NOTES ON MODELING INTERSPIKE INTERVAL DENSITIES AND THEIR TRANSFORMATION BY SYNAPTIC TRANSMISSION
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We consider a monosynaptic pre- and postsynaptic cell population situation in which there are many presynaptic cells acting on one postsynaptic cell (convergence). In this situation presynaptic spikes are generating EPSP's that arrive at times $t_{-1}, t_0, t_1, \cdots$ and which generate a total EPSP in the postsynaptic cell soma of the form

$$E(t) = E_0 \sum_{i=0}^{n} e^{-(t-t_i)/\tau}.$$

We assume that the differences in arrival times, $t_n - t_{n-1} \equiv \Delta_n$ are identically distributed with the density f and are independent of one another.

Assume that the maximum amplitude of the inputs is a constant $\theta_1 > \theta_0$ and that the threshold function $\theta(t)$ for the production of post-synaptic spikes has the form

$$\theta(t) = \frac{\theta_0}{1 - e^{-(t-t_a)/\tau_\theta}} \tag{1}$$

where t_a is the absolute refractory time of the post-synaptic cell and τ_θ is the decay constant of the threshold in the relative refractory state back to θ_0. We further assume that $\tau_\theta < \tau$ which means that the only possible intersection point of $\theta(t)$ and $E(t)$ is on the upstroke of the EPSP's.

Assume that an action potential was generated in the post synaptic cell at time $t = 0$. To calculate the time $t = T_{AP}$ of occurrence of the next post synaptic spike we solve the equation $\theta(T_{AP}) = \theta_1$ and denote this solution by

$$T_{AP} = T_{AP}(\theta_0, \theta_1, t_a, \tau).$$

The next post synaptic spike will occur at a time $T_* = t_n$ where $T_n \geq T_{AP}$ and $T_{n-1} < T_{AP}$. Our goal is to find the density function f_* for the distribution of the random variable T_* given by

$$T_* = t_n \qquad \text{if} \qquad t_{n-1} < T_{AP} \leq t_n. \tag{2}$$

Let

$$p_n = \text{Prob}\ (t_{n-1} < T_{AP} \leq t_n)$$

and denote by f_n the density of the distribution of the t_n. Then we argue that the density f_* is given by

$$f_*(x) = \sum_{n=1}^{\infty} p_n f_n(x). \tag{3}$$

Next we are going to calculate f_n and p_n. The first is easy since

$$t_n = \Delta_n + t_{n-1},$$

and consequently

$$f_n(x) = \int_0^x f(y) f_{n-1}(x-y)dy \qquad \text{with} \qquad f_1(x) = f(x) \tag{4}$$

To calculate the p_n note that

$$\begin{aligned} p_n &= \text{Prob } (t_n \geq T_{AP} \text{ and } t_{n-1} < T_{AP}) \\ &= \int_0^{T_{AP}} \text{Prob } (t_n \geq T_{AP} | t_{n-1} = x) f_{n-1}(x)dx \qquad (5) \end{aligned}$$

Clearly,

$$\begin{aligned} \text{Prob } (t_n \geq T_{AP} | t_{n-1} = x) &= \text{Prob } (t_{n-1} + \Delta_n \geq T_{AP} | t_{n-1} = x) \\ &= \text{Prob } (\Delta_n \geq T_{AP} - t_{n-1} | t_{n-1} = x) \\ (6) \qquad &= \text{Prob } (\Delta_n \geq T_{AP} - x) \text{ since } \Delta_n \text{ and } t_{n-1} \text{ are independent.} \end{aligned}$$

Therefore for $x \leq T_{AP}$ we have

$$\begin{aligned} \text{Prob } (t_n \geq T_{AP} | t_{n-1} = x) &= \text{Prob } (\Delta_n \geq T_{AP} - x) \\ &= \textstyle\int_0^{T_{AP}-x} f(y)dy. \qquad (7) \end{aligned}$$

Combining equation (7) with (5) we have

$$p_n = \int_0^{T_{AP}} \left\{ \int_0^{T_{AP}-x} f(y)dy \right\} f_{n-1}(x)dx,$$

or

$$p_n = \int_0^{T_{AP}} \left\{ \int_0^{T_{AP}-y} f_{n-1}(y)dx \right\} f(y)dy. \tag{8}$$

In summary from equations (3), (4), and (8) we have

$$f_*(x) = \sum_{n=1}^{\infty} p_n f_n(x) \tag{9}$$

$$f_n(x) = \int_0^x f(y) f_{n-1}(x-y)dy \qquad \text{with} \qquad f_1(x) = f(x) \tag{10}$$

$$p_n = \int_0^{T_{AP}} \left\{ \int_0^{T_{AP}-y} f_{n-1}(x)dx \right\} f(y)dy \tag{11}$$

$$T_{AP} = t_a + \tau_\theta \log \left[\frac{\theta_1}{\theta_1 - \theta_0} \right] \tag{12}$$