DENSITIES AND COLLAPSED DENSITIES IN HIGH DIMENSIONAL DYNAMICAL SYSTEMS FILE: COLLDEN.TEX 27 MAY, 1995; 26 SEPTEMBER, 1995

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THE PROBLEM

We have the following question to consider. Why does the collapsed density of one coupled map lattice (CML)–a manifestly high dimensional dynamical system–mimic the behaviour predicted for an ensemble of CML's? Or, more precisely, does it? Should it?

FORMULATION

We consider an "ensemble" of M CML's, and each of these M CML's consists of N units. Then, for the j^{th} CML. the activity at any time is described by the vector

$$x^j = (x_1^j, \cdots, x_N^j).$$

Suppose, for the sake of argument, that the state space of each of the N elements of the CML is X and is finite. Form a partition A_l on X such that

$$\cup_{l=1}^m A_l = X \qquad A_l \cap A_{l'} = \emptyset, \quad l \neq l'.$$

Then the state space of one of the CML's in our ensemble is just given by

$$\mathcal{X} = X^N.$$

A HISTOGRAM OF STATE VARIABLES IN ONE CML

Now note that the fraction of the N units of the j^{th} CML in A_l is given by

$$F_l^j = \frac{1}{N} \sum_{i=1}^{N} 1_{A_l}(x_i^j). \tag{1}$$

Using this idea and notation, we could write an histogram approximation to the "collapsed density" of the j^{th} CML as

$$\begin{aligned} \mathcal{F}^j(x) &= \sum_{l=1}^{m} F_l^j 1_{A_l}(x) \\ &= \sum_{l=1}^{m} \left\{ \frac{1}{N} \sum_{i=1}^{N} 1_{A_l}(x_i^j) \right\} 1_{A_l}(x). \end{aligned} \tag{2}$$

Since j is arbitrary, we might as well take $j = 1$ so the notation is more suggestive of the collapsed density of one CML and write

$$\begin{aligned} \mathcal{F}^1(x) &= \sum_{l=1}^{m} F_l^1 1_{A_l}(x) \\ &= \sum_{l=1}^{m} \left\{ \frac{1}{N} \sum_{i=1}^{N} 1_{A_l}(x_i^1) \right\} 1_{A_l}(x). \end{aligned} \tag{3}$$

COLLAPSED DENSITY FROM AN ENSEMBLE DENSITY

Suppose we know that, analytically, the **ensemble** density for our system is given by $f(x_1, \cdots, x_N)$. Then we can define the **collapsed density** by

$$\bar{f}(x) = \int_X \cdots \int_X f(x_1, \cdots, x_N) \prod_{p=1}^{N} \delta(x_p - x) dx_p$$