

ZEROPOINT FLUCTUATIONS AND DARK ENERGY IN JOSEPHSON JUNCTIONS

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We discuss the possibility that vacuum fluctuations underlying dark energy could lead to measurable effects for the quantum noise spectrum in Josephson junctions [C. Beck, M. C. Mackey, *Phys. Lett. B* **605**, 295 (2005)]. We argue that a recent discussion of Jetzer and Straumann [*Phys. Lett. B* **606**, 77 (2005)] relating the measured noise spectrum in Josephson junctions to van der Waals forces is incorrect. The measured noise spectrum in Josephson junctions is a consequence of the fluctuation dissipation theorem and the Josephson effect and has nothing to do with van der Waals forces. Consequently, the argument of Jetzer and Straumann does not shed any light on whether dark energy can or cannot be measured using superconducting Josephson devices. We also point out that a more recent paper of Jetzer and Straumann [*Phys. Lett. B* **639**, 57 (2006)] claiming that ‘zeropoint energies do not show up in any application of the fluctuation dissipation theorem’ violates the standard view on the subject.

Keywords: Dark energy; zeropoint energy; fluctuation dissipation theorem; quantum noise; Josephson junctions; cutoff.

1. Introduction

Recently we hypothesized that if vacuum fluctuations underly dark energy then this effect could be detected experimentally using resistively shunted Josephson junctions [1]. Our suggestion was based on an experiment by Koch et al. [2], who have shown that superconducting Josephson devices have a noise spectrum consistent with theoretical predictions [3] based on a generalized treatment of quantum fluc-

tuations by Callen and Welton [4]. Subsequently, our paper was criticized by Jetzer and Straumann [5, 6], who claimed there is no basis for our hypothesis.

In this note we argue that the logic behind the Jetzer and Straumann criticism [5] is misleading. Their paper [5] is based on an equilibrium van der Waals model that is not applicable to our system. We also deal with a new version of their criticism [6] and show that the view expressed in [6], namely that the noise in Josephson junctions has nothing to do with zeropoint energies, is in apparent contrast to the standard treatments dealing with quantum noise in Josephson junctions [7–9].

Our conclusion is that the arguments presented by Jetzer and Straumann do not shed any light on a possible relation between quantum noise and dark energy. Rather, experimental tests are necessary, which will be performed in the near future [13, 14].

2. The Data and the Theory

Koch et al. [3] derived the power spectrum $S(\omega)$ (units of A^2/Hz) describing the measured current noise in a resistively shunted Josephson junction in the form

$$S(\omega) = \frac{4}{R} \left[\frac{\hbar\omega}{2} + \frac{\hbar\omega}{\exp(\hbar\omega/kT) - 1} \right], \quad (1)$$

where R (ohms) is the shunt resistor and T is the absolute temperature. The experimental work of Koch et al. [2] convincingly demonstrated that Equation (1) fits the experimental data $S(\omega)$ as a function of $\omega = 2\pi\nu$ between $\nu = 0$ and $\nu = 6 \times 10^{11}$ Hz at 1.6 and 4.2 K.

From a formal point of view, the expression in brackets of Equation (1) is the mean energy

$$\bar{U}(\nu, T) = \frac{1}{2}h\nu + \frac{h\nu}{\exp(h\nu/kT) - 1}, \quad (2)$$

of an oscillator with frequency ν at temperature T . For low temperatures the spectrum $S(\omega)$ is dominated by the linear term in ω , which can be attributed to the effects of vacuum (zero-point) fluctuations [8]. As the temperature is increased the second term, which is identical to the ordinary Bose-Einstein statistics, plays an ever larger role in $S(\omega)$.

3. The Hypothesis and the Criticism

If we take the customary expression for the energy per unit volume at a frequency ν and temperature T

$$\rho(\nu, T) = \frac{8\pi\nu^2}{c^3} \bar{U}(\nu, T) \quad (3)$$

then

$$\rho(\nu, T) = \frac{8\pi\nu^2}{c^3} \left[\frac{1}{2}h\nu + \frac{h\nu}{\exp(h\nu/kT) - 1} \right] \quad (4)$$

In Equation (4) the first term

$$\rho_{vac}(\nu) = \frac{4\pi h\nu^3}{c^3} \quad (5)$$

is due to the zero-point fluctuations, while the second term

$$\rho_{rad}(\nu, T) = \frac{8\pi h\nu^3}{c^3} \frac{1}{\exp(h\nu/kT) - 1} \quad (6)$$

is simply the photonic black body spectrum. Equation (4) suffers from the embarrassing prediction that there should be an infinite amount of energy per unit volume, since

$$\lim_{\nu_c \rightarrow \infty} \int_0^{\nu_c} \rho(\nu, T) d\nu$$

is divergent. Indeed, writing

$$\rho(\nu, T) = \rho_{vac}(\nu) + \rho_{rad}(\nu, T), \quad (7)$$

it is easily seen that the divergence is a consequence of the temperature independent vacuum fluctuation term because

$$\int_0^{\infty} \rho_{rad}(\nu, T) d\nu = \frac{\pi^2 k^4}{15\hbar^3 c^3} T^4 \quad (8)$$

simply yields the customary Stefan-Boltzmann law. To circumvent this divergence, we suggested in [1] that Equation (5) is only valid up to a certain cutoff frequency ν_c so that the total energy associated with $\rho_{vac}(\nu)$ is given by

$$\int_0^{\nu_c} \rho_{vac}(\nu) d\nu = \frac{4\pi\hbar}{c^3} \int_0^{\nu_c} \nu^3 d\nu = \frac{\pi\hbar}{c^3} \nu_c^4. \quad (9)$$

We noted that a future experiment could examine whether the measured vacuum fluctuations in Fig. 1 of [1] might be a signature of dark energy. If so, one would expect to see a cutoff in the measured spectrum at

$$\nu_c \simeq (1.69 \pm 0.05) \times 10^{12} \text{ Hz}, \quad (10)$$

where this value of ν_c is obtained by setting

$$\frac{\pi\hbar}{c^3} \nu_c^4 \simeq \rho_{dark} = (3.9 \pm 0.4) \text{ GeV/m}^3 \quad (11)$$

(ρ_{dark} is the currently observed dark energy density in the universe [1]).

Jetzer and Straumann [5] have criticized the hypothesis of [1] based on two different points. In their own words:

- Point 1. “... the spectral density originally comes from a simple rational expression of Boltzmann factors, which are not related to zero-point energies.”

To illustrate their point, Jetzer and Straumann consider a simplified model of the van der Waals force between two harmonic oscillators and calculate the response of the system to distance changes. Their result is independent of zero-point energies of the two oscillators and from this they conclude that the same also holds for the measured spectrum (1) in Josephson junctions.

- Point 2. “...the absolute value of the zero-point energy of a quantum mechanical system has no physical meaning when gravitational coupling is ignored. All that is measurable are changes of the zero-point energy under variations of system parameters or of external couplings, like an applied voltage.”

Based on this general statement, Jetzer and Straumann claim that experiments based on Josephson junctions are unable to detect dark energy since only differences in vacuum energy would be physically relevant.

Here we argue in Section 4 that Point 1 is misleading since the observed spectra in Josephson junctions have nothing to do with van der Waals forces. In Section 5 we argue that Point 2 is theoretically unclear (since the quantum noise in Josephson junctions has not been shown to be renormalizable) but experimentally testable.

4. Point 1

The justification of Point 1 of Jetzer and Straumann [5] is based on an equilibrium statistical mechanical model for the van der Waals interaction between two identical harmonic oscillators. The authors point out that a simple transformation can decouple the oscillators. The ground state of the decoupled system is the sum of the zero-point energies of the two decoupled oscillators and the corresponding van der Waals force is independent of the zero-point energies of the original oscillators.

Our response to Point 1 is based on the following four observations.

1. The simple model discussed in [5] is neither a valid description of the shunting resistor nor of the Josephson junction. Jetzer and Straumann make computations for van der Waals forces, whereas the measured spectra in the Josephson junctions are a consequence of a completely different effect, the ac Josephson effect [10]. Oversimplified *theoretical* models may not shed any light on the origin of *measured* noise spectra in Josephson junctions.
2. What is *measured* in the experiment of Koch et al. [2] is the spectrum of current fluctuations in the resistive shunt, mixed down at the Josephson frequency. The fact that the *experimental* data in [2] is so closely fit by Equation (1) is an indication that at low temperatures there is a significant correspondence between the behaviour of this superconducting device and the prediction of the corresponding theoretical treatment.

Jetzer and Straumann claim, on the basis of their simplified model for van der Waals forces, that the linear term $\hbar\omega/2$ in Equation (1) cannot be due to vacuum fluctuations. It may be a matter of semantics to argue about what to call the source of this term, but their contention contradicts the received wisdom [8,9,11] which clearly singles out zero-point fluctuations as the source underlying the linear term $\hbar\omega/2$ in the spectrum.

3. Arguments for why vacuum (zero-point) fluctuations have a measurable effect in Josephson junctions have been given by various authors, e.g. [11]. Namely, a driven Josephson junction is a non-equilibrium system, and non-equilibrium systems can be influenced by vacuum fluctuations in a measurable way. For example, zero-point fluctuations can cause excited atoms to return

to the ground state, thus producing an experimentally detectable effect. The argument against this observation presented in [5] is based on equilibrium statistical mechanics and does not incorporate non-equilibrium effects.

4. What is really at the root of the measured noise spectra in resistors is the fluctuation dissipation theorem [4,9,12] which precisely predicts a power spectrum as given by Equation (1). This spectrum has been experimentally confirmed by Koch et al. [2] up to frequencies of 0.6 THz. All textbooks [8,9] and classical papers [4,12] on the subject emphasize the fact that the linear term in the spectrum is induced by zero-point fluctuations.

Based on these points, we find Point 1 made by Jetzer and Straumann to be unconvincing.

5. Point 2

Turning to Point 2, it is clear that experiments involving van der Waals forces or the Casimir effect can only probe differences in vacuum energy. This is well known and related to the fact that QED is a renormalizable theory. Adding an arbitrary constant to the vacuum energy density leaves the physical predictions of this theory invariant. The correct conclusion is that experiments based on the Casimir effect have no chance of measuring the absolute value of vacuum energy.

The Josephson junction experiment, however, exploits a different effect which apparently has nothing to do with the Casimir effect. The theory of dissipative non-equilibrium quantum systems, such as driven Josephson junctions, is much less well understood than the Casimir effect. Whether the dissipative quantum theory underlying resistively shunted Josephson junctions can be renormalized is presently unclear. Hence the absolute value of vacuum energy may well have physical meaning for these kinds of superconducting quantum systems.

To illustrate this point, assume that only differences in vacuum energy are relevant for the Josephson junction experiment of Koch et al., as Point 2 of Jetzer and Straumann suggests. It should then be possible to add an arbitrary constant (with the dimension of energy) to Equation (2), without changing the physical predictions of the theory. In our case the underlying theory is provided by the fluctuation dissipation theorem [4,8,9] which predicts in complete generality that the mean square fluctuations $\langle V^2 \rangle$ of the voltage in the shunting resistor are given by

$$\langle V^2 \rangle = \frac{2}{\pi} \int \bar{U}(\omega/2\pi, T) R(\omega) d\omega \quad (12)$$

where $\bar{U}(\nu, T)$ is given by Equation (2) and $R(\omega)$ is the shunting resistor. If we change \bar{U} by an additive constant C to

$$\tilde{U}(\omega/2\pi, T) = \frac{1}{2} \hbar \omega + C + \frac{\hbar \omega}{\exp(\hbar \omega / kT) - 1}, \quad (13)$$

the result would contradict the results of the Koch et al. [2] experiment. Any $C \neq 0$ would imply voltage fluctuations in the resistor different from those actually measured. Hence we obtain a contradiction if we apply point 2 of Jetzer and Straumann to our system.

We thus conclude that Point 2 is unclear from a theoretical point of view, and further that the resolution of this question cannot be decided on purely theoretical grounds. Rather, further experimental investigation is necessary. In [1] we suggested an experimental check to see whether a cutoff in the measurable spectrum could be observed near the critical frequency $\nu_c = 2\pi\omega_c \approx 1.7$ THz corresponding to dark energy density. If such a cutoff is observed, it would indeed be the *new physics* underlying this cutoff that makes the system couple to gravity and make the absolute value of vacuum energy physically relevant. Virtual photons that are not gravitationally active may well exist beyond this cutoff, it is just the gravitationally active part of vacuum fluctuations that would cease to exist at ν_c .

A repeat of the Koch experiment, based on new types of Josephson junctions operating in the THz region, will now be carried out by Warburton [13] and Barber and Blamire [14]. These new experiments will measure the noise spectrum up to frequencies exceeding the predicted critical frequency $\nu_c \approx 1.7$ THz corresponding to the inferred dark energy density, using both nitride and cuprate based Josephson junctions. This is an interesting experimental project since the fluctuation dissipation theorem and its potential contribution to dark energy density has never been tested before at these high frequencies.

6. Zeropoint Energies and the Fluctuation Dissipation Theorem

Jetzer and Straumann have recently published a new version of their criticism [6]. They now consider the fluctuation dissipation theorem rather than an equilibrium van der Waals model, thus adopting our point of view of what the relevant dynamics should be. However, their main conclusion, printed in italics in their concluding remarks, is still erroneous in our opinion. We quote

[Jetzer-Straumann, [6], p.58] ‘Zero-point energies do not show up in any application of the fluctuation dissipation theorem’.

Based on the above statement, Jetzer and Straumann again strongly criticize our hypotheses.

Here we want to point out that the above statement of Jetzer and Straumann, on which their entire criticism is based, is in sharp contrast to the common interpretation taken in the field. To illustrate this point, let us provide a few quotations to show how the universal term

$$H_{uni} := \left[\frac{1}{2}\hbar\omega + \frac{\hbar\omega}{e^{\hbar\omega/kT} - 1} \right] \quad (14)$$

occurring in the fluctuation dissipation theorem is usually interpreted physically (the emphasis in *italics* below is added):

[Landau-Lifshitz [7], p. 386] ‘It should be noted that the factor in the braces of eq. (14) is the *mean energy* (in units of $\hbar\omega$) of an oscillator at temperature T ; the term $\frac{1}{2}\hbar\omega$ corresponds to the *zero-point oscillations*.’

[Kogan [9], p. 55] ‘Eq. (14) describes the mean number of quanta (discrete excitations) of an oscillator with frequency ω at temperature

T. The r.h.s. is the mean energy of this oscillator. It consists of *ground state energy* $\hbar\omega/2$ (it is called *zero-point energy*, or the energy of zero-point vibrations) and the mean energy of the oscillator's excitations.'

[Gardiner [8], p. 5] 'The spectrum rises linearly with increasing ω because of the first term in eq. (14), which arises from the *zero-point fluctuations* in the *harmonic oscillators*...'

The main conclusion of Jetzer and Straumann in [6], quoted above, contradicts the standard view. All sources clearly emphasize the fact that the linear term of the function H_{uni} that occurs in the fluctuation dissipation theorem, connecting fluctuation spectra with dissipation, has the physical meaning of a zero-point energy of a suitable quantum mechanical oscillator. We thus think that the view of Jetzer-Straumann expressed in [6] is untenable.

On a closer inspection of [6], the reason why the authors arrive at their non-standard view is immediately apparent. In [6], the role of the universal function H_{uni} occurring in the fluctuation dissipation theorem and the system Hamiltonian H_{sys} describing the quantum system under consideration is confused. The authors re-derive in [6] the well-known fact that the fluctuation dissipation theorem is valid for arbitrary Hamiltonians H_{sys} , in particular for those where an arbitrary additive constant is added to H_{sys} . However, their argument relates to the system Hamiltonian H_{sys} and not to H_{uni} . The idea that we proposed in [1] and further worked out in [15] was to test in future experiments [13, 14] whether the zero-point term occurring in the universal Hamiltonian H_{uni} has any relation to dark energy. If that is the case, a cutoff must be found in Josephson experiments. The zero-point term in H_{uni} cannot be removed by adding arbitrary constants to it, as shown in section V. The line of reasoning of Jetzer and Straumann in [6] is highly misleading in this context, since they add constants to a different Hamiltonian, H_{sys} , which has nothing to do with the universal Hamiltonian we consider, H_{uni} . In particular, the considerations in [6] provide no insight into the physical interpretation of the vacuum energy associated with H_{uni} , which is invariant and universal.

Jetzer and Straumann state in [6] that our insertion of an arbitrary constant C in eq. (13) is wrong. However, they fail to explain to the reader that we did this for the sole purpose of *deriving a contradiction* of the Jetzer-Straumann suggestion in [5], namely to shift the zero-point energy of our system by adding an additive constant. So certainly this equation is wrong, because it was our purpose to derive a contradiction.

One remark is at order. Models of dark energy always require new physics in one way or another. The question is where and in which form this new physics enters. The class of models that can be tested with Josephson junctions associate dark energy with ordinary electromagnetic vacuum energy [15–17]. Clearly, in order to reproduce the correct dark energy density in the universe, for these types of models there must be a phase transition point at around 1.7 THz where virtual photons lose their gravitational activity. Virtual photons can still persist at higher frequencies (hence ordinary QED is still valid), just their *gravitational activity* ceases to exist at higher frequencies in these types of models, by means of a phase transition describing a change of gravitational behaviour of virtual photons at high frequencies.

This phase transition is the new physics associated with the model. Since the zero-point term of H_{uni} cannot be renormalized away, the above phase transition might be observable in dissipative quantum systems described by the fluctuation dissipation theorem. For this reason we think it is very interesting to experimentally test the fluctuation dissipation theorem at high frequencies, to either confirm or rule out these types of dark energy models.

For a Ginzburg-Landau model describing the above phase transition, see [17]. For a possible technical application of the zeropoint term in the fluctuation dissipation theorem, see [18].

7. Conclusions

We have argued in this note that the objections of Jetzer and Straumann [5] to the hypothesis formulated in [1] are not applicable to our system. The arguments presented in [5] are based on a model for the van der Waals forces between two harmonic oscillators, which have nothing to do with the measured noise spectra in Josephson junctions. Moreover, the arguments presented in the more recent paper [6] are in apparent contrast to the standard textbook view on quantum noise in Josephson junctions.

We further contend that the *only* way to really test the hypothesis that there is a cutoff in the frequency spectrum of measurable vacuum fluctuations is to actually do the experiment. Appeal to theoretical arguments extended to situations in which the theory has not been verified do not shed any light on the (so far unknown) nature of dark energy.

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