs of $B$.

For operation with hot water, valve (10) is closed and valve (11) is open. Water is supplied to the boiler/superheater combination by connecting a water pump, located in a large container, to the inlet $A$ with a length of plastic hose. The hot water from the boiler goes through the inside tubes of passes 1 and 2 as before. Then with a plastic hose connecting the outlet at $B$ with the inlet at $E$, the hot water will continue to flow through passes $3,4,5$, and 6 and exit through the flowmeter at $F$. The rate of hot water flow is controlled with a valve on the flow meter. The hot water flow rate, like the cold water flow rate, must be obtained directly. Hot water flow through passes 3 and 4 and/or 5 and 6 is accomplished by opening and closing valves $5,6,7$, and 8 appropriately.

A 220 volt single phase power supply is used to heat the water in the boiler/superheater combination. The current is controlled by a variac to
supply about 6.5 amperes. Two 7 ampere circuit breakers are incorporated as a precautionary measure.

### 5.3 Instrumentation

Nineteen Chromel-Constantine thermal couples are located at various positions throughout the exchanger as shown in Figure 5.1; eight measure the hot fluid temperature in the inside tube and eleven the cold fluid temperature in the annulus. The temperature recorded by an individual thermocouple is displayed on the digital Omega meter to within $\pm 0.1$ degree. The location of the thermocouple corresponds to the number selected with the switches on the control panel, and the number on the switches correspond to the numbered positions shown in Figure 5.1.

A voltmeter and ammeter are also located on the control panel. They are used to determine the power into the boiler/preheated combination.

### 5.4 Procedure

### 5.4.1 Preliminary Measurements

1. Barometric Pressure
2. Room Temperature

### 5.4.2 Operation for Hot and Cold Water Flows

1. Large container of water with immersion pumps is filled.
2. Heat exchanger configuration is chosen with appropriate hot and cold water valves turned on or off.
3. Immersion pumps are connected on the hot and cold water inlets and containers placed to collect the hot and cold water at the outlets.
4. The immersion pumps are started and the flowmeter valves initially opened completely in order to let all the air bubbles out of the heatexchanger. Sometimes the pumps may have to be shaken in order to release the collectd air in their inlets.
5. When no more air bubbles are detected in the pipes, the flowmeter valves are adjusted to the given flow rates.
6. The boiler heater is switched on and for most tests the variac set to give a maximum of 6.5 amps . The ammeter and voltmeter respond quickly to any changes in the variac setting. Once the variac is set it is necessary to wait about 15 minutes for the system to reach steady conditions.
7. When steady conditions have been achieved - i.e. a periodic check on one or two temperature readings can be used to establish when steady conditions have been achieved; at least two sets of temperature readings should be taken at appropriate locations for the chosen configuration.
8. The heat exchanger configuration is reset (with appropriate valves turned on or off), or the flow rate of the hot and/or cold fluid is changed, and step 7 is repeated.

### 5.4.3 Operation for Steam and Cold Water Flows

1. The boiler water supply should be filled with water. It is most important to ensure that the boiler/superheater combination does not run out of water during the entire test. If it does the boiler will burn out. The sight glass on the boiler should be continuously checked to ensure an adequate water level throughout the steam test. Superheated steam in the neighborhood of $125^{\circ} \mathrm{C}$ at atmospheric pressure can be obtained.
2. Adjust the cooling water flow in the annulus to ensure condensation of the steam in passes 1 and 2. If it is too low, then vapor can be seen to ${ }^{\circ}$ issue from the exit at $B$.

### 5.5 Results

1. It is suggested that for each test run you plot the temperature profiles through the heat exchanger in addition to recording the flow rates and temperatures. In this manner, any errors are easily spotted and there will be no confusion in calculating the correct log mean temperature difference for parallel or counterflow configurations.
2. The heat transfer rate should be calculated for the hot fluid and cold fluid. Any discrepancy should be discussed. These values may then be used to calculate the overall heat transfer coefficient and the heat exchanger effectiveness.
3. The overall heat transfer coefficient should be obtained theoretically knowing the Reynolds number for the tube and annulus and using an appropriate Nusselt number correlation. This may only be done for the first three passes where the tube material and diameters are constant.

### 5.6 Theory

The theoretical analysis of concentric tube heat exchangers for either parallel or counterflow arrangements is covered in most basic Heat Transfer texts. The three basic heat transfer conservation equations are:

$$
\begin{gather*}
q=\dot{m}_{h} c_{p_{h}}\left(T_{h_{\text {in }}}-T_{h_{\text {out }}}\right)  \tag{5.1}\\
q=\dot{m}_{c} c_{p_{c}}\left(T_{c_{\text {in }}}-T_{c_{\text {out }}}\right)  \tag{5.2}\\
q=U A \delta T_{L M} \tag{5.3}
\end{gather*}
$$

or replacing equation (5.1) with the following for steam

$$
\begin{equation*}
q=\dot{m}_{s}\left(\text { enthalpy steam }{ }_{\text {in }}-c_{p} T_{h_{\text {out }}}\right) \tag{5.4}
\end{equation*}
$$

where

$$
\begin{aligned}
& q=\text { heat transfer rate of the exchanger }(\mathrm{W}) \\
& \dot{m}=\text { mass flow rate of fluid }(\mathrm{kg} / \mathrm{s})
\end{aligned}
$$

$c_{p}=$ specific heat of the fluid $\left(\mathrm{J} / \mathrm{kg}-{ }^{\circ} \mathrm{C}\right)$
$T=$ temperature $\left({ }^{\circ} \mathrm{C}\right)$
$U=$ overall heat transfer coefficient ( $\mathrm{W} / \mathrm{m}^{2}-{ }^{\circ} \mathrm{C}$ )
$A=$ active surface area of the exchanger ( $\mathrm{m}^{2}$ )
$\delta T_{L M}=\frac{T_{x=0-T_{x}}}{\ln \frac{T_{x}=0}{T_{x=2}}}$.
Also the subscripts have the following meanings:
$h=$ hot fluid
$c=$ cold fluid
in = into the heat exchanger
out =out of the heat exchanger
$x=0$ : one end of the heat exchanger
$x=L$ : other end of the heat exchanger.
The enthalpy of the superheated steam and/or saturated water is obtained form the steam tables.

The effectiveness ( $\epsilon$ ) of a heat exchanger is a measure of its efficiency

$$
\epsilon=\frac{q}{q_{\max \text { possible }}}
$$

where

$$
q_{\max \text { possible }}=\left(\dot{m}_{h} c_{p}\right)_{\min }\left(\delta T_{\text {max }}\right)
$$

Refer to your heat transfer text for the details concerning the calculation of $\delta T_{\text {max }}$.

The value of the overall heat transfer coefficient $U$ may be obtained from

$$
A_{0} U=\frac{1}{\frac{1}{A_{i} h_{\mathrm{i}}}+\frac{\ln \tau_{0} / \tau_{1}}{2 \pi k l}+\frac{1}{A_{0} h_{0}}}
$$

where
$k=$ thermal conductivity of the inner tube material
$r_{0}$ and $A_{o}=$ radius and area of the outside surface of the inner tube
$r_{\mathrm{i}}$ and $A_{\mathrm{i}}=$ radius and area of the inside surface of the inner tube
$l=$ length of the heat exchanger
$h_{0}$ and $h_{\mathrm{i}}=$ convective heat transfer coefficients on the outside and inside of the inner tube.

Values of $h_{\mathrm{o}}$ and $h_{\mathrm{i}}$ are obtained from the appropriate Nusselt number correlation as a function of Reynolds number and Prandtl number. Difficulty may be encountered in finding an appropriate correlation from basic Heat Transfer texts since the Reynolds number is close to the transition region between laminar and turbulent flow. If the flow is not clearly in the laminar region, i.e. $R e<2000$, then it is suggested that turbulent correlations be used to determine the Nusselt number and hence the convective heat transfer coefficient.

Table 5.1: Tube dimensions \& materials.

| $\begin{gathered} \text { Pass } \\ n 0 . \end{gathered}$ | $\begin{aligned} & \text { Wetted } \\ & \text { length } \\ & \text { of } \\ & \text { inner } \\ & \text { pipe } \\ & (\mathrm{E}) \end{aligned}$ | Inner dismeter of inner pipe (") | Outer diameter of inner pipe (") | $\begin{aligned} & \text { Inner } \\ & \text { pipe } \\ & \text { material } \end{aligned}$ | Inner diameter of outer pipe (") | Outer pipe material |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 34.25 | 0.320 | 0.375 | Cu | 0.550 | Cu |
| 2 | 31.375 | 0.320 | 0.375 | Cu | 0.550 | Cu |
| 3 | 25.375 | 0.320 | 0.375 | Cu | 0.550 | Cu |
| 4 | 25.375 | 0.180 | 0.250 | Cu | 0.550 | Cu |
| 5 | 25.375 | 0.320 | 0.375 | Steel | 0.550 | Cu |
| 6 | 25.375 | 0.180 | 0.250 | Steel | 0.550 | Cu |



Fig. 5.1: Schematic of the heat exchanger.

## Chapter 6

## H2- Transient Convective Heat Transfer

### 6.1 Objectives

1. To study the convective heat transfer from a stationary heated sphere to air flowing at a uniform velocity and constant temperature over the sphere.
2. To determine the variation of the average Nusselt number with the Reynolds number.
3. To compare the results of this experiment with those given in the form of empirical correlations in published literature.

### 6.2 Apparatus

Acrylic Transparent Tube -
Sheet Metal -
Bell Shaped Nozzle -
Aluminum Sphere -
Jig
Steel Support Rod
Electric Oven

ID:
29.21 cm honeycomb form
ID: $\quad 22.86 \mathrm{~cm}$
diameter $\simeq 2.54 \mathrm{~cm}$

### 6.3 Apparatus Setup

Air is forced through the thick walled acrylic tube by means of a fan driven by an electric motor. A variable angle baffle is used to control the flow rate. A sheet metal honeycomb is located in the plastic tube to straighten out the flow, thus making the velocity relatively uniform over the tube cross-section. Further uniformity is achieved by forcing the air flow through a bell-shaped converging nozzle at the exit of the plastic flow tube.

An aluminum, brass, or steel sphere is used as the heated body in this experiment. A jig and a thin steel support rod are used to position the sphere in the oven and in the air stream.

### 6.4 Instrumentation

## - Thermocouples and Chart Recorder

Two iron-constantan thermocouples (Type J), one in the air stream and the other positioned at the center of the aluminum sphere, are used to measure the temperature difference between the sphere and the airstream ( $T-T \infty$ ). A chart recorder is used to record the variation of this temperature difference with time.

## - Potentiometer

The thermocouple inside the sphere is also connected to a calibrated potentiometer. This potentiometer is used to monitor the temperature of the sphere while it is heated in the oven to prevent overheating.

## - Pitot-Static Tube

A pitot-static tube is used to measure the air stream velocities in the exit plane of the bell nozzle. Using a traverse mechanism fitted with a vernier scale, the pitot can be positioned at various points along the diameter of the bell nozzle to determine the velocity profile.

- Inclinable Manometers
- Mercury Thermometer
- Mercury Barometer
- Micrometer


### 6.5 Procedure

### 6.5.1 Preliminary Checks \& Measurements

1. Check paper feed continuity, pens and zero setting of the chart recorder.
2. Short the input channels to the chart recorder, and crefully "zero" the response. Then, reattach the cable for reading velocity.
3. Level and adjust the zero reading of the manometer. Do this with great care. Then, set the inclination to 1:2.
4. Record the atmospheric pressure in mm Hg.
5. Record the room temperature.
6. Measure sphere diameter.

### 6.5.2 Experimental Procedure

1. Start the fan and open the air baffles fully. Allow two minutes for the flow to attain reasonably steady conditions.
2. Lower pitot-static tube to the bottom of the traverse mechanism and lock it in place. The stagnation pressure port should be roughly at the center of the nozzle outlet in this position.
3. Place the aluminum sphere inside the heating oven. Monitor the temperature of the sphere, and make sure it does not exceed $180^{\circ} \mathrm{C}$.
4. While the solid sphere is heating, measure the temperature of the air stream, using a thermometer.
5. Record manometer reading (actual value, do not adjust for inclination at this stage).
6. When the solid sphere temperature reaches $170^{\circ} \mathrm{C}$, turn on chart record (chart feed switch, and record switch). Retract pitot-static tube away from the mouth of the nozzle.
7. Once the solid sphere temperature is close to $180^{\circ} \mathrm{C}$, retract the oven, and position the sphere at the center of the nozzle outlet plane.
8. When the chart recorder reading reaches 0.5 mV , stop the chart feed, and flip the "record" switch to "standby". Mark run number on the chart.
9. Adjust the baffle opening to the next marked position (lower air flow rate). Repeat steps 2 to 8.
10. Do a maximum of six runs, if possible, in a total of 75 minutes. Do at least four runs.
11. Fill out the supplied data recording sheets for each run.
12. Use the computer programs, RUN and CORREL, to do the required computations. REMEMBER: Program RUN is designed for the aluminum sphere only.

### 6.5.3 Instructions for Using the Computer Programs "RUN" and "CORREL"

1. Get out of "Windows" to the "DOS" shell.
2. Get to the "Root" directory.
3. Change to the directory named "MECHLABI". Then type "OS386" and "Enter".
4. Type "UP RUN" and "Enter".
5. Type in required inputs, for each run, using the information on the data recording sheets.
6. Once the program has executed, check the output on the screen, and then print out the contents of the file "RESULTS".
7. Repeat steps 5 and 6 for each run.
8. Fill out the " Nu vs $\mathrm{Re}^{\prime}$ data sheet provided to you.
9. Type "UP CORREL" and "Enter".
10. Type in required inputs.
11. After program execution, print out contents of the file "NUVSRE".

### 6.6 Results

1. First justify, then apply the lumped parameter analysis of unsteady heat conduction to determine the average heat transfer coefficient for each of the air velocities used in the experiment. Prove $k_{1}=2$ from first principals.
2. Establish a correlation between the Nusselt number and the Reynolds number. Present a suitable plot of this correlation based on the experimental data.
3. Compare graphically the results with those of Raithby and Whitaker.
4. Identify and discuss the sources of error in this experiment.

### 6.7 Theory

Transient forced convective heat transfer problems are widely encountered in "modern" applications such as the space-shuttle orbiter during reentry or heat exchangers during start-up of nuclear reactors as well as in "classical". applications like the cooling of a metal casting in an air stream.

This experiment involves the analysis of transient forced convective heat transfer from a heated metal sphere to air flowing under atmospheric conditions (Figure 6.1).

To simplify the solution of the problem, the following assumptions will be made:

1. The external thermal resistance between the surface of the metal sphere and the surrounding air stream is much larger than the internal thermal resistance of the system. This leads to the assumption that the sphere temperature is only a function of time( t ) and not of the spatial coordinates $(x, y, z)$ or the sphere. Experimental evidence shows that when the dimensionless ratio of the internal resistance to the external resistance is less than 0.1 , the error introduced by the assumption of constant sphere temperature at any instant of time is less than $5 \%$. You are to lend justification to this assumption using the experimental data and the thermal conductivity of the sphere.
2. The free stream velocity, $U_{\infty}$, is assumed to be constant once it is fixed. The effect of turbulent fluctuations in the flow will effect the heat transfer rate. Your discussion should probe this issue.

With consideration to the above assumptions, an energy balance for the sphere over a small time interval $d t$ is:

$$
\left[\begin{array}{c}
\text { The change in internal energy } \\
\text { of the sphere during dt }
\end{array}\right]=\left[\begin{array}{c}
\text { The net heat flow from } \\
\text { the sphere to the } \\
\text { air stream during dt }
\end{array}\right]
$$

or in mathematical terms,

$$
\begin{equation*}
-\rho c_{p} V d T=\bar{h} A\left(T-T_{\infty}\right) d t \tag{6.1}
\end{equation*}
$$

where
$c_{p}=$ the specific heat of the sphere
$\rho=$ density of the sphere
$V=$ volume of the sphere
$T=T_{w}=$ wall temperature $=$ average temperature of the sphere
$A=$ surface area of the sphere
$d T=d\left(T-T_{\infty}\right)=$ temperature change during time interval $d t$
$\bar{h}=$ average convective heat transfer coefficient
Taking the initial temperature of the sphere at $t=0$ to be $T=T_{i}$, equation 6.1 can be integrated and arranged as

$$
\begin{equation*}
\ln \frac{T-T_{\infty}}{T_{i}-T_{\infty}}=-\frac{\bar{h} A}{\rho c_{p} V} t \tag{6.2}
\end{equation*}
$$

Note that once the variation of temperature $v s$ time and properties of the sphere are monitored, the only remaining unknown, $\bar{h}$, can easily be determined for each of the air velocities used in the experiment. When repeated for a number of different mass flow rates, as you have done during this experiment, the relation between the average convective heat transfer coefficient $\bar{h}$ and the free stream velocity $U_{\infty}$ can be determined empirically. As in the case of most convective heat transfer problems, this relationship can be expressed non-dimensionally in the form

$$
\begin{equation*}
N u=K_{1}+K_{2} R e^{k_{3}} \tag{6.3}
\end{equation*}
$$

where

$$
N u=\text { Nusselt number }
$$

$R e=$ Reynolds number
$K_{1}, K_{2}, K_{3}=$ empirical coefficients
Hence by plotting $\ln \left(N u-K_{1}\right)$ vs. $\ln (R e)$, it is possible to obtain the empirical coefficients of equation 6.3 for this experiment.

Two studies similar to this experiment have been performed by Raithby and Whitaker, whose correlations are listed below:

Raithby correlation

$$
\begin{equation*}
N u=2+.210 R e^{.606} \tag{6.4}
\end{equation*}
$$

Whitaker correlation

$$
\begin{equation*}
N u=2+P r^{.4}\left[.4 R e^{1 / 2}+.06 R e^{2 / 3}\right]\left[\frac{\mu_{s t}}{\mu_{w}}\right]^{0.25} \tag{6.5}
\end{equation*}
$$

Biot Number: To check if the lumped parameter assumption is valid, the Biot Number should be calculated.

$$
\begin{equation*}
\mathrm{Bi}=\frac{\bar{\hbar} V / A}{k}=\left(\frac{\bar{h} r_{o} / 3}{k}\right)_{\text {sphere }}<0.1 \tag{6.6}
\end{equation*}
$$

where

$$
\begin{aligned}
& k=\text { thermal conductivity of the sphere } \\
& r_{0}=\text { radius of the sphere. }
\end{aligned}
$$



Fig. 6.1: Forced convection heat transfer from a sphere.

## Chapter 7

## H3- Turbulent Internal Forced Convection

### 7.1 Objectives

1. To study fully developed turbulent forced convection heat transfer from a pipe, with a constant wall heat flux, to air flowing at a steady rate inside a pipe.
2. To establish the variation of the average Nusselt number with the Reynolds number, for a fixed Prandtl number.
3. To compare the results with those given in the form of empirical correlations in the published literature.

### 7.2 Apparatus

The main test section of the experiment consists of a 2.405 cm ( 0.947 in .) I.D. $\times 2.540 \mathrm{~cm}(1.000 \mathrm{in}$.) O.D. $\times 6.096 \mathrm{~m}(20 \mathrm{ft}$.) long stainless steel pipe. This length is long enough to ensure both thermally and hydrodynamically fully developed flow over the second half of the pipe. The pipe is heated electrically by passing a constant current through its walls. This produces an essentially constant specified heat fiux thermal boundary condition at the interface between the air flow and the pipe wall. The current is supplied by an arc welder that is connected to each end of the test section by copper
electrodes. The heated pipe is wrapped with 4.44 cm ( 1.75 in .) thick, standard fiber glass pipe insulation in an attempt to make heat loss from the pipe to the external environment negligible. Tefion couplings and a double O-ring seal are used to link the inlet and exhaust sections to the test section. The exhaust section consists of a 2.202 cm ( 0.867 in .) I.D. $\times 1.016 \mathrm{~m}$ ( 40 in.) long steel pipe. The test section air flow is supplied by a rotary vane air pump operating in the blowing mode. The air flow is regulated by means of a gate valve located at the inlet to the air pump. After the air leaves the pump it passes through a counter flow heat exchanger that uses cold water as the cooling fluid. The water flow rate is controlled by a gate valve mounted on the wall near the counter flow heat exchanger. The air then flows through the connecting piping to the inlet of the heated test section.

### 7.3 Instrumentation

The pipe wall of the heated test section is instrumented with 28 ChromelConstantan thermocouples (Type E) along its length. Knowing the relative position of each thermocouple (see Table 7.1), the temperature readings can be used to determine the temperature gradient along the pipe wall. There are also two separate thermocouples used to measure bulk air temperatures, one located at the inlet to the heated test section, and the other at the outlet of the exhaust section. All thermocouple outputs are read using a thermal couple switch capable of connecting any one of a possible forty thermocouples to a digital temperature meter located on the panel beside the arc welder.

The power supplied to the system from the arc welder is measured using a digital voltmeter and a digital ammeter located on the top of the unit.

A pitot stagnation tube is positioned at the centre of the exhaust exit pipe diameter. The stagnation tube is connected to an inclinable reservoirtype manometer, filled with methyl alcohol (sp. gr. $=0.81$ ), that is used to measure the exhaust flow maximum stagnation gauge pressure.

### 7.4 Procedure

### 7.4.1 Preliminary Checks

1. Thermocouples

- power on the digital temperature meter
- all thermocouples working

2. Manometers

- leveling
- zero setting
- tilt angles (start at 1:1)
- all connections

3. Power Measurement Instruments

- power on
- digital readout

4. Cooling Water System

- flow control valve

5. Air Flow System

- air blower connections, power supply, etc.
- flow control valve


### 7.4.2 Preliminary Measurements

1. Atmospheric Pressure
2. Room Temperature
3. Manometer bank water level for atmospheric pressure (i.e. no flow conditions)

### 7.4.3 Main Experiment

1. Open the cooling water flow control valve until a constant flow of cooling water can be seen flowing out of the system into the drain.
2. Open the air flow control valve as far as is possible without "blowing out" either of the manometers. This will establish the maximum air flow rate possible. Next determine a reasonable minimum air flow rate keeping in mind that this experiment is concerned with turbulent flows. Divide this flow rate range into at least six appropriate intervals (six flow rates).
3. Set the air flow control valve to obtain the maximum air flow rate established in Step 2.
4. Turn on the arc welder to the low power setting and adjust the power output to a level of 300 Watts by monitoring the current and voltage output.
5. Monitor the test section wall temperatures to determine when steady state conditions have been achieved.
6. Record the stagnation gauge pressure at the outlet of the exhaust section.
7. Record the readings of all the thermocouples numbered 3 to 33 inclusive, excluding number 32 (see Table 7.1).
8. Decrease the air flow rate to the next level established in Step 2 and repeats Steps 5 to 7.
9. At the end of the experiment shut down the power supply to all equipment, starting with the arc welder; close the water flow control valve, and inform the demonstrator or technician in charge of the experiment.

### 7.5 Results

1. Plot the measured values of the pipe wall temperatures, and calculate the temperature gradient in the thermally fully developed region (i.e.
the region where the temperature gradient becomes linear) for each flow rate. Using this gradient and the measured test section inlet bulk air temperature, calculate the difference between the test section wall temperature and the bulk air temperature in the fully developed region: Note that in the fully developed region, with the constant wall heat flux boundary condition $\frac{d T_{b}}{d x}=\frac{d T_{w}}{d x}$. Knowing this temperature difference, the measured power input, and the test section dimensions, the resulting heat transfer coefficient can be calculated.
Test section Reynolds numbers are to be calculated based on the average mass flow rate. Using the measured maximum stagnation gauge pressure value, assuming that $P_{\text {atm }}=P_{\text {static }}$ at the exit, Bernoulli's equation can be used to calculate the maximum flow velocity [Ref. 1]. For the range of flow rates considered in this particular experiment, the average flow velocity that would be obtained by a ten point loglinear pitot scan [Ref. 2] can be calculated to within about $1 \%$ by the following correlation:

$$
\begin{equation*}
\frac{V_{\max }}{V_{\operatorname{avg}}}=1.21 \tag{7.1}
\end{equation*}
$$

Evaluating fluid properties at the arithmetic mean of the bulk air temperatures, the corresponding Nusselt numbers and Reynolds numbers can be calculated.
2. Establish a correlation between the Nusselt number and the Reynolds number. The suggested form of this correlation is:

$$
\begin{equation*}
N u=C R e^{n} \tag{7.2}
\end{equation*}
$$

Note that this correlation assumes that the Prandtl number remains essentially constant throughout this experiment. Present a suitable plot of this correlation and the experimental data points.
3. Compare your results with those obtained using the Colburn correlation, the Kays and Perkins correlation, and the Seider-Tate correlation [Refs. 3, 4]. These are given in Table 7.2. Other results in the-literature [Refs.3-5] may also be used in this comparison if you think they are relevant. Both tabular and graphical comparisons are desirable.
4. Identify and discuss the sources of error in this experiment: obtain quantitative estimates whenever possible [Ref. 6].

### 7.6 Theory

- Thermally and hydrodynamically fully developed turbulent duct flow [Refs. 1, 3-5].
- Equation of state of air.
- Fluid flow measurements using a pitot stagnation tube [Refs. 1, 2, 6].

The aforementioned topics are covered in the HEAT TRANSFER course. The following references may prove helpful.

### 7.7 References

1. White, F.M., Fluid Mechanics, McGraw-Hill, 1979.
2. Winernitz, F.A.L., and Fischl, C.F., "A simplified integration technique for pipe-fiow measurements", Water Power, pp. 225-234, June 1957.
3. Handbook of Heat Transfer, Chapter 7, McGraw- Hill, 1973.
4. Holman, J.P., Heat Transfer, $6^{\text {th }}$ edition, McGraw-Hill, 1986.
5. Krieth,F., and Black, W.Z., Basic Heat Transfer, Harper and Row Publishers, 1980.
6. Holman, J.P., Experimental Methods for Engineers, $3^{\text {rd }}$ edition, McGrawHill, 1978.

Table 7.2: Heat transfer correlations (see references for details).

| Correlation | Name: Reference |
| :--- | :--- |
| $N u=0.023 R e^{0.8} P r^{1 / 3}$ | Colburn: [Ref. 3] |
| $N u=0.022 R e^{0.8} P r^{0.6}$ | Kays-Perkins: [Ref. 3] |
| $N u=0.027 R e^{0.8} P r^{1 / 3}\left(\mu / \mu_{w}\right)^{0.14}$ | Seider-Tate: [Ref. 4] |

Table 7.1: Thermocouple locations.

| Therne. couple No. | Location frea start of upstrean and of heated cest section $\text { ( } \mathbf{a}$ | Sype of measurement |
| :---: | :---: | :---: |
| 3 | 0.000 | -bulk alr cemperature ar inlet to heated cest section |
| 4 | 0.203 | -cest section vall temperature |
| 5 | 0.606 | - |
| 6 | 0.610 | - - |
| 7 | 0.813 | - - |
| 8 | 1.016 | - - |
| 9 | 2.422 | - - |
| 10 | 1.623 | - - |
| 11 | 1.829 | - - |
| 12 | 2.032 | - |
| 13 | 2.235 | - - |
| 14 | 2.438 | - - |
| 15 | 2.642 | - - |
| 26 | 2.845 | - |
| 17 | 3.048 | - - |
| 18 | 3.252 | - |
| 19 | 3.454 | - - |
| 20 | 3.658 | - - |
| 21 | 3.862 | - - |
| 22 | 6.064 | - - |
| 23 | 4.267 | - - |
| 24 | 4.670 | - - |
| 25 | 4.674 | - - |
| 26 | 4.877 | - - |
| 27 | 5.080 | - - |
| 28 | 5.283 | - - |
| 29 | 5.486 | - - |
| 30 | 5.690 | - |
| 31 | 5.893 | - |
| 32 | -•••• | - Lenore chis reaperacure seadias |
| 33 | $\cdots \cdots$ | - elr ceaperacure at erhanes section exit |

## Part III

## Thermodynamics

Efficiency is a word that is often bandied about carelessly leading to ambiguity and meaningless assertions. So then, what is really meant by efficiency? How does one decide what is efficiency? What is efficiency measuring? As will be discovered in the four labs that follow, efficiency is really a catchall term for a variety of parameters that measure a system's performance. Thus, there are many types of efficiencies, each one having its own significance. How they are applied, calculated, and interpreted is of the utmost importance to any mechanical engineer.

In the four following labs, very real and vital situations will be encountered in which the appropriate performance parameters will be calculated. The internal combustion engine remains the backbone of today's society, and is justifiably the focus of two of the labs. The two most prevalent types of internal combustion engines will be studied; they are the compression ignition engine and the spark ignition engine. The compression ignition engine that is used is the classic Ruston-Hornsby Diesel engine. Climate control is also a significant contemporary issue. The processing of cutting edge materials, the development of high speed computers, and the maintenance of comfortable conditions in large buildings all require an understanding air conditioning principles. To this end a model air conditioning plant will be evaluated in lab T4. Finally, at the heart of just about every thermodynamic system from refrigeration units to gas-turbine power plants is the compressor. An understanding of its operation and performance parameters are essential to the design of more economical and environmentally sound systems than currently exist. To obtain an appreciation for this, a two-stage air compressor will be analyzed in lab T3.

## Chapter 8

## T1- Ruston-Hornsby Diesel Engine

### 8.1 Objective

To determine the constant speed operating characteristics of the engine.

### 8.2 Apparatus

The Ruston-Hornsby is a 4 -stroke, single cylinder, water cooled diesel engine governed to operate at speeds between 350 and 450 rpm . The engine produces a maximum power of $9.7 \mathrm{~kW}(13 \mathrm{hp})$. It is an old, traditional diesel engine.

Specifications:
-bore: $\quad 15.24 \mathrm{~cm}$ (6.0 in.)
-stroke:
-clearance volume:
-prony brake arm length:
-tare weight of brake arm:
-P-V diagram calibration factor:
29.21 cm ( 11.5 in .)
$364 \mathrm{~cm}^{3}$ (22.2 in. ${ }^{3}$ )
96.52 cm ( 38 in .)
$5.10 \mathrm{~kg}(11.25 \mathrm{lb})$
$1379 \mathrm{kN} / \mathrm{m}^{3} / \mathrm{cm}$
( $508 \mathrm{psi} / \mathrm{in}$ )
-Higher Heating Value of Diesel (HHV): $\quad 45,891 \mathrm{kN} / \mathrm{kg}$
(19,730 BTU/lb)
-Volumetric Efficiency (assumed constant): $80 \%$

### 8.3 Instrumentation

- Barometer
- Sling Psychrometer
- Thermometers at:
- air inlet
- exhaust outlet
- water inlet
- water outlet
- engine temperature
- Hand Tachometer for rpm
- Load Balance for Prony Brake
- Cooling Water Flow Meter and Balance Tanks (flow water is calibrated with balance tanks initially)
- Cylinder Pressure Transducer and piston displacement together with Oscilloscope to obtain $p-V$ diagrams. Camera to fix $p-V$ diagram and Bit Pad to get area of $p-V$ diagram.
- Fuel Meter with Stop Watch


### 8.4 Procedure

The engine is started with compressed air. It should be running for some time before the lab starts to reach a steady operating state. The dead, or tare, weight of the prony brake arm is given. It can be checked at the end of the experiment when the engine is not running by rotating the flywheel by hand in one direction then the other. The tare weight is the average of the
two measured forces.

At the beginning and end of the experiment, the following readings should be taken:

- Barometric Pressure
- Wet and Dry Bulb Air Temperature

The experiment proceeds in the following manner:

1. Initially the load on the prony brake will be set at 4.54 kg ( 10 lbs .) at a governed speed somewhere between 350 and 450 rpm . The technician or TA will adjust the friction on the prony brake and fuel flow to maintain constant rpm and load.
2. The fuel flow rate, cooling water flow rate, and engine temperature should be checked over a 5 to 10 minute period to ensure steady state conditions have been reached. Then the following readings should be taken as quickly as possible:

- Brake Load (W)
- rpm (N)
- Temperatures at
(a) engine ( $\mathrm{T}_{\mathrm{e}}$ )
(b) exhaust $\left(\mathrm{T}_{\mathrm{g}}\right)$
(c) cooling water in $\left(\mathrm{T}_{\mathbf{w}_{i}}\right)$
(d) cooling water out ( $\mathrm{T}_{\mathbf{w}_{o}}$ )
- Water Flow Rate ( $\dot{m}_{w}$ )
- Fuel Flow Rate ( $\dot{m}_{f}$ )
- $p-V$ diagram picture

3. The technician or TA will then adjust the brake load, and fuel flow if necessary, to obtain a new operating condition, and step 2 will then be repeated for three more loads to obtain four approximate load conditions at constant rpm:
(a) $\mathrm{W}=4.54 \mathrm{~kg}(10 \mathrm{lbs}$.
(b) $W=9.07 \mathrm{~kg}(20 \mathrm{lbs}$.
(c) $\mathrm{W}=13.61 \mathrm{~kg}(30 \mathrm{lbs}$.$) .$
(d) $\mathrm{W}=18.14 \mathrm{~kg}(40 \mathrm{lbs}$.

### 8.5 Results

It is most convenient to record the primary data on an EXCEL spreadsheet and do the following calculations (in British or Metric units):

1. Brake Power ( kw or hp )
2. Indicated Power ( kw or hp )
3. Air to Fuel Ratio
4. Specific Fuel Consumption (kg/BHP or lbs./BHP)
5. Mechanical Efficiency
6. Overall Thermal Efficiency
7. Ideal Efficiency
8. Heat Balance ( $k J$ or BTU)

Finally, the following graphs should be plotted:

1. Air to Fuel Ratio vs. Brake Power
2. Specific Fuel Consumption vs. Brake Power
3. Efficiencies 5, 6, and 7 (above) vs. Brake Power
4. Heat Balance vs. Brake Power

### 8.6 Theory

### 8.6.1 Brake Power

Brake Power $=$ power delivered to output shaft $\left(\frac{\mathrm{N}-\mathrm{m}}{\mathrm{s}}\right)$


Fig. 8.1

$$
\mathrm{BP}=\text { Torque } \times \text { Radians } / \text { sec. }=\mathrm{W} \times 2 \pi \ell \times \mathrm{N}
$$

Note: $1000 \mathrm{~N}-\mathrm{m} / \mathrm{s}=1 \mathrm{~kW}$
$550 \mathrm{ft}-\mathrm{lb}_{f}=1 \mathrm{hp}$

### 8.6.2 Indicated Power

Indicated Power $=$ power developed in the cylinder on the piston ( $\frac{\mathrm{N}-\mathrm{m}}{\mathrm{s}}$ )
Note: Friction Loss = Indicated Power - Brake Power


Fig. 8.2

Force $=$ pressure $\times$ area $=P \times A$

$$
\mathrm{IP}=\text { Force } \times \frac{\text { stroke }}{\text { sec }}=\mathrm{P} \ell \mathrm{AN}^{\prime}
$$

where

$$
\begin{aligned}
\mathrm{P}= & \text { mean effecive pressure } \\
& \text { (average pressure in cylinder) } \\
\ell= & \text { stroke of piston } \\
\mathrm{A}= & \pi(\text { bore })^{2} / 4 \\
\mathrm{~N}^{\prime}= & \text { number of powerstrokes } / \text { sec } \\
& (1 / \text { revolution for } 2 \text { stroke } \\
& 1 / 2 / \text { revolution for } 4 \text { stroke) }
\end{aligned}
$$

The mean effective pressure is obtained from the $p-V$ diagram.

$$
\operatorname{MEP}\left(\frac{\mathrm{N}}{\mathrm{~m}^{2}} \text { or } \mathrm{psi}\right)=\frac{\text { Area }}{\text { length }} \times \text { Calibration Factor }
$$



Fig. 8.3

### 8.6.3 Air to Fuel Ratio (A/F)

$$
A / F=\frac{\text { actual mass flow rate of air }}{\text { mass flow rate of fuel }}
$$

The actual mass flow rate of air through the engine is not measured directly. It is obtained by assuming a constant volumetric efficiency of $80 \%$.

$$
\eta_{\mathrm{vol}}=0.8 \equiv \frac{\text { actual mass flow rate of air }}{\text { theoretical flow rate of air }}
$$

where the theoretical mass flow rate of air is the density of air times the volume drawn into the cylinder by the piston per unit time. Here the density of air drawn into the cylinder is assumed to be constant at the inlet room barometric pressure and temperature. An ideal gas with negligible water vapor is assumed and

Actual Mass Flow of Air $\left(\mathrm{kg} / \mathrm{s}\right.$ or $\left.\mathrm{lb}_{\mathrm{m}} / \mathrm{s}\right)=0.8\left(\frac{\mathrm{P}}{\mathrm{RT}}\right)\left[\right.$ stroke $\left.\times \frac{\pi}{4}(\text { bore })^{2}\right]\left(\frac{\mathrm{N}^{\prime}}{2}\right)$
where $\mathrm{R}=53.3 \frac{\mathrm{ftlb}}{\mathrm{lb}_{\mathrm{f}}{ }^{\circ} \mathrm{R}}$ or $286 \frac{\mathrm{~J}}{\mathrm{~kg} \mathrm{~K}}$. Also note that $1 \mathrm{~J}=1 \mathrm{Nm}$. The Mass Flow Rate of Fuel is obtained directly by measuring the time it takes to consume a given weight of fuel.

### 8.6.4 Specific Fuel Consumption (SFC)

$$
\mathrm{SFC}=\frac{\text { rate of fuel consumption }}{\text { output power }}=\frac{\dot{m}_{f}}{\mathrm{BP}}
$$

The units for SFC are $\frac{\mathrm{kg}}{\mathrm{kW} \mathrm{hr}}$ or $\frac{\mathrm{lb}}{\mathrm{hp} \mathrm{hr}}$.

### 8.6.5 Mechanical, Overall, and Ideal Efficiency

Mechanical Efficiency

$$
\eta_{m}=\frac{B P}{I P}
$$

## Overall Thermal Efficiency

$$
\eta_{0}=\frac{\text { Output }}{\text { Input }}=\frac{\mathrm{BP}}{\dot{m}_{f} \times \mathrm{HHV}}
$$

## Ideal Efficiency

The ideal diesel cycle has the following processes shown in Figure 8.4.
Referring to Figure 8.4, the ideal efficiency is

$$
\begin{equation*}
\eta_{\mathrm{id}}=\frac{Q_{\mathrm{H}}-Q_{\mathrm{L}}}{Q_{\mathrm{H}}}=1-\frac{c_{V}\left(\mathrm{~T}_{4}-\mathrm{T}_{1}\right)}{c_{\mathrm{P}}\left(\mathrm{~T}_{3}-\mathrm{T}_{2}\right)} . \tag{8.1}
\end{equation*}
$$



Fig. 8.4
Under isentropic conditions ( $s=$ constant)

$$
p V^{\gamma}=\text { constant }
$$

where $\gamma=\frac{c_{p}}{c_{V}}=1.4$ and

$$
\frac{\mathrm{T}_{2}}{\mathrm{~T}_{1}}=\left(\frac{V_{1}}{V_{2}}\right)^{\gamma-1}
$$

and

$$
\frac{\mathrm{T}_{3}}{\mathrm{~T}_{4}}=\left(\frac{V_{4}}{V_{3}}\right)^{\gamma-1}
$$

where $p V=$ RT. Substitute the above into 8.1 to obtain

$$
\eta_{\mathrm{id}}=1-\frac{r^{1-\gamma}\left(r_{c}^{\gamma}-1\right)}{\gamma\left(r_{c}-1\right)}
$$

where $r=\frac{V_{1}}{V_{2}}$ (the compression ratio) and $V_{2}=$ clearance volume and $\left(V_{1}-V_{2}\right)=$ piston displacement. Also $r_{c}=\frac{V_{3}}{V_{2}}$ (the cut-off ratio) and $V_{3}$ is estimated from the $p-V$ diagram.

### 8.6.6 Heat Balance

The Heat Balance is an accounting of where the total heat from the fuel goes.

1. Energy supplied by the fuel

$$
\mathrm{E}_{f}=\dot{m}_{f} \times \mathrm{HHV}
$$

2. Useful energy produced by the engine

$$
\mathrm{E}_{\mathrm{out}}=\mathrm{BHP}
$$

3. Energy lost by the exhaust gases

$$
E_{\text {exhaust }}=\dot{m}_{\mathrm{a}} c_{p}\left(\mathrm{~T}_{\text {exhaust }}-\mathrm{T}_{\text {room }}\right)
$$

where $c_{p}=1.03 \mathrm{~kJ} / \mathrm{kg}-\mathrm{K}=0.25 \mathrm{BTU} / \mathrm{lb}_{m}-{ }^{\circ} \mathrm{R}$
4. Energy lost to cooling water

$$
\mathrm{E}_{\text {water }}=\dot{m}_{w} c_{p_{w}}\left(\mathrm{~T}_{w_{\text {out }}}-\mathrm{T}_{w_{i n}}\right)
$$

where $c_{p_{w}}=4.18 \mathrm{~kJ} / \mathrm{kg}-\mathrm{K}=1.00 \mathrm{BTU} / \mathrm{lb}_{\mathrm{m}}{ }^{-}{ }^{\circ} \mathrm{R}$
5. Energy lost to friction

$$
\mathrm{E}_{\text {friction }}=\mathrm{IHP}-\mathrm{BHP}
$$

6. Energy losses unaccounted for.

$$
f)=a)-[b)+c)+d)+e)]
$$

## T2 - Two Stage Piston Air Compressor

## Objective

This lab will measure the operating characteristics of a two-stage compressor as the output supply pressure is varied. Measurements of the power input and various states of air throughout the compressor cycle will permit the efficiency of the device to be computed.

## Apparatus



1. Trolley
2. Drive Motor
3. Acoustic attenuator
4. Pressure vessel 2nd stage
5. Pressure transducer
6. Manometer
7. Safety valve
8. Regulating valve
9. Inlet pressure vessel
10. Nozzle for volume flow measurement
11. Differential pressure transducer
12. Switch Cabinet with Digital Displays
13. Resistance thermometer
14. Pressure vessel and intermediate cooler for 1st stage
15. First Compressor Stage
16. 2-Stage piston compressor
17. Second Compressor Stage

The following is a process diagram for the compressor unit:


PI indicates a pressure reading, TI indicates a temperature reading. If there is a number next to the reading, its value will appear on the digital display with the corresponding number. PD indicates a pressure difference measurement.

Ambient air is drawn into Tank 1 through a measuring nozzle that permits the volumetric flow rate of air through the device to be calculated. Tank 1 acts as a settling chamber that is very near to atmospheric. The first stage compressor brings the gas to $\mathrm{T}_{2}, \mathrm{p}_{2}$. The gas is then fed to an intercooler at constant pressure, which lowers the temperature to $\mathrm{T}_{3}$. The gas then goes to the second stage compressor, exiting at $\mathrm{T}_{4}$ and $\mathrm{p}_{4}$. The gas exiting the second stage goes though another heat-rejecting cooler and then into the compressed air Tank 2. The pressure of Tank 2 is set by a throttling valve that is controlled by the operator. The air is exhausted through a sound absorber. Safety valves prevent the compressor from over-pressurizing any component in the system.

## Instrumentation

## Pressure:

The pressures are recorded by pressure transducers. As a check, there are also some manometers on the pressure vessels. The digital displays indicate the absolute pressure at the measuring points in bar. The manometers on the tanks indicate the over-pressure at the measuring points in bar.
The differential pressure at the inlet is recorded by a differential pressure transducer connected to a Venturi nozzle.

Temperature:
Four resistance thermometers (Pt100) with transducers measure the temperature; this is indicated on the digital displays, in degrees Celsius.

Rating:
The effective power of the compressor motor is measured using a transducer and is indicated on digital displays, in kilowatts.

Here is a layout of the switch cabinet with displays and controls:


1. $\mathrm{p}_{1}$ - Inlet pressure
2. $\mathrm{T}_{1}$ - Inlet temperature
3. $\mathrm{p}_{2}$ - Pressure after 1st compressor stage
4. $\mathrm{T}_{2}$ - Temperature after 1st compressor stage
5. $\mathrm{p}_{4}$ - Pressure vessel pressure
6. $\mathrm{T}_{3}-$ Temperature before $2^{\text {nd }}$ compressor stage
7. dp - Differential pressure across Venturi nozzle
8. $\mathrm{T}_{4}$ - Temperature after 2 nd compressor stage
9. Emergency stop switch
10. Master switch
11. Electric motor switch
12. Electrical output

## Experimental Procedure

1. Turn on the compressor.
2. Allow compressor to run for several minutes until the pressure in Tank $2\left(p_{4}\right)$ has reached a steady value.
3. Using the throttle valve, regulate the pressure $\mathrm{p}_{4}$ to the desired value (between 6 and 12 bar).
4. Allow the compressor to run for 3 to 5 minutes, such that the pressure $\mathrm{p}_{4}$ has reached a steady value, before recording.
5. Repeat steps 3 and 4 for a total of five runs.
6. Turn off the compressor.

Pressures and temperatures, along with the electric power supplied to the compressor, should be read from the display panel (shown above) and recorded in a table as follows:

| $\begin{aligned} & \text { p1 } \\ & \text { (bar) } \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { T1 } \\ & \left({ }^{\circ} \mathrm{C}\right) \\ & \hline \hline \end{aligned}$ | $\begin{aligned} & \text { p2 } \\ & \text { (bar) } \end{aligned}$ | $\begin{aligned} & \mathrm{T} 2 \\ & \left({ }^{\circ} \mathrm{C}\right) \\ & \hline \hline \end{aligned}$ | $\begin{aligned} & \mathrm{T} 3 \\ & \left({ }^{\circ} \mathrm{C}\right) \\ & \hline \hline \end{aligned}$ | p4 (bar) | $\begin{aligned} & \text { T4 } \\ & \left({ }^{\circ} \mathrm{C}\right) \\ & \hline \hline \end{aligned}$ | $\Delta p$ (mbar) | $\begin{aligned} & \hline \mathrm{P} \\ & (\mathrm{~kW}) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |

## Results

1- Plot the P-V diagrams for all five runs.
2- Calculate the polytropic compression indices for both the low and high pressure cylinders. Validate your results by fitting a power function to your data for the first and second compression stages (i.e. $p V^{\gamma}=$ constant)
3- Calculate the efficiency of the compressor for all five runs. Comment.

## Theory

## 1 - Introduction

Air compressors are widely used for gas transportation/distribution and gas processing (separation, refrigeration, etc.). Compressed air is also used as a mechanical power source (pneumatic drills, wrenches, etc.). Where there is a risk of explosion due to flammable gases, for example, in mining or in the chemical industry, compressed air is used instead of electrical energy. Reciprocating compressors use a piston-in-cylinder arrangement and are capable of very high final pressures, especially if used in multi-stage operation.

## 2 - Function

Compressors, through the input of energy from prime movers, such as electric motors and combustion engines, pump compressible fluids from regions of lower pressure to regions of higher pressure. This compression not only raises the pressure but also the temperature of the fluid; this heat of compression is partly dissipated by cooling ribs on the outside cylinder wall. The following figure shows the basic layout of a piston compressor.


1. Cylinder cover
2. Pressure valve
3. Intake valve
4. Cylinder
5. Piston
6. Piston pin
7. Connecting rod
8. Crankshaft
9. Crankcase
10. Oil sump

The processes in the compressor can best be explained by examining the sequence of steps the piston goes through alongside the corresponding p-v diagram. Note that the following $\mathrm{p}-\mathrm{v}$ diagrams are rotated by $90^{\circ}$ to the right so that they correspond to the piston stroke.


## Compression

The piston starts from point 1 , bottom dead centre (BDC), and moves up, reducing the volume and increasing the pressure and temperature of the air in the closed cylinder.

## Expulsion

Compression causes the pressure in the cylinder to increase until point 2, where the exhaust valve opens as the pressure equals or slightly exceeds the pressure set downstream of the compressor. The piston continues its motion, discharging air at constant pressure $\mathrm{p}_{2}$ and temperature $\mathrm{T}_{2}$ until it reaches top dead center (point 3)

## Return expansion

After point 3, the piston begins to expand the air remaining in the cylinder and the exhaust valve closes. The pressure drops.

## 3 - Two-Stage Compression

As pressure ratios during compression increase, the maximum temperatures during compression increase as well, causing problems with maintaining lubrication and valve damage. To reach higher degrees of compression, multistage compression with intercooling is used. Intercooling rejects heat from the compressed air after the first stage compressor so that excessive temperatures are not generated in the second stage of compression. The cooler air also has greater density (occupies less specific volume), so
the piston in the second stage does not need to perform as much work as if the gas were directly compressed from the initial to the final pressure. In the following p-v diagram, the gas is isentropically compressed from 1 to 2 , allowed to cool between 2 to 1 II, and is isentropically compressed from 1II to 2II. For single stage compression, the gas would be isentropically compressed from 1 to $2^{\prime}$. The area between the two curves is the work saved by using a two stage cycle.


## 4 - Intake volume

The volumetric flow rate through the system is given by the pressure differential across the venture nozzle:

$$
\dot{V}=A_{d} \cdot \sqrt{\frac{2 \cdot \Delta p}{\rho}}
$$

with $\dot{V}$ in $\mathrm{m}^{3} / \mathrm{s}, \Delta p$ in $\mathrm{Pa}, \rho$ in $\mathrm{kg} / \mathrm{m}^{3}$ and $\mathrm{A}_{\mathrm{d}}=1131 \cdot 10^{-4} \mathrm{~m}^{2}$

The density $\rho$ of the air depends on the temperature and pressure:

$$
\rho=\frac{100 \cdot p_{0}}{287 \cdot(T+273)}
$$

with $\rho$ in $\mathrm{kg} / \mathrm{m}^{3}, \mathrm{p}_{0}$ in mbar and T in ${ }^{\circ} \mathrm{C}$.

## 5 - Polytropic index of Compression

The compression process in both the first and second stage compressors can be described by a polytropic relation $p V^{n}=$ constant, where $n$ is given by

$$
n=\frac{\ln \left(\frac{p_{\text {in }}}{p_{\text {out }}}\right)}{\ln \left(\frac{V_{\text {out }}}{V_{\text {in }}}\right)}=\frac{\ln \left(\frac{p_{\text {in }}}{p_{\text {out }}}\right)}{\ln \left(\frac{T_{\text {out }}}{T_{\text {in }}}\right)+\ln \left(\frac{p_{\text {in }}}{p_{\text {out }}}\right)}
$$

for both the low and high pressure cylinders. For an isentropic compression, the polytropic exponent should equal the isentropic coefficient ( $k=1.4$ for air).

## 6 - Calculation of the efficiency

For a compressor that delivers air at the same temperature as the inlet, the theoretical amount of power required to compress the air is called the isothermal power. This is given by:

$$
\dot{W}_{\text {iso }}=\dot{m} R T \cdot \ln \left(\frac{p_{4}}{p_{1}}\right)=p_{1} \dot{V}_{1} \cdot \ln \left(\frac{p_{4}}{p_{1}}\right)
$$

with p in Pa and $\dot{V}$ in $\mathrm{m}^{3} / \mathrm{s}$
We can then define the isothermal efficiency as:

$$
\eta=\frac{P_{\text {hydr }}}{P_{\text {electr }}}
$$

## Chapter 10

## T3-

## Small Gas Turbine Demonstration Unit

### 10.1. Introduction

Using the ET792 Gas Turbine Demonstration Unit, the function and behaviour of a scale model gas turbine can be demonstrated and studied. Gas turbine plant are used to generate mechanical and electrical energy:

- Driving electricity generators in power stations
- Driving compressors and pumps in oil and gas production
- Driving ships, locomotives and heavy vehicles
- For the propulsion of aircraft with propeller and jet engines
Gas turbines are always used if high power density, low weight and quick starting are required. Contrary to piston engines, as fluid flow machines they permit high material flow rates in small dimensions. In this way light and, at the same time, powerful drives can be realised.
As the moving parts of a gas turbine only perform rotary motion, almost vibration free running can be achieved if the turbine is well balanced. Disadvan-
3 tages are the high gas speeds and high noise emissions due to the simultaneous connection to the atmosphere.
In comparison to steam turbines, gas turbines work at higher temperatures but with lower pressures. The high temperatures particularly in the area of the turbine require particularly heat resistant materials.

The demonstration unit is a two shaft system with radial compressors and turbines. The power turbine drives a generator for generating electrical power. In addition, the system can be used to demonstrate a single shaft jet engine. All components necessary for the operation of the system are combined in a compact arrangement on a trolley on castors. Operation with propane gas ensures clean, odour-less operation and good exhaust gas quality.
The system is of straightforward construction and is specially designed for training purposes. The control and display of all important process parameters is combined on a control panel..A clearly laid out process schema on the control panel eases the allocation of the measured values and assists in the performance of the experiment.
An optionally available PC data acquisition card with analysis software facilitates the on-line logging of all relevant process parameters and their graphic display.
Apart from the pure demonstration of the operating behaviour, it is also possible to perform qualitative investigations, such as the determination of the electrical power output, the spec. gas consumption or the system efficiency.


Fig. 1.1 Large Indusirial Gas Turbine . Plant with 56MW Power Ratin!


Fig. 12

Jet Engine for the Propulsion of an Aircratt


Fig. 1.3 2-Shaft Vehicle Turbine with 200kW Power Rating

## 10. 2. System Description

The demonstration unit contains a complete gas turbine system with the following subsystems:

- Gas generator consisting of a compressor (1), turbine (2), combustion chamber (3) and intake system with muffler (4)
- Power turbine (5) with exhaust muffler (6) and belt drive to the generator (it is also possible to fit a steel pipe with thrust nozzle instead of the power turbine).
- Fuel system consisting of a main valve (7), rapid action stop valve (8), pressure regulator (9), control valve (10) and burner nozzle
- Ignition system with ignition plug and ignition transformer
- Lubricating system consisting of a tank (11), oil pump (12), oil filter (13); pressure regulator (14) and thermostatically regulated oil cooler (15)
- Generator (16) with converter, ballast resistors and power indicator
- Starter system with starting fan (17) and chan-ge-over damper (18)
- Measuring instruments and controls with temperature, flow rate, speed and pressure measuring points and associated displays. These also include the safety components such as temperature and pressure limiters, oil pressure and oil temperature monitoring.


Fig. 2.1 Process Schema of the Gas Turbine Demonstration Unit

Function
10.2.1. Gas Generator

The core of the system is formed by the gas generator. This consists of a radial turbine with directly coupled radial compressor and a combustion chamber. The turbine and compressor, together with the bearing housing in-between, form a compact unit. This unit is normally used as a turbocharger on turbocharged engines.
The air drawn in is injected into the light alloy spiral housing by the compressor wheel rotating at high speed (1) (70000-130000 rpm). Here the speed of the air is converted into pressure. The compressed air is further slowed down in the diffuser (2) and then fed to the combustion chamber (3). At the inlet to the combustion chamber, part of the air is drawn off and fed to the front of the combustion chamber pipe (4). This primary air is used as combustion air for the fuel. A turbulence generator (5) sets the air in rotation and slows it down such that fuel (propane) injected in gaseous form via the combustion nozzle (6) can burn with a stable flame. The combustion pipe is cooled from outside


Fig. 2.4 Exhaust Gas Turbocharger


Fig. 2.2 Function Schema of a Gas Turbine


Fig. 2.3 Gas Generator .
by the secondary air. This is fed to the combustion chamber via holes (7) to cool the very hot combustion gases (approx. $2000^{\circ} \mathrm{C}$ ) to the permitted turbine inlet temperature of $600-800^{\circ} \mathrm{C}$. An ignition plug ( 8 ) is used to ignite the air-fuel mixture on starting.
From the combustion chamber, the combustion gases flow into the spiral housing on the turbine and are accelerated for entry into the radial wheel (9). In the turbine, the gases give up their energy to the wheel to drive the compressor. During this process, the gases are expanded and cooled to a large degree. They leave turbine at around $600^{\circ} \mathrm{C}$
and can either be fed to the power turbine that follows, or to the thrust nozzle.
The turbine and compressor wheels are fitted to a common shaft (10) such that they are overhanging. The shaft is mounted in the bearing housing in plain bearings. Due to the very high speeds, the bearings have a floating intermediate sleeve. The lubricating oil for the pressure lubrication is also used as a coolant for the bearings that are subjected to high temperatures.

[^0]
### 10.2.20 Jet Pipe and Thrust Nozìle

In operation as a jet engine, the exhaust gases are expanded in the thrust nozzle (1) and, at the same time, accelerated. The gases exit freely from the jet nozzle. As the gas generator is mounted elastically on leaf springs (2), the thrust can be measured using an electrical force sensor (3).


Fig. 2.5 Thrust Measurement

## 1c. 2.3 - Fuel System

Propane is used for the fuel. This has the advantage that in the case of ignition failure, no unburned fuel can collect in the system.
In addition, the low delivery pressure to the combustion chamber provides an effective self-regulating mechanism for the speed. With increasing speed, the pressure in the combustion chamber also increases. The lower pressure difference to the gas nozzie pressure automatically reduces the fuel feed. In this way the hazard normally present with gas turbines does not exist, and it is not necessary to use a complex speed regulator. The main gas valve is fitted at the inlet of the gas to the system. After the main gas valve, the supply pressure is indicated. The rapid action stop valve shuts down the system immediately in the event of hazardous operating states. The nozzle pressure is adjusted using the pressure reducing valve. This is also indicated. The gas flow is indicated on a variable-area flowmeter. The regulator valve for the system power output then follows. From here the gas flows to the combustion chamber and is injected using a four-hole nozzie.

### 10.2.4 - Lubricating System

The turbines place high requirements on the supply of lubricating oil. The oil collected in the lubricating oil tank is pumped through a primary oil filter and an oil cooler to the turbines by an electrically driven gear pump. A pressure limiting valve limits the oil pressure to max. 4 bar. An oil pressure switch interrupts the supply of gas for combustion as soon as the oil pressure drops below 2 bar. The oil temperature in the tank is indicated. The feed of gas for combustion is also interrupted at more than $100^{\circ} \mathrm{C}$. The oil cooler is thermostatically regulated; the flow rate for the cooling water is regulated as a function of the oil temperature.
The return oil from the turbines runs back to the tank under gravity. Oil vapour produced is diverted to the intake and burnt in the gas turbine.
10.2.5 Starting and Ignition System

The starting system consists of a powerful fan and damper arrangement. On starting, the normal intake is closed using a damper and the connection to the starting fan opened. The damper arrangement is operated using a rotary knob on the front
panel. The starting fan replaces the compressor, which on starting does not yet function, and provides the air necessary for initial combustion. Once the turbine has reached a certain minimum speed, the compressor takes over the function and the starting fan can be switched off.
The ignition system consists of an ignition plug and an ignition transformer for the necessary ignition voltage. The ignition system is operated using a push button. At the same time, the feed of gas for combustion is enabled via the rapid action stop valve. The ignition system must be operated until ignition has occurred in the combustion chamber. This is indicated by increasing turbine inlet temperature. At more than $600^{\circ} \mathrm{C}$, the gas feed remains enabled and the ignition can be switched off.

3 - Physical Layout
All system components are arranged in a steel frame. The frame is closed towards the front by the front panel. The other sides are closed by removale perforated sheets. These ensure good cooling and ventilation together with protection against contact with hot or rotating parts.
The following figure shows the layout of the components with the front panel removed.


1 Intake funnel with measuring orifice
2 Intake muffler
3 Starting fan
4 Change-over damper
5 Gas generator
6 Power turbine
7 Generator
8 Exhaust muffler
9 Exhaust outlet
10 Ignition plug
11 Combustion chamber
12 Gas nozzle
13 Oil tank
14 Oil filter
15 Oil cooler thermostat
16 Cooling water inlet and outlet
17 Braking resistors
18 Belt tensioner
19 Switch box
20 Oil retum pipes
21 Connecting pipe to power turbine
22 Jet pipe and nozzle
23 Leal spring mounting 24 Force transducer

Fig 3.1 Physical Layout of the Gas Turbine System

## 4. Theory

## ic. 4. 1- The Open Gas Turbine Process

The functional gas turbine model uses an open circulatory process in which the working medium is extracted from the surrounding environment and later returned to it.
During this process the working medium, air, is subjected to the following changes of state:

- Adiabatic compression of the cold air, using a compressor (1), from ambient pressure $p_{1}$ to pressure $\mathrm{P}_{2}$ and the associated in increase in temperature from $\mathrm{T}_{1}$ to $\mathrm{T}_{2}$.
- Isobaric heating of the air from $T_{2}$ to $T_{3}$ through the addition of heat. Heat is added by burning fuel with the oxygen from the air in the combustion chamber (2).
- Adiabatic expansion of the hot air in a turbine
(3) from pressure $p_{2}$ to $p_{1}$ with a temperature drop from $\mathrm{T}_{3}$ to $\mathrm{T}_{4}$.

In a closed circulatory process the working medium would need to be isobarically cooled back down to the inlet temperature $\mathrm{T}_{1}$. With an open

## Depiction on the Total Heat-Entropy Diagram

To be able to better evaluate the relationships in the circulatory process, it is useful to depict the process on a total heat-entropy diagram, the socalled $T$, $s$ diagram. On this diagram the temperature of the working medium is plotted against the spec. entropy.

On the T,s diagram, the amounts of heat can be depicted as areas. The useful work is given by the difference between the amount of heat supplied, the area $a, 2,3, b$, and the amount of. heat drawn off, area $4, \mathrm{~b}, \mathrm{a}, 1$.

Using the $T, s$ diagram, questions on the thermal efficiency and capacity of the process to perform work can be investigated. Both the temperature ratios and also the pressure ratio $\pi=\frac{\mathrm{P} 2}{\mathrm{p}_{1}}$ are of significance here.


Fig. 4.1 SimpleOpen Gas Turbine Plant
circulatory process the residual heat is also given off to the surrounding environment.
The mechanical power extracted using the turbine is partly used to drive the compressor, and is partly available as useful power. This power can be utilised, for example, to operate a generator (4).


Fig. 4.2 T,s-Diagram of the Gas Turbine Process

## Thermal Efficiency

The thermal efficiency is given by the relationship of the heat supplied to the mechanical work. Assuming that the working medium has a constant capacity to perform work, the thermal efficiency is given by:

$$
\dot{\eta_{t+n}}=1-\frac{T_{1}}{T_{2}}=1-\frac{1}{\pi^{\frac{k-1}{k}}} .
$$

With an average value for $\kappa=1.4$ this equation

## Specific Capacity to Perform Work

The following relationship applies for the spec. capacity to periorm work:

$$
w_{N}=c_{p} T_{3}\left(1-\frac{1}{\pi^{0.285}}\right)-c_{p} T_{1}\left(\pi^{0.285}-1\right) .
$$

It can be seen that, apart from the pressure ratio, the intake and turbine inlet temperature are of significance. The intake temperature is generally determined by the ambient conditions. The turbine inlet temperature $T_{3}$ should be selected such that it is as high as possible. In practice it is limited by the thermal stability of the turbine blades. Thus the pressure ratio $\pi$ is also in this case the decisive factor.

## Depiction on the $p, v$ Diagram

The circulatory process can also be depicted on a $p, v$ diagram. Here especially the compression and expansion process are clearly depicted.

Mechanical work also results in a closed area here. Contrary to the T,s diagram, in this case the areas depict the mechanical work. It can be seen that on the supply of heat between 2 and 3 , the spec. volume of the gas, that is the density, reduces. The excess useful power of the turbine results from it being able to process a higher volume for the same pressure difference as on the compressor.
yields:

$$
\eta_{\mathrm{th}}=1-\frac{1}{\pi^{0.285}}
$$

It can be seen that the efficiency is only dependent on the pressure ratio $\pi$. The highest temperature in the process, the turbine inlet pressure $T_{3}$ does not have any influence on the thermal efficiency.

The power output of the plant is given by multiplying the specific capacity to perform work with the mass flow rate used.

$$
P_{N}=\dot{m} w_{N}
$$

The relationships given here are applicable to a single shaft plant, they are thus not directly applicable to our gas turbine. However they do give general pointers for understanding the operating behaviour of the plant. For example, the pressure ratio is a function of the square of the speed. Thus the plant is significantly more efficient at high speeds.


Fig. 4.3 p.v Diagram of the Gas Turbine Process

## 4. 2-Snaft Plant



Fig. 4.5 2-Shatt Gas Turbine Plant


P


Turbine Work tor Compressor $\mathbf{v}$


Fig. 4.4 p.v-Diagram of a 2 shatt plant

A 2-shatt plant has two independent turbines. The first turbine (1) only drives the compressor (2). As the entire pressure drop available is not needed for this process, there is enough energy for a second turbine connected in series (3). This second turbine, also termed the power turbine, generates the useful power.
The advantage of this arrangement is that a change in the load or the speed of the power turbine has little or no effect on the compressor, this continues to run at the optimal speed in the best efficiency region. Even the jamming of the power turbine cannot cause damage to the plant. For this reason, 2-shaft plant is preferred for vehicle and ship drives with widely varying load requirements. In addition the speed of the drive can be better matched.
As the first turbine together with the compressor and the combustion chamber only generates the working gas for the power turbine, this group of components is also called the gas generator.
For a 2-shaft plant the p,v diagrams on the left are applicable.
The compressor turbine only needs to raise the power demanded by the compressor. This results in the area wv being the same as wT. As the spec. volume $v$ is larger than on the compressor, the compressor turbine requires a smaller pressure drop to point 4. This does not result in the working gas being expanded all the way to pressure p1. The remaining pressure drop to point 5 can then. be used in the power turbine and converted into the useful work wN.

As approfrizte to their pressure regions, a differentiatier is also made between the turbines in the form of high pressure (HP) and low pressure turbines (LFj).

The air mass flow miL in $\mathrm{kg} / \mathrm{s}$ is calculated as follows:

$$
\ddot{m}_{L}[\mathrm{~kg} / \mathrm{s}]=\frac{P_{0}}{1000} \cdot \frac{T_{0} P_{1}}{P_{.0}\left(T_{1}+273\right)} \cdot \text { display }[/ / \mathrm{s}]
$$

with $\rho_{0}=1.199 \mathrm{~kg} / \mathrm{m}^{3}, T_{0}=293 \mathrm{~K}, \rho_{0}=1.013 \mathrm{bar}$

$$
\dot{m}_{L}[\mathrm{~kg} / \mathrm{s}]=0.347 \cdot \frac{\mathrm{p}_{1}}{\left(T_{1}+273\right)} \cdot \text { display }[1 / \mathrm{s}]
$$

Here $T_{1}$ is the intake temperature in ${ }^{\circ} \mathrm{C}$ and $p_{1}$ the ambient pressure in bar.

The gas mass flow mig in $\mathrm{g} / \mathrm{s}$ is caiculated as follows::

$$
\dot{m}_{G}[g / \mathrm{s}]=\frac{1000}{3600} \cdot \frac{T_{0}\left(P_{D}+1\right)}{P_{0}\left(T_{G} \div 273\right)} \cdot \text { display }[\mathrm{kg} / \mathrm{h}]
$$

with $T_{0}=273 \mathrm{~K}$, and $p_{0}=2.0 \mathrm{bar}$

$$
\dot{m}_{G}[g / \mathrm{s}]=37.91 \cdot \frac{\mathrm{P}_{\mathrm{D}}+1}{T_{\mathrm{G}}+273} \cdot \text { display }[\mathrm{kg} / \mathrm{h}]
$$

Here $T_{G}$ is the gas temperature in ${ }^{\circ} \mathrm{C}$ and $p_{D}$ the nozzle overpressure in bar.

## - Determination of the Operating Point

 of the CompressorThe operating points of a compressor are characterised by air flow rate miL and pressure ratio $\pi$. The air flow rate miL was calculated in Section 6.3.2. The pressure ratio $\pi$ is calculated from the ratio of the intake pressure $p_{1}$ and outlet pressure $\mathrm{p}_{2}$.

$$
\pi=\frac{p_{2}}{p_{1}}
$$

The outlet pressure must be calculated from the turbine inlet pressure and combustion chamber pressure difference:

This yields values for the load points: By plotting the operating points on the compressor map, it can be assessed whether the compressor is optimally designed. Of particular interest are the efficiency and distance from the pump limit.

## - Determination of the Power Turbine Output Power

To calculate the effective power output of the power turinine from the electrical power output, the efficiency of the generator and the belt drive must be taken into account. This is $\eta_{\mathrm{el}}=74 \%$.
For full load it is found

$$
P_{\mathrm{Te}}=\frac{\mathrm{P}_{\mathrm{e} t}}{\eta_{\mathrm{et}}}
$$

In addition, the measured output power must be reduced to sea level and $15^{\circ} \mathrm{C}$ in accordance with ISA.

$$
P_{\text {Tred }}=\sqrt{\frac{288}{T_{1} \div 273}} \frac{1.013}{P_{1}} P_{T e} .
$$

Determination of the Spec.

## Fuel Consumption

The specific fuel consumption is calculated from the amount of fuel supplied and the useful power output. Here a differentiation is made between shaft power turbine with PTred and jet engine with Sred. For the shaft power turbine the following applies:

$$
\mathrm{b}_{\mathrm{e}}=\frac{\dot{\mathrm{m}_{\mathrm{G}}} \cdot 3.6}{\mathrm{P}_{\text {Tred }}}
$$

## - Determination of the Power Efficiency

The power efficiency of the turbine is calculated from the specific fuel consumption and lower calorific value $h_{u}$ of the fuel. For propane the following applies

$$
\mathrm{h}_{\mathrm{u}}=46369 \mathrm{~kJ} / \mathrm{kg}
$$

$$
p_{2}=p_{3}-\left(p_{3}-p_{2}\right)
$$

This yields for the power efficiency
$\therefore \quad \eta_{e}=\frac{3600}{b_{e} \cdot h_{u}}=0.0238$.
It is to be noted that the efficiency of a two-shaft system increases continuously with effective power. The absolute fuel consumption $\dot{m} g$ is independent of the output power of the power turbine at constant gas generator speed.

- Determination of the Air Ratio

The air ratio is given by the quotient of the actual amount of air drawn in and the amount of air necessary for the stoichiometric combustion of the fuel.
The amount of air necessary to burn propane is:

$$
L_{\min }=15.23 \frac{\mathrm{~kg}_{\mathrm{L}}}{\mathrm{~kg}_{G}} .
$$

From which the air ratio at full load is

$$
\lambda=\frac{1}{L_{\min }} \frac{\dot{m}_{L}}{\dot{m}_{G}}
$$

## RESULTS:

1. Do a maximum 6 runs and Minimum 4 runs (for different loads).
2. Fill The Working Sheet (Al)
3. Provide one sample of calculations.
4. Determine the compressor performance (A2).
5. Plot and discuss the relation between the following:

- $\dot{m}_{L}, \dot{m}_{c}$ with $\mathrm{P}_{\mathrm{Tred}}$.
- $\mathbf{b}_{e}$ with $\dot{m}_{G}$
- $\eta_{t h}, W_{n}$ with $\pi$.
- $\eta_{e}$ with PTred .
- $P_{\text {Tred }}$ with $b_{e}$

6. Discuss the performance of the system.

All controls and displays are fitted to the front panel. The function of the individual elements is largely self-explanatory due to printed labels.


## Controls

A. Main gas valve

3 Gas pressure regulator
C Gas control valve
D Rapid shut down button
$E$ Ignition button
F Staring fan switch
6 Change-over damper rotary knob
1 for starting air
H Potentiometer for generator load
I Master switch
$j$ Emergency Stop switch

Displays
K Gas supply pressure (bottle pressure)
2 Gas nozzle pressure
5 Differential pressure, combustion chamicer pi- po
7 Pressure before the turbine pu
9 Pressure before the power turbine pa
II Air flow rate, speed at the inlet
12 Gas flow rate
L Oil pressure indicator lamp
M Ignition indicator lamp
$1, \overline{3}, 4$ Temperatures, air inlet $T_{1}$, gas inlet $T g$ and
after compressor $T_{2}$
6 Temperature, turbine inlet $T_{3}$
B/D Temperatures, power turbine inlet $T_{4}$ and power turbine outlet Ts
17 Oil temperature
13 Speed, gas generator ni
14 Speed, power turbine ni
:5 Power, generator PeI
Is Thrust S

## Chapter 11

## T4- Air Conditioning Plant

### 11.1 Objectives

1. To illustrate the principles of air conditioning.
2. To determine the overall coefficient of performance of a refrigeration cycle.
3. To evaluate energy transfer in a steady flow process that includes heating, cooling, work transfer, and mixing.
4. To understand the psychrometric chart.

Note: A superscript asterisk (*) indicates that this part of the lab is temporarily not functional, or not performed.

### 11.2 Apparatus

Pre-heater (two) -nominally at $110 \mathrm{~V}: 0.5 \mathrm{~kW}$ 1.0 kW
-both extended fin electric heating elements
Steam generator and distributor*:3 stages of 1.5 kW

Cooling Unit (refrigerant R12):
-Compressor speed: 2850 rpm
-swept volume: $\quad 21 \mathrm{~cm}^{3} / \mathrm{rev}$
-power factor: (typically) 0.9
Re -heater -same specifications as the pre-heater

Axial flow fan -variable speed control

Damper

### 11.3 Apparatus Setup

A schematic of the Recirculating Air Conditioning Unit A770 is given in Figure 11.1. The untreated air passes in series through:
(i) an air measuring intake orifice
(ii) a mixing zone (where it may be mixed with recirculated air)
(iii) a pre-heater
(iv) a humidifier supplied with steam from the boiler*
(v) a cooler/dehumidifier with a precipitate water outlet
(vi) a re-heater
(vii) an axial flow fan with variable speed control which changes the mass flow rate of air
(viii) an air measuring duct orifice
(ix) a damper which controls the quantity of air discharged to the atmosphere. (Any air not discharged is recirculated and mixes with untreated air in (ii).)

### 11.4 Instrumentation

- Five Wet- and Dry-Bulb Temperature Sensors

The sensors, located at positions A-E in Figure 11.1, are used in conjunction with the psychrometric chart to determine the condition of the air.

- Two Thin Plate Orifices and Manometers

The combination of these devices is used to obtain pressures used in the mass flow rate calculations.

- Refrigerant Flow Meter and Two Pressure Gauges
- Voltmeters and Ammeters


### 11.5 Procedure

### 11.5.1 Experimental Procedure

1. Before turning on the power supply, turn on the water supply to the boiler and the main water supply (on the wall), and make sure that the drain valve is closed*. Wait until the water level in the gauge glass stabilizes (leave water supply open)*.
2. Rotate the fan speed controller fully clockwise.
3. Switch on the electrical supply (unit main switch).
4. Make sure all wet-bulb sensors are filled with distilled water throughout the whole experiment.
5. When recirculating the air, make sure that the damper is locked at the selected position.
6. Before taking any measurements, make sure that steady-state conditions have been reached. This takes about 10 minutes. You will be asked to run the air conditioning plant at a number of fixed conditions depending on:
(a) the room air condition;
(b) the air flow rate, which depends on the fan speed;
(c) the recirculation, which depends on the damper position;
(d) the heating;
(e) the heating and humidification*;
(f) the cooling;
(g) the cooling and dehumidification;
(h) the cooling, dehumidification, and reheating.
7. Obtain the current and voltage readings by pressing the upper row of switches. No more than one switch should be pressed at any time.
8. Enter the results in the supplied observation sheet (Figure 11.4). Do not forget to change the wet-bulb temperatures from screen values to sling values. Refer to the Theory section for details. It is necessary to subtract and additional $1^{\circ} \mathrm{C}$ from the sling wet-bulb temperature $\mathrm{T}_{8}$.

### 11.5.2 Shutdown Procedure

Before switching the apparatus off, perform the following:

1. move the damper to the zero recirculation position;
2. switch off the pre-heater, boiler ${ }^{*}$, re-heater, and compressor;
3. allow the fan to run at maximum speed for five minutes. Open the boiler drain valve to empty the boiler reservoir*.

### 11.6 Results

1. Draw a control surface around each component used in the experiment and make an energy balance for it by utilizing measured thermal and flow properties, as well as power supplied to it, if any.
2. Discuss the reason(s) for any discrepancy between the two sides of the "energy balance" equation for each of the above "open systems".
3. For the refrigeration plant, the overall coefficient of performance (COP) should be calculated. It is defined as the following:

$$
\text { Overall COP }=\frac{\text { Cooling Rate }}{\text { Power Input }}=\frac{\dot{m}_{\tau}\left(h_{13}-h_{12}\right)}{E I_{c} \cos \phi}
$$

where

$$
\begin{aligned}
& \dot{m}_{\tau}=\text { refrigerent mass flow rate } \\
& \cos \phi=\text { power factor }=0.9
\end{aligned}
$$

4. The effect of the fan should be discussed.
5. For heat loss from the duct, the overall heat transfer coefficient can be taken as $U=7.0 \mathrm{~W} / \mathrm{m}^{2}-\mathrm{K}$.
6. The mass flow rate of the inlet air $\left(\dot{m}_{A}\right)$, the air in the duct $\left(\dot{m}_{B}\right)$, and the recirculated air ( $\dot{m}_{R}$ ) are calculated using

$$
\begin{gathered}
\dot{m}_{A}=0.0757 \sqrt{Z / V_{A}} \\
\dot{m}_{B}=0.0757 \sqrt{Y / V_{B}} \\
\dot{m}_{R}=\dot{m}_{B}-\dot{m}_{A}
\end{gathered}
$$

where
$Z=$ orifice pressure differencial at the intake in $\mathrm{mm} \mathrm{H}_{2} \mathrm{O}$
$Y=$ orifice pressure differential in the duct in $\mathrm{mm} \mathrm{H} \mathrm{H}_{2} \mathrm{O}$
$V=$ specific volume of the air in $\mathrm{m}^{3} / \mathrm{kg}$ (use the psychrometric chart)
$\dot{m}_{R}=$ mass flow rate of the recirculated air.

The theoretical analysis for orifice plates can be found in experiment F1.

### 11.7 Theory

### 11.7.1 Introduction

Fresh air contains about $23 \%$ oxygen and $76 \%$ nitrogen by mass. The remainder is composed of small quantities of other gases and vapors. Of these, the most important is water vapor. Although the water vapor content is usually very small, typically less than $2 \%$, it has a considerable effect on the rate of evaporation and heat transfer from moist surfaces and materials. The vapor content of the atmosphere is loosely referred to as the humidity.

Humidity has several specific definitions. They are the following:
(i) Humidity Ratio or Specific Humidity, $\omega$, which is the ratio

$$
\omega=\frac{\text { mass of water vapor in a given mixture }}{\text { mass of dry air in a given mixure }}
$$

(ii) Percentage Relative Humidity, $\phi$, is the ratio

$$
\phi=\frac{\text { partial pressure of the water vapor in a given mixture } \times 100}{\text { saturation pressure of water vapor at the mixture temperature }}
$$

(iii) Percentage Saturation, $\mu$, is the ratio

$$
\mu=\frac{\text { mass of water vapor in a given atmosphere } \times 100}{\text { mass of water vapor to saturate the atmospere at the same temperature }}
$$

### 11.7.2 The Psychrometric Chart

Several methods may be used to determine the humidity ratio and the relative humidity experimentally. The most general instrument used to measure the humidity ratio is the hygrometer which contains moisture absorbing chemicals. An air sample is introduced into the hygrometer until the water vapor is completely absorbed. The amount of water vapor present in the sample may be determined from the difference in the weight of the chemicals before
and after exposure. Electric transducers composed of resistors that change resistance with the relative humidity may be used to measure the relative humidity. However, for the purposes of this experiment, wet- and dry-bulb temperatures will be used in conjunction with the psychrometric chart to determine the humidities. For a detailed discussion of these properties, refer to your thermodynamics book.

The wet-bulb temperature is read from a wet-bulb thermometer, which is an ordinary thermometer whose temperature sensing tip is enclosed by a wick moistened with water. The dry-bulb temperature is the temperature indicated by an ordinary temperature sensing device. If the surrounding air is not saturated, the water in the wick of the wet-bulb thermometer evaporates and the temperature measured will be less than that of the dry-bulb temperature. The wet-bulb temperature, thus, depends on the rates of heat and mass transfer between the wick and the air.

The psychrometric chart (Figure 11.2) facilitates the determination of a number of properties of the air/vapor mixture. Given any two independent properties, a state point may be marked on the psychrometric chart, and from this the following related properties may be determined:

1. dry-bulb temperature
2. wet-bulb temperature (sling)
3. specific volume
4. specific humidity
5. specific enthalpy
6. percentage saturation.

Several items should be noted when reading the psychrometric chart.

1. The specific enthalpy scale gives the enthalpy of the dry air plus the associated water vapor: $h_{a}+\omega h_{v}$, where $h_{a}$ is the specific enthalpy of the dry air and $h_{v}$ is the specific enthalpy of the water vapor. The mixture enthalpy, then, has units of $\mathrm{kJ} / \mathrm{kg}$ of dry air. Also, the reference
state of the specific enthalpy of the mixture is somewhat peculiar. The specific enthalpy of the dry air, $h_{a}$, is measured relative to $0^{\circ} \mathrm{C}$, not OK as in the air tables. The specific enthalpy of the water vapor, $h_{v}$, is evaluated as the specific enthalpy of the saturated vapor, $h_{g}$, at the dry-bulb temperature.
2. A psychrometric chart is valid only for the pressure stated on the chart.
3. At normal atmospheric conditions, the relative humidity is approximately equal to the percentage saturation.
4. In all cases it is necessary to convert the wet-bulb temperature from a screen temperature to a sling temperature before entering the state point on the psychrometric chart. The sling wet-bulb temperature is indicated by a wet-bulb thermometer placed in an air stream moving at $3.5 \mathrm{~m} / \mathrm{s}$ or more. The term comes from an instrument called a sling psychrometer which contains a dry-bulb thermometer mounted together with a wet-bulb thermometer on a holder. The psychrometer is whirled around in the air inducing air to flow over the wick of the wetbulb thermometer causing the water in the wick to evaporate. Once equilibrium is reached the wet- and dry-bulb temperatures are read. In the air conditioning unit used in this experiment, the air velocity is usually between 1 and $2 \mathrm{~m} / \mathrm{s}$, less than the velocity needed to be considered a sling temperature. A wet-bulb temperature taken in this velocity range is called a screen value. This value will be lower than the sling value, and therefore, as stated above, the wet-bulb temperature must be converted to a sling value before consulting the psychrometric chart. This is accomplished by using Figure 11.3.


Fig. 11.1: Schematic of the apparatus.

$$
96
$$

## $\angle b$




Fig. 11.3: Relationship between "screen" and "sling" wet bulb temperatures.


Fig. 11.4: Observation sheet.

## Part IV

## Vibrations

Vibration analysis and control are increasingly important as technology races forward. Vibrations are caused by many phenomena, such as fluid flow and combustion. The study of vibrations, touches a multitude of disciplines. In fact, vibration analysis is important to physicists and mathematicians, as well as, to engineers. As such, vibration analysis has evolved into a very sophisticated and elegant field where the most current tools for analysis are available, such as chaos and fractal theory.

In the following three chapters, an introduction into the world of experimental vibration analysis is presented. The situations presented are relatively simple; however, this does not diminish their importance. They are extremely pertinent since solutions to more complicated vibration problems are often based on the solution of simple problems.

## Chapter 12

## V1- Rotating Unbalance

### 12.1 Objectives

1. To study the forced vibrational behaviour of a damped mechanical system undergoing oscillatory motion.
2. To determine the natural frequency and damping coefficient of the system by several methods.
3. To construct a tuned dynamic vibration absorber and examine its effect on the system.

### 12.2 Apparatus

Steel beam - Dimensions:

| length - | 0.8130 m |
| :--- | :--- |
| width - | 0.0254 m |
| depth - | 0.0127 m |
| density - | $7833 \mathrm{~kg} / \mathrm{m}^{3}$ |
| Youngs modulus - | $207 \times 10^{9} \mathrm{~Pa}$ |

Steel disks (two) - Dimensions: thickness 6.35 mm

- Unbalanced holes: diameter (a) 19.0 mm
diameter (b) 15.0 mm
eccentricity 38.1 mm
102

| Steel Spring - | - Constant: | $3400 \mathrm{~N} / \mathrm{m}$ |
| :--- | :--- | :--- |
|  | - Mass: | 0.382 Kg |

Viscous Damper -Oil filled
-Two modes: low damping ( $\delta=0.13$ ) - aligned holes
high damping ( $\delta=0.45$ ) - holes non aligned

Motor - Total weight: $\quad 5.3 \mathrm{~kg}$
(Electric motor, disks, damper and spring support)

- Additional Weight: 4.09 Kg ( 9 lbs )


### 12.3 Apparatus Setup

The mechanical system of this experiment consists of a flexible, steel beam pin-jointed at both ends to a rigid frame, as in Figure 12.1. Supported at the midpoint of the beam is the electric motor which rotates the two unbalanced disks in the longitudinal plane of the beam. The force exerted by the unbalanced rotation of the disks causes the beam to vibrate. Also attached to the beam at the same position as the motor is the mechanical damper. The damper consists of two sliding metal plates which vibrate in a tube of oil. The plates have holes that can be aligned to adjust the level of damping, as in Figure 12.2.

Finally, the spring provided may be attached to the beam at the same position as the damper. By adding weights to the bottom of the spring a tuned dynamic vibration absorber may be constructed.

### 12.4 Instrumentation

## - Photonic Sensor

A photonic sensor is a device used to measure amplitude of oscillation. By emitting a ray of light onto the beam and measuring its reflected intensity, the amplitude of oscillation can be determined. The intensity
of light is related to the distance between the beam and the sensor, as seen in Figure 12.3. The reflected light received by the sensor is converted into an analogue voltage signal. By sending this signal to an oscilloscope, a trace is produced which is related to the displacement of the beam.

It is important to ensure the sensor head does not touch the beam; thus the variation obtained for the calibration curve will be similar to that to the right of the cut-off line in Figure 12.3. The photonic sensor is a non-linear device that behaves linearly in certain regions(CD and EF in Figure 12.4). The sensor should be operated in region CD. However, region EF should be used when the excitation frequency $\omega \simeq \omega_{n}$ for the low damping tests.

## - Stroboscope

In addition to measuring the amplitude of the beam vibration, it is also important to measure the phase difference between the input force and the beam displacement. By shining a strobe light (triggered to flash at the same frequency as the electric motor) at a moving pointer on the unbalanced disks, the phase angle can be read on a stationary protractor. The triggering device for the stroboscope is an electric circuit with a set of contact points; one of these points is fixed to the oscillating beam, and the second to the micrometer screw. If the micrometer is adjusted so that the points just close at the maximum displacement of the beam, the stroboscope will flash at the same instant as this displacement.

### 12.5 Procedure

### 12.5.1 Preliminary Calculations

1. Calculate equivalent stiffness of the beam.
2. Calculate equivalent mass of the beam.
3. Calculate theoretical natureal frequency of the system.
4. Calculate the non-dimensional damping ratio $\zeta$ for low \& high damping.
5. Prepare a spreadsheet on Excel 4.0 to plot the theoretical amplitude, $\chi$, given by Equation 12.21, and the phase angle $\phi$ (Equation 12.22), for both low and high damping.
6. Calculate the mass of the viberation absorber.

### 12.5.2 Preliminary Measurements

1. Calibrate the photonic sensor and find a linear range to perform the experiment. To do this:
a) turn off the system
b) set screw vernier as close as possible to the reflecting surface (without touching).
c) obtain static output (DC) on oscilloscope using the cursors (change the voltage scale to get the best accuracy when necessary); do NOT change any other setting on the oscilloscope
d) displace photonic sensor by 0.010 in
e) repeat c) and d) in the range 0-0.25 in
f) plot points to find best linear ranges
g) operate sensor in the middle of appropriate range for the rest of the experiment
h) set oscilloscope to (AC).

### 12.5.3 Experimental Procedure

1. Set damping to low.
${ }^{-}$2. Be certain sensor head does not touch beam at the resonance.
2. Set frequency to 700 rpm on control indicator.
3. Read amplitude from oscilloscope in volts using the cursors (peak to peak).
4. Measure the phase angle with stroboscope; be certain sensor just touches trigger to activate.
5. Increment frequency by 25 rpm on the control indicator.
6. Repeat steps 4 to 6 to the frequency 1000 rpm .
7. Repeat steps 3 to 7 for the high damping case.
8. (a) Calculate absorber mass necessary to tune to experimental resonance frequency.
(b) Construct and attach vibration absorber to setup.
9. Set system to high damping and repeat steps 3 and 4 using the same increments up to the frequency 1100 rpm .
10. Turn off the electric motor and conduct an impact test on the beam:
a) set damping to low
b) set oscilloscope to "single" so waveform can be captured in the memory
c) give beam a sharp blow
d) print the screen using "Screen dump"
e) on the screen using the cursors again, measure directly the period and various peaks to find the damping ratio
f) set damping to high and repeat b) to e)

### 12.6 Results

1. Find the amplitude vs. frequency of the system for both damping levels (plot on the same graph the theoretical and the experimental results and discuss the differences).
2. Plot the phase angle vs. frequency of the system for both damping levels.
3. Find the damped and undamped natural frequencies.
4. Obtain the damping levels of the system.
5. Plot the amplitude vs. frequency of the system with the vibration absorber for high damping (plot the results both with and without the absorber on the same graph).
6. Discuss the errors in both theoretical and experimental results.

### 12.7 Theory

Vibrational analysis is concerned basically with the forces exerted on bodies undergoing oscillatory motion. Any system that possesses mass and elasticity is capable of vibration. Therefore, most engineering systems require some vibrational consideration.

There are essentially two types of vibrations, free and forced.

Free vibration occurs under the influence of the forces inherent to the system itself and when no external forces are present. A system under free vibration will oscillate at one of its natural frequencies, which is a property of the mass and stiffness of that system (this is true if there is no damping, otherwise it will vibrate at the damped natural frequency).

Forced vibration occurs due to external excitation forces. If the excitation is oscillatory, the system will oscillate at the excitation frequency.

To properly analyze a system, the equation of motion must be formulated with respect to a fixed coordinate system. The number of independent coordinates required to describe the motion is called the degrees of freedom of the system. A rigid body has six degrees of freedom (three components of position and three angular orientations). Although a continuous, elastic body will require an infinite number of degrees of freedom, with certain approximations the system can be made dynamically equivalent to one having finite degrees of freedom. Many vibration problems can be treated fairly accurately by reducing the degrees of freedom to one.

Consider now a single degree of freedom system with viscous damping, as shown in Figure 12.5. The general form of the equation of motion is:

$$
\begin{equation*}
m \ddot{x}+F_{d}+k x=F(t) \tag{12.1}
\end{equation*}
$$

where $\quad F(t)$ - excitation force
$F_{d}$-damping force

Viscous damping is expressed as $F_{d}=c \dot{x}$ where $c$ is a constant of proportionality. If the excitation force $F(t)=0$ in Eqn. 12.1, we have the case of free damped vibration, with the equation of motion reducing to:

$$
\begin{equation*}
m \bar{x}+c \dot{x}+k x=0 \tag{12.2}
\end{equation*}
$$

The natural frequency of such a system is found by assuming the damping to be negligible, thus reducing the equation of motion to

$$
\begin{equation*}
m \ddot{x}+k x=0 \tag{12.3}
\end{equation*}
$$

or

$$
\begin{equation*}
\bar{x}+\frac{k}{m} x=0 \tag{12.4}
\end{equation*}
$$

From this equation, the circular natural frequency is defined as

$$
\begin{equation*}
\omega_{n}^{2}=\frac{k}{m} \tag{12.5}
\end{equation*}
$$

and the natural frequency (in cycles/sec or Hz ) is given by

$$
\begin{equation*}
f_{n}=\frac{\omega_{n}}{2 \pi} \tag{12.6}
\end{equation*}
$$

To determine the solution of Eqn. 12.2, we assume $x=e^{s t}$ and substitute this into the original equation. Through algebraic manipulation, the solution will be of the form

$$
\begin{equation*}
x=A e^{\left(s_{1} t\right)}+B e^{\left(s_{2} t\right)} \tag{12.7}
\end{equation*}
$$

where

$$
\begin{equation*}
s_{1,2}=\frac{-c}{2 m}+\sqrt{\left(\frac{c}{2 m}\right)^{2}-\frac{k}{m}} \tag{12.8}
\end{equation*}
$$

where A and B are constants obtained from initial conditions. From this solution it is possible to determine the physical properties of the system.

### 12.7.1 Critical Damping

Critical damping is the limiting case for which oscillatory motion will occur. This is the value of $c$ obtained when the quantity in the square root sign of Eqn. 12.8 vanishes, thus changing the roots $s_{1}$ and $s_{2}$ from complex to real, .implying no oscillatory motion. It is defined as

$$
\begin{equation*}
c_{c}=2 m \omega_{n}=2 \sqrt{k m} \tag{12.9}
\end{equation*}
$$

### 12.7.2 Damping Ratio

It is convenient to express any damping value in terms of the critical damping by the non-dimensional damping ratio defined as

$$
\begin{equation*}
\zeta=\frac{c}{c_{c}} \tag{12.10}
\end{equation*}
$$

### 12.7.3 Oscillatory Motion

When $\zeta<1$, the system is said to be underdamped and oscillatory motion will occur, as in Figure 12.6.

### 12.7.4 Non-Oscillatory Motion

When $\zeta>1$, the system is said to be overdamped, and the motion consists of an exponentially decreasing function, as in Figure 12.7.

### 12.7.5 Critically Damped Motion

When $\zeta=1$, the system is said to be critically damped. Depending on the initial velocity of the system, one of the responses seen in Figure 12.8 will be obtained.

### 12.7.6 Logarithmic Decrement

By measuring the decay of successive amplitudes, the amount of damping in a system can be found. Referring to Figure 12.9, logarithmic decrement is defined as

$$
\begin{equation*}
\delta=\ln \frac{\chi_{1}}{\chi_{2}}=\frac{2 \pi \zeta}{\sqrt{1-\zeta^{2}}} \tag{12.11}
\end{equation*}
$$

### 12.7.7 Theoretical Envelope

The theoretical envelope corresponds to the curve that contains all maxima of another curve. For example, in Figure 12.9, it includes the different points $x_{1}, x_{2}$, etc... Recalling the solution of free vibrations, it is easy to prove that the theoretical envelope is given by

$$
\begin{equation*}
x=x_{1} \exp \left(-\zeta \omega_{n} t\right) \tag{12.12}
\end{equation*}
$$

where $x_{1}$ corresponds to the first maximum (see Figure 12.9).

### 12.7.8 Forced Vibration

Harmonic vibration is often encountered in engineering systems. Consider the system of Figure 12.5 under the influence of a harmonic force $F=F_{0} \sin \omega t$, as in Figure 12.10. The equation of motion for this system can be found to be

$$
\begin{equation*}
m \ddot{x}+c \dot{x}+k x=F_{0} \sin \omega t \tag{12.13}
\end{equation*}
$$

The solution to this equation is the superposition of the free and forced responses. The free response is of the form of Eqn. 12.7 with the real part of $s_{1,2}$ being negative, so that it will die out when $t$ increases. The forced

$$
110
$$

response will have a steady state oscillation of the same frequency as that of the excitation force. This forced response will be of the form

$$
\begin{equation*}
x=\chi \sin (\omega t-\phi) \tag{12.14}
\end{equation*}
$$

where
$\chi$-amplitude of oscillation
$\phi$ - phase angle(lag) of the displacement with respect to the excitation force.

By substitution of Eqn. 12.14 into Eqn. 12.13, and after some algebraic manipulation, these values can be found to be

$$
\begin{gather*}
\chi=\frac{F_{0}}{\left[\left(k-m \omega^{2}\right)^{2}+(c \omega)^{2}\right]^{1 / 2}}  \tag{12.15}\\
\phi=\tan ^{-1}\left[\frac{c \omega}{k-m \omega^{2}}\right] \tag{12.16}
\end{gather*}
$$

By introducing the physical properties of Eqns. 12.5, 12.9, and 12.10, the non-dimensional forms of amplitude and phase can be determined:

$$
\begin{gather*}
\frac{k \chi}{F_{0}}=\left[\frac{1}{\left(1-\left(\frac{\omega}{\omega_{n}}\right)^{2}\right)^{2}+\left(2 \zeta\left(\frac{\omega}{\omega_{n}}\right)\right)^{2}}\right]^{1 / 2}  \tag{12.17}\\
\phi=\tan ^{-1}\left[\frac{2 \zeta\left(\frac{\omega}{\omega_{n}}\right)}{1-\left(\frac{\omega}{\omega_{n}}\right)^{2}}\right] \tag{12.18}
\end{gather*}
$$

### 12.7.9 Rotating Unbalance

A common source of harmonic vibration is the unbalance of rotating machinery. If the center of gravity of a rotating mass is not located at the geometric axis of rotation, an unbalanced force is created. This force is

$$
\begin{equation*}
F=m e \omega^{2} \sin \omega t \tag{12.19}
\end{equation*}
$$

This, once again, applies to a single degree of freedom system. The system of this experiment consists of a rigid, damped beam subjected to a rotating unbalance ( $m$ ), and can be represented as in Figure 12.11. Letting $x$ be the displacement of the non-rotating mass ( $M-m$ ), the displacement of $m$ is $x+e \sin (\omega t)$. The equation of motion is thus formulated as

$$
\begin{equation*}
(M-m) \ddot{x}+m(x+e \sin \omega t) \cdot=-k x-c \dot{x} \tag{12.20}
\end{equation*}
$$

which can be rearranged as

$$
\begin{equation*}
M \bar{x}+c \dot{x}+k x=\left(m e \omega^{2}\right) \sin \omega t \tag{12.21}
\end{equation*}
$$

This is exactly the form of Eqn. 12.13 with $F_{0}=m e \omega^{2}$. Therefore, the steady state solutions for amplitude and phase are

$$
\begin{gather*}
\chi=\frac{m e}{M}\left[\frac{\left(\frac{\omega}{\omega_{n}}\right)^{2}}{\left[\left(1-\left(\frac{\omega}{\omega_{n}}\right)^{2}\right)^{2}+\left(2 \zeta\left(\frac{\omega}{\omega_{n}}\right)\right)^{2}\right]^{1 / 2}}\right]  \tag{12.22}\\
\phi=\tan ^{-1}\left[\frac{2 \zeta\left(\frac{\omega}{\omega_{n}}\right)}{1-\left(\frac{\omega}{\omega_{n}}\right)^{2}}\right] \tag{12.23}
\end{gather*}
$$

### 12.7.10 Vibration Absorber

As can be seen from Eqn. 12.22, the closer the excitation frequency is to the natural frequency the greater the amplitude of oscillation. At the point when the two are close, a condition of resonance is encountered, where dangerously large oscillations may result. It can be proved that maximum amplitude
occurs when the forcing frequency is equal to the damped natural frequency, defined by

$$
\begin{equation*}
\omega_{d}=\omega_{n} \sqrt{1-\zeta^{2}} \tag{12.24}
\end{equation*}
$$

In many cases, $\zeta$ is small, so that the $\omega_{d} \approx \omega_{n}$. It is common practice in engineering to design equipment and systems so as to avoid resonance. When design criteria call for the natural and excitation frequencies to be in the same region, it is possible to avoid resonance by utilization of a vibration absorber, as in Figure 12.12. The mass and stiffness of the absorber are chosen so that its natural frequency is equal to the excitation frequency of the initial system

$$
\begin{equation*}
\omega^{2}=\frac{k_{2}}{m_{2}} \tag{12.25}
\end{equation*}
$$

Substituting $\omega_{11}^{2}=\frac{k_{1}}{m_{1}}$ and $\omega_{22}^{2}=\frac{k_{2}}{m_{2}}$ the equation for the amplitude of the initial system with no damping can be found through algebra to be

$$
\begin{equation*}
\chi_{1}=\frac{m e}{m_{1}} \frac{\left(\frac{\omega}{\omega_{11}}\right)^{2}\left[1-\left(\frac{\omega}{\omega_{22}}\right)^{2}\right]}{\left[1+\frac{k_{2}}{k_{1}}-\left(\frac{\omega}{\omega_{11}}\right)^{2}\right]\left[1-\left(\frac{\omega}{\omega_{22}}\right)^{2}\right]-\frac{k_{2}}{k_{1}}} \tag{12.26}
\end{equation*}
$$

The tuned absorber will cause two resonant frequencies to occur on either side of the original resonant frequency, and hence reduce the motion of $M$ at this point to zero, as seen in Figure 12.13.

### 12.7.11 Support Motion

In many instances, the excitation of the single degree of freedom system is due to the motion of the support surface, which, as in Figure 12.14, may be undergoing harmonic motion.

By letting $x$ be the displacement of the mass from a fixed reference point, the equation of motion for the system can be formulated as

$$
\begin{equation*}
m \bar{x}+c \dot{x}+k x=k y \tag{12.27}
\end{equation*}
$$

By assuming solutions of the form

$$
\begin{equation*}
y=Y e^{i \omega t} \tag{12.28}
\end{equation*}
$$

$$
\begin{equation*}
x=\chi e^{i(\omega t-\phi)} \tag{12.29}
\end{equation*}
$$

and substituting them into Eqn. 12.27, the absolute value of the amplitude ratio can be determined as

$$
\begin{equation*}
\left|\frac{\chi}{Y}\right|=\left[\frac{1}{\left(1-\left(\frac{\omega}{\omega_{n}}\right)^{2}\right)^{2}+\left(2 \zeta\left(\frac{\omega}{\omega_{n}}\right)\right)^{2}}\right]^{1 / 2} \tag{12.30}
\end{equation*}
$$



Fig. 12.1: The apparatus.


Fig. 12.2: Position of marker lines on mechanical damper.

a) Schematic
b) Calibration curve

Fig. 12.3: Fotonic probe.


Fig. 12.4: Fotonic sensor response.

$$
115
$$




Fig. 12.5: Damped free vibrations.
Fig. 12.6: Underdamped system.


Fig. 12.7: Overdamped system.
Fig. 12.8: Critical damping.


Fig. 12.9: Determining the logarithmic decrement.

$$
\$ 16
$$



Fig. 12.10: Forced Vib. Fig. 12.11: Rotating unbalance. Fig. 12.12: Vib. absorber.


Fig. 12.13: Amplitude response of a tuned vibration absorber.


Fig. 12.14: Support motion.

## Chapter 13

## V2- Vibrations of Continuous Systems

### 13.1 Objectives

1. To determine the first three transverse natural frequencies and mode shapes of a given continuous system.
2. To compare the results with those from the published literature.

### 13.2 Apparatus

Aluminum beam - Dimensions: length 32 cm width $\quad 1.5 \mathrm{~cm}$ depth $\quad 0.4 \mathrm{~cm}$
Youngs modulus: $6.9 \times 10^{10} \mathrm{~N} / \mathrm{m}^{2}$
Density: $\quad 2.7 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}$
Lumped mass - Weight: . . 051 kg
Location: varies for each experiment

### 13.3 Apparatus Setup

The system of this experiment consists of a rectangular, aluminum beam pin-jointed at both ends and carrying a sliding mass which can be positioned along the beam. This system is supported by a small frame and attached to the source of vibration.

### 13.4 Instrumentation

## - Photonic Sensor

The photonic sensor is attached to a screw mechanism which can travel horizontally above the beam. See experiment V1 for a detailed explanation of sensor's function.

- Vibration Exciter

The $B \& K$ vibration exciter unit delivers the forced vibration to the beam through a rotating unbalance mechanism.

- Power Amplifier
- Function Generator
- Oscilloscope


### 13.5 Procedure

### 13.5.1 Preliminary Checks

1. Instrumentation
-connected properly
-all power supplies on
2. Amplifier
-if overload light is on, switch off and reduce gain.
-restart.
3. Fotonic Sensor
-be certain vibrating beam does not hit probe.
-it is essential that probe is located at same mean distance from the beam for each measurement, otherwise changing the sensitivity of the sensor and causing erroneous results.
4. Lumped Mass
-clamped properly at given position.

### 13.5.2 Preliminary Measurements

1. Calibrate photonic sensor(See Exp. V1).
2. Obtain approximate values for the first three natural frequencies by performing a rough frequency scan with the photonic sensor.

### 13.5.3 Experimental Procedure

1. Do a frequency scan to locate the first three natural frequencies. (Choose an appropriate position for the sensor depending on frequency desired).
2. For 2 natural frequencies, (mode 1 and 3 ) determine the mode shape by measuring the amplitude of oscillation at pre-measured intervals along the beam.

### 13.6 Results

1. Obtain the first three theoretical natural frequencies.
2. Obtain the first three theoretical normalized mode shapes.
3. Plot the experimental data to determine the maximum amplitude position by fitting a smooth curve.
4. Obtain normalized experimental mode shapes using the maximum values obtained in 3.
5. Plot on the same graph the theoretical and experimental results, and discuss the differences.
6. Discuss the errors in both theoretical and experimental results.

### 13.7 Theory

The vibrating system of this experiment can be idealized as a beam pinned at both ends and carrying a point mass as in Figure 13.1. To understand the motion of the beam due to lateral vibration, consider the forces on the elemental mass of the beam in Figure 13.2. Summing the forces and moments and rearranging terms, two important relationships are obtained:

$$
\begin{gather*}
\frac{\partial V}{\partial x}=p(x, t)  \tag{13.1}\\
\frac{\partial M}{\partial x}=V \tag{13.2}
\end{gather*}
$$

Equation 13.1 relates the rate of change of shear stress to the loading of the beam. Equation 13.2 relates the rate of change of the bending moment to the shear stress. By combining these equations and performing the appropriate operations, the following relation is obtained:

$$
\begin{equation*}
\frac{\partial^{2} M}{\partial x^{2}}=\frac{\partial V}{\partial x}=p(x, t) \tag{13.3}
\end{equation*}
$$

The bending moment is also related to the curvature by the flexure equation:

$$
\begin{equation*}
M=E I \frac{\partial^{2} y}{\partial x^{2}} \tag{13.4}
\end{equation*}
$$

$E$ - Youngs modulus
$I$ - moment of inertia of cross-section around neutral axis where $I=\frac{1}{12} b h^{3}$ for a rectangular section.

The loading of the beam is also equal to the inertia load due to its mass and acceleration. Since the inertia force is in the opposite direction as $y$ in Figure 13.2, the following can be found:

$$
\begin{equation*}
p(x, t)=-\rho A_{0} \frac{\partial^{2} y}{\partial t^{2}} \tag{13.5}
\end{equation*}
$$

where $A_{0}$ is the cross sectional area.
By substitution of equations 13.4 and 13.5 into equation 13.3 and assuming that $y(x, t)=Y(x) \cos (\omega t)$, the differential equation of motion for the beam can be formulated as:

$$
\begin{equation*}
\frac{d^{4} Y}{d x^{4}}-\lambda^{4} Y=0 \tag{13.6}
\end{equation*}
$$

where

$$
\begin{equation*}
\lambda^{4}=\frac{\rho A_{0} \omega^{2}}{E I} \tag{13.7}
\end{equation*}
$$

are the characteristic roots of the equation. The general solution of this homogeneous differential equation may be expressed as

$$
\begin{equation*}
Y(x)=A \cosh (\lambda x)+B \sinh (\lambda x)+C \cos (\lambda x)+D \sin (\lambda x) \tag{13.8}
\end{equation*}
$$

The ratios between the constants $A, B, C, D$ and the characteristic root $\lambda$ are obtained from the boundary conditions of the beam. The system of this experiment can be assumed to consist of two adjacent beams subjected to certain continuity conditions at the point of contact, that is, where the point mass is located on the beam. Thus one solves for two displacement functions, $Y_{A} C(x)$ and $Y_{B} C(x)$. The following boundary conditions must be met:

1. The displacement is zero at $x=0$ and $x=l$.
2. The bending moment is zero at $x=0$ and $x=l$.
3. The displacement, shape and bending moment are continuous at $x=$ a.
4. The shear forces in the two beams at $x=a$ differ by an amount equal to the inertia force associated with the point mass.

These four boundary conditions, together with equation 13.3, can be utilized to obtain the ratio between the constants $A, B, C$ and $D$ of the solution.

To obtain the frequency $f$, equation 13.9 must be solved by iteration for $\lambda$ (see Fig. 13.3).

$$
\begin{align*}
f(\lambda)= & 2 \sin \lambda \sinh \lambda-\alpha \lambda[\sin \lambda \xi \sin \lambda(1-\xi) \sinh \lambda- \\
& \sinh \lambda \xi \sinh \lambda(1-\xi) \sin \lambda]=0 \tag{13.9}
\end{align*}
$$

where $\alpha=\frac{m}{\rho A_{0} l}, \xi=\frac{a}{l}$ and $m$ is the lumped mass.
Thus the natural frequency of the system can be found as

$$
\begin{equation*}
\omega_{n}^{2}=\lambda_{n}^{4}\left[\frac{E I}{\rho A_{0} l^{4}}\right] \tag{13.10}
\end{equation*}
$$

where $n=1,2,3 \ldots$
The mode shape of the system is its shape at an instant of time. The mode shapes for the first three natural frequencies are to be obtained in this experiment. The $n^{\text {th }}$ mode shape is given as

$$
\begin{equation*}
Y_{n}(x)=C_{n}\left[\sinh \left(\frac{\lambda_{n} x}{l}\right)-\sigma_{n} \sin \left(\frac{\lambda_{n} x}{l}\right)\right] \tag{13.11}
\end{equation*}
$$

for $0 \leq x \leq a$ and

$$
\begin{equation*}
Y_{n}(x)=D_{n}\left[\sinh \lambda_{n}\left(1-\frac{x}{l}\right)-\bar{\sigma}_{n} \sin \lambda_{n}\left(1-\frac{x}{l}\right)\right] \tag{13.12}
\end{equation*}
$$

for $a \leq x \leq l$
where

$$
\begin{gathered}
\sigma_{n}=\frac{\sinh \lambda_{n} \sin \lambda_{n}(1-\xi)}{\sin \lambda_{n} \sinh \lambda_{n}(1-\xi)} \\
\overline{\sigma_{n}}=\frac{\sinh \lambda_{n} \sin \lambda_{n} \xi}{\sin \lambda_{n} \sinh \lambda_{n} \xi} \\
D_{n}=\frac{C_{n} \sinh \lambda_{n} \xi}{\sinh \lambda_{n}(1-\xi)}
\end{gathered}
$$

for $n=1,2,3 \ldots$
Note that $Y_{n}$ yields only the shape, not the absolute values, unless $C_{n}$ and $D_{n}$ are known. It is suggested that the deflection shapes are normalized by choosing the maximum deflection as unity and then calculate $C_{n}$ and $D_{n}$. (In practice $C_{n}$ can be taken equal to one, the maximum value is then found, and the curve is normalized with respect to this value.)

If there were no point mass, only a rod pinned at both ends, the mode shapes would be similar to those seen in Figure 13.3.


Fig. 13.1: Vibrating system.


Fig. 13.2: Forces on an elemental mass.
124


Fig. 13.3a: First frequency mode shape.


Fig. 13.3b: Second frequency mode shape.


Fig. 13.3c: Third frequency mode shape.

$$
125
$$

## Chapter 14

## V3- Single Degree of Freedom Systems

### 14.1 Objectives

1. To study the free vibration of a simple spring-mass-damper system.
2. To study the forced vibration due to unbalanced rotation and support motion of the system.
3. To compare the results with those found in the published literature.

### 14.2 Apparatus

Steel Frame -
Slotted Disks (two) Springs - mass 0.3 Kg , Unbalanced System -

Mass: $\quad 1.7 \mathrm{~kg}$
Mass: $\quad 1.0 \mathrm{~kg}$ each
Stiffness: $\quad 1.1 \mathrm{kN} / \mathrm{m}$
Mass: $\quad 1.28 \mathrm{~kg}$

### 14.3 Apparatus Setup

The apparatus consists basically of a rigid frame with vertical mass carriage guides, an upper spring mounting, and a lower mounting for the dashpot. A
picture of the apparatus is shown in Figure 14.1. The mass carriage, to which various slotted weights may be attached, is constrained by rollers running on the vertical guides to provide a single degree of freedom. The upper end of the spring is attached to the frame and the lower end to the mass carriage. The upper point of attachment is adjustable so that the equilibrium position of the carriage may be varied. The adjustable oil dashpot provides controlled damping and can be attached to the carriage by means of a thumb screw.

A unit consisting of two counter-rotating unbalanced disks are attached to the carriage to provide the harmonic disturbing force in the vertical direction. Alternatively, a connecting rod and crank unit provide a means of applying a periodic displacement to the point of support of the spring.

### 14.4 Instrumentation

- Variable speed motor and control unit
- Frequency indicator meter
- Pen recorders

Two pen recorders are attached to the mass carriage frame. The continuous paper recorder for amplitude and frequency measurement consists of a drum driven by a synchronous motor and a roll of paper. The paper passes over a plough so it can fall vertically. A small weight clipped to the end of the paper provides the tension to enable the drum to control the paper speed. It can be assumed that the paper slips at $2 \mathrm{~cm} / \mathrm{s}$. The rotating drum recorder for phase measurement consists of a drum around which a strip of recording paper is attached, driven by the main drive unit at the applied forcing frequency.

### 14.5 Procedure

### 14.5.1 Preliminary Checks

1. Dashpot
-oil level should be slightly above hole in the splash cover
2. Amplitude and frequency recorder
-adjust pen so that tip just touches the paper
-be sure carriage is at the limit stops before switching on
3. Phase recorder
-ensure the overlap is such that the pen moves over the step smoothly without tearing the paper

### 14.5.2 Experimental Procedure

(A) Damped Free Vibration

* asterisk indicates that step is performed by T.A.
*1. Place the number of slotted weights on the mass carriage and lock in position.
*2. Adjust the vertical position of the carriage so it is at the center of the paper roll.

3. Fit the pen in the pen holder.
4. Draw carriage down to the limit stops, switch on the recorder and release the carriage.
5. When the oscillations have stopped, switch off the recorder.
6. Open the damper by turning the sleeve $1 / 2$ turn and repeat the experiment.
7. Continue in the same manner in increments of $1 / 2$ turns until about 3 turns have been made. Tear off the paper recordings.
(B) Forced Vibration: Rotating Unbalance
*1. With the carriage resting on the lower limit stops, carefully insert the unit carrying the counter-rotating discs, with the drive pinion positioned at the lower left and clamp in place.
*2. Connect the small end of the flexible drive shaft to the unit and the other end to the main drive connection situated below the phase recorder. Ensure that the square drive end fits snugly into their respective housings before screwing the knurled sleeves.
*3. Connect the dashpot to the carriage as in part (A).
*4. Connect the lower end of the spring to the mass carriage. Adjust the height of the carriage to position it in the center of the paper roll.
8. Open the damper by turning the sleeve 4 turns from the fully closed position.
9. Obtain a free-vibration trace as in part (A).
10. With the speed control set at zero, switch on the motor circuit. Increase the speed slowly to 1.5 Hz . Allow the vibrating mass to reach steady state vibration and then switch on the amplitude recorder, allowing sufficient length of trace to be able to determine amplitude and frequency. Switch off the recorder.
11. Increase the frequency in small steps, up to 5.0 Hz repeating procedure (7) for each frequency (smaller frequency increments should be taken near resonance).
12. Switch off the motor. From the paper recordings, obtain the amplitude ratio versus frequency curve.
(C) Forced Vibration: Support Motion
*1. Release the upper spring support by removing the thumb nut, and using this nut, attach one end of the connecting rod to the support. Attach the other end to the eccentric stud located on the drive unit.
*2. Lock the two slotted weights in position. With the eccentric stud in the $90^{\circ}$ position, center carriage on amplitude recorder paper roll.
*3. Set damping level such that no oil splashing occurs at resonance.
13. With the speed set to zero, switch on the motor circuit, and repeat as in part (B), step 6.
14. Attach a strip of graph paper around the phase recorder drum with adhesive tape. Fit pen into holder about 2 mm clear of the paper and trace the support motion on the phase recorder at the lowest possible frequency, (this will be your reference trace ( $\phi=0$ )).
15. Starting at 1.5 Hz obtain a trace on amplitude recorder as in step 7 of part (B).
16. At the same frequency as part 6, make a trace on the phase recorder using the small lever to allow the pen to make contact with the paper. Use a different pan colour for each frequency (repeat step 5 if traces get too crowded).
17. Increase the frequency in small steps up to the maximum frequency (5 Hz ) of the excitation and for each frequency, repeat steps 6 and 7.

### 14.6 Results

The following plots or parameters should be obtained based on experimental data:

1. The displacement vs. time from the free vibration tests for each damping setting.
2. The damping coefficient for each (underdamped) curve in part (A).
3. For the last trace of part (A), the theoretical amplitude envelop and comparison with experiment.
4. The non-dimensional amplitude vs. frequency ratio ( $M X / m e$ vs. $\omega / \omega_{n}$ ) for part (B) (note: first obtain an average value for the produce ' $m e$ ' from the experimental data).
5. The amplitude ratio vs. frequency ratio and phase vs. frequency ratio $\left(X / Y\right.$ and $\phi$ vs. $\left.\omega / \omega_{n}\right)$ for part C.
6. Obtain theoretical curves for comparison to results in $4 \& 5$ above.

Note: To obtain results and compare for the forced vibration cases, one must know the damping ratios. These may be obtained from the free vibration traces (step 6 in part (B), and step 4 in part (C)).

Discuss the results stating sources of errors and weakness in theory. Give reasons for any discrepancies between theoretical and experimental results.

### 14.7 Theory

See theory of experiment V1.


Fig. 14.1: The apparatus.


[^0]:    a Shatt
    b Compressor wheel c Turbine wheel
    d Air inlet
    e Compressed air outlet
    f Exhaust gas inlet

