Nonlinear Dynamics
26.1–5

STATISTICAL PROPERTIES OF NETWORKS OF COUPLED NEURAL ELEMENTS

John G. Milton[1], Po Hsiang Chu[2] and Michael C. Mackey[3]

From 1) The University of Chicago, 2) DePaul University and 3) McGill University.

ABSTRACT

The statistical properties of a 2-D network of neural elements coupled diffusively is examined. Computer simulations indicate that if each network element in the absence of coupling generates an asymptotically stable limiting density (ASLD), then in the presence of coupling the network generates an ASLD provided that it is sufficiently large.

INTRODUCTION

The measurement of the statistical properties of neuronal spike trains is an important topic in neuro-physiological investigations [1-2]. Although most authors have emphasized the variety of spike train patterns that can be observed, it is equally true that many neurons in a given region of cortex produce very similar spike train patterns [1]. The traditional approach has been to model the input-output relations of a neuron in terms of stochastic point processes and then determine the inter-spike probability density function of the output which arise from a superposition of random spike trains which may or may not be interact (for review see [2]).

An alternate possibility is that neural spike train patterns are generated through nonlinear chaotic dynamics [3-4]. Then the distinction between stochastic and deterministic processes is blurred because, for example, given any one-dimensional probability density it is possible to construct an infinite number of deterministic processes whose iterates are chaotic and which generate the prescribed density [5-6]. Indeed, non-linear chaotic deterministic processes have recently been used to interpret the statistics of patient survival time data [6] and ion channel kinetics [7].

Here we study the statistical properties of N^2 neural elements coupled in a NxN network with dynamics given by

$$y_{t+1} = (1-\varepsilon)F(y_t(i,j)) - \frac{\varepsilon}{4}C_t \qquad (1)$$

where

$$C_t = F(y_t(i-1,j)) + F(y_t(i+1,j)) + F(y_t(i,j-1)) + F(y_t(i,j+1))$$

where y_t, y_{t+1} is the inter-spike interval of the i,j-th element at times t, t+1, $\varepsilon \in [0,1]$ is the coupling constant, i,j = 1,...,N and the boundary conditions are periodic. The function F is non-linear and describes the dynamics of a neural element. It is chosen such that

$$x_{t+1} = F(x_t) \qquad (2)$$

generates an ASLD [5-6]. This ASLD corrresponds to the distribution of inter-spike intervals that would be measured experimentally.

METHODS

Two forms of F were studied: 1) the quadratic map

$$x_{t+1} = 4x_t(1-x_t) \qquad (3)$$

for which it is possible to derive the limiting density analytically ([5]; shown in Fig. 1A); and 2) the recurrent inhibitory loop map

$$x_{t+1} = A - Bx_t\frac{\phi^n}{x_t^n+\phi^n} \qquad (4)$$

Eq. (4) is derived as the singular limit of a delay-differential equation model for recurrent inhibition [8] which reproduces the statistics of CA1 hippocampal neuron spike trains [9]. Computer simulations suggest that (4) produces an ASLD when A=1, B=5, n=3, ϕ=.35 (Fig. 1B). Computer simulations were used to calculate the inter-spike probability densities as a function of n and ε for F given by (3) and (4). All programs were written in C.

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RESULTS

Figure 1 compares the interspike histograms generated from eq. (1) with $\varepsilon = 0.25$ for N=1 (A,B) and N=10 (C,D). Identical interspike histograms were obtained for different initial conditions and for different initial densities. In the case of N=10, each element of the network reproduced the identical interspike histogram. Moreover, the identical interspike histogram was generated by measuring all NxN network elements. These observations suggest that the network of neural elements described by (1) produces an ASLD. Identical ASLD's were obtained for N > 10 (range studied 10-1000). Qualitatively similar results were obtained for other values of ε.

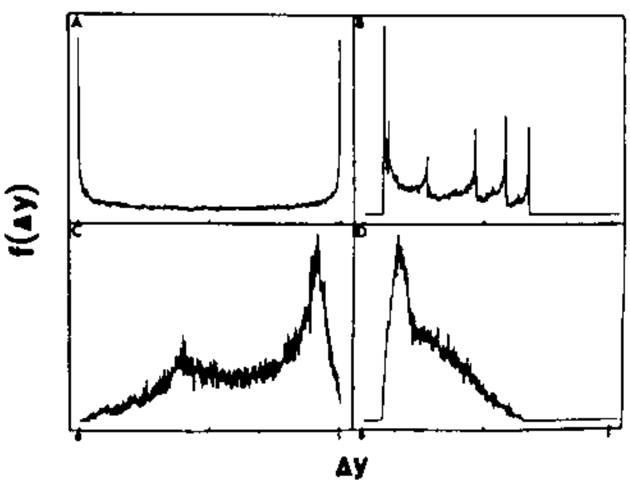

Figure 1: Inter-spike histograms (ASLD's) computed from eq. (1) when N x (# iterations) = 30,000. See text for details.

CONCLUSIONS

Our results suggest that if F generates an ASLD then a network of such elements with diffusive coupling also generates an ASLD provided that the network is sufficiently large. Verification of this conjecture requires analytical study. The ASLD in the presence of coupling differs from that in the absence of coupling. Coupled lattice networks such as eq. (1) have been studied extensively in a physics context as models of spatiotemporal complexity [10]. The present study is the first to study these networks from a statistical point of view. The possibility that networks can generate ASLD's may explain the observation that the statistical properties of neurons in the cortex can be so similar. Moreover our observations lend support to the concept of distribution coding in the nervous system [2].

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Address inquiries to: Dr. John G. Milton, Dept. Neurology, Box 425, The University of Chicago Hospitals, 5841 South Maryland Ave., Chicago, Ill. 60636 (Tel: (312)-702-1780, e-mail: splace@ace.bsd.uchicago.edu.

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