Available online at www.sciencedirect.com
SCIENCE DIRECT
PHYSICS LETTERS B
Physics Letters B 605 (2005) 295–300
www.elsevier.com/locate/physletb

Could dark energy be measured in the lab?

Christian Beck [a], Michael C. Mackey [b,1]

[a] *School of Mathematical Sciences, Queen Mary, University of London, Mile End Road, London E1 4NS, UK*
[b] *Centre for Nonlinear Dynamics in Physiology and Medicine, Departments of Physiology, Physics and Mathematics, McGill University, Montreal, PQ, Canada*

Received 7 October 2004; accepted 22 November 2004

Available online 25 November 2004

Editor: P.V. Landshoff

Abstract

The experimentally measured spectral density of current noise in Josephson junctions provides direct evidence for the existence of zero-point fluctuations. Assuming that the total vacuum energy associated with these fluctuations cannot exceed the presently measured dark energy of the universe, we predict an upper cutoff frequency of $\nu_c = (1.69 \pm 0.05) \times 10^{12}$ Hz for the measured frequency spectrum of zero-point fluctuations in the Josephson junction. The largest frequencies that have been reached in the experiments are of the same order of magnitude as ν_c and provide a lower bound on the dark energy density of the universe. It is shown that suppressed zero-point fluctuations above a given cutoff frequency can lead to $1/f$ noise. We propose an experiment which may help to measure some of the properties of dark energy in the lab.

PACS: 74.81.Fa; 98.80.-k; 03.70.+k

Keywords: Zero-point fluctuations; Dark energy; Josephson junctions

1. Introduction

In his "second theory" of black-body radiation, Planck [1] (cf. also [2]) found the average energy of a collection of oscillators at temperature T and frequency ν to be

$$\bar{U}(\nu, T) = \frac{1}{2}h\nu + \frac{h\nu}{\exp(h\nu/kT) - 1}. \tag{1}$$

The first (temperature independent) term is now referred to as the zero-point energy and commonly related to vacuum fluctuations. The second term gives rise [1,3] to the Planck black body spectrum

$$\rho(\nu, T) = \frac{8\pi h\nu^3}{c^3}\frac{1}{\exp(h\nu/kT) - 1} \tag{2}$$

E-mail addresses: c.beck@qmul.ac.uk (C. Beck), mackey@cnd.mcgill.ca (M.C. Mackey).
URLs: http://www.maths.qmul.ac.uk/~beck, http://www.cnd.mcgill.ca/people_mackey.html.
[1] Also: Mathematical Institute, University of Oxford, 24-29 St Giles', Oxford OX1 3LB, UK.

0370-2693/$ – see front matter
doi:10.1016/j.physletb.2004.11.060

that is relatively flat for $h\nu \ll kT$ and which approaches zero for $h\nu \gg kT$.

In spite of early convictions by some investigators that the zero-point energy term in Eq. (1) would not have any experimental correlate, this has not been the case. Indeed, the zero-point term has proved important in explaining X-ray scattering in solids [4]; understanding of the Lamb shift between the s and p levels in hydrogen [5,6]; predicting the Casimir effect [7–9]; understanding the origin of van der Waals forces [7]; interpretation of the Aharonov–Bohm effect [10,11]; explaining Compton scattering [5]; and predicting the spectrum of noise in electrical circuits [12–15]. It is this latter effect that concerns us here.

Koch et al. [13] measured the frequency spectrum of current fluctuations in Josephson junctions. At low temperatures and high frequencies the experimental spectrum is dominated by zero-point fluctuations, confirming the physical relevance of the zero-point term in Eq. (1) up to frequencies of the order $\nu_{\max} = 6 \times 10^{11}$ Hz. Here we reanalyze their experimental results in light of recent astronomical estimates of dark energy density in the universe [16–19].

Our hypothesis is that the signature of zero-point fluctuations measured by Koch et al. imply a non-vanishing vacuum energy density in the universe. This vacuum energy would have large scale gravitational effects, and cannot exceed the measured dark energy density of the universe as determined in astronomical measurements [16,17]. On this basis we predict a cutoff frequency (ν_c) for the zero-point fluctuations in Josephson junction experiments, which is only slightly larger than the maximum frequency $\nu_{\max}$ reached in Koch et al.'s 1982 experiment. Future experiments, based on Josephson junctions that operate in the THz region [20,21], could thus help to clarify whether this cutoff exists and whether the dark energy of the universe is related to the vacuum fluctuations that play a role in the Josephson junction experiments.

2. Estimating a cutoff frequency for zero-point fluctuations

If Planck [1] and Nernst [3] had used the relation $\rho(\nu,T) = 8\pi h\nu^2 \bar{U}(\nu,T)/c^3$, then instead of Eq. (2) they would have obtained

$$\begin{aligned}\rho(\nu,T) &= \frac{8\pi\nu^2}{c^3}\left[\frac{1}{2}h\nu + \frac{h\nu}{\exp(h\nu/kT)-1}\right] \\ &= \frac{4\pi h\nu^3}{c^3}\left[1 + \frac{2}{\exp(h\nu/kT)-1}\right] \\ &= \frac{4\pi h\nu^3}{c^3}\coth\left(\frac{h\nu}{kT}\right). \end{aligned} \tag{3}$$

Eq. (3), which is correct from the perspective of quantum electrodynamics [22], predicts that if all frequencies ν are taken into account then there should be an infinite energy per unit volume since

$$\lim_{\nu_c\to\infty}\int_0^{\nu_c} \rho(\nu,T)\,d\nu$$

diverges. To avoid this one could introduce a cutoff frequency $\nu_c < \infty$.

Split the total energy density into

$$\rho(\nu,T) = \rho_{\mathrm{vac}}(\nu) + \rho_{\mathrm{rad}}(\nu,T), \tag{4}$$

where

$$\rho_{\mathrm{vac}}(\nu) = \frac{4\pi h\nu^3}{c^3} \tag{5}$$

is due to zero-point fluctuations, and

$$\rho_{\mathrm{rad}}(\nu,T) = \frac{8\pi h\nu^3}{c^3}\frac{1}{\exp(h\nu/kT)-1} \tag{6}$$

corresponds to the radiation energy density generated by photons of energy $h\nu$. Integration of (5) up to ν_c yields

$$\int_0^{\nu_c}\rho_{\mathrm{vac}}(\nu)\,d\nu = \frac{4\pi h}{c^3}\int_0^{\nu_c}\nu^3\,d\nu = \frac{\pi h}{c^3}\nu_c^4, \tag{7}$$

while integration of (6) over all frequencies yields the well-known Stefan–Boltzmann law

$$\int_0^{\infty}\rho_{\mathrm{rad}}(\nu,T)\,d\nu = \frac{\pi^2k^4}{15\hbar^3c^3}T^4. \tag{8}$$

Suppose Eq. (5) is valid only up to a cutoff frequency ν_c, due to new but as yet unknown physics. How might we determine ν_c? We propose using estimates of the dark energy density to place an upper limit on the value calculated from Eq. (7).

Current estimates [16,17] indicate that dark energy constitutes 73% of all energy in the universe. To calculate the dark energy density ρ_{dark} we need the critical energy density ρ_c of a flat universe (the data of [17] indicate that the universe is flat), which is $\rho_c = 10.539 h^2_{\text{Hubble}}\ \text{GeV/m}^3 = 10.539 \times (0.71 \pm 0.04)^2\ \text{GeV/m}^3$. Finally, we have

$$\rho_{\text{dark}} = 0.73\rho_c = (3.9 \pm 0.4)\ \text{GeV/m}^3. \tag{9}$$

If we set

$$\frac{\pi h}{c^3}\nu_c^4 \simeq \rho_{\text{dark}} \tag{10}$$

then

$$\nu_c \simeq (1.69 \pm 0.05) \times 10^{12}\ \text{Hz}. \tag{11}$$

3. Measurements of zero-point fluctuations in Josephson junctions

The behavior of a resistively shunted Josephson junction is modeled as a particle that moves in a tilted periodic potential, and the effect of the noise current is to produce random fluctuations of the tilt angle [12]. This situation is captured by the stochastic differential equation

$$\frac{\hbar C}{2e}\ddot{\delta} + \frac{\hbar}{2eR}\dot{\delta} + I_0 \sin\delta = I + I_N. \tag{12}$$

Here δ is the phase difference across the junction, R is the shunt resistor, C the capacitance of the junction, I is the mean current, I_0 the noise-free critical current, and I_N is the noise current. As shown in [12,23], the junction noise current should have a spectral density given by

$$S(\nu) = \frac{2h\nu}{R}\coth\left(\frac{h\nu}{kT}\right) = \frac{4h\nu}{R}\left(\frac{1}{2} + \frac{1}{\exp(h\nu/kT) - 1}\right). \tag{13}$$

The first term in Eq. (13) is due to vacuum fluctuations, and the second one is due to ordinary Bose–Einstein statistics. This predicted spectral behaviour has been experimentally verified in the work of [13] measuring the current noise in a resistively shunted Josephson junction at two different temperatures. Further, the computed cutoff frequency (11) is less than one order of magnitude larger than the highest frequency used in these experiments. Fig. 1 shows how well the predicted form of the power spectrum (13) is experimentally verified up to frequencies of order 6×10^{11} Hz (note that no fitting parameters are used in this figure). For more recent theoretical work on the quantum noise theory of Josephson junctions, see [24,25].

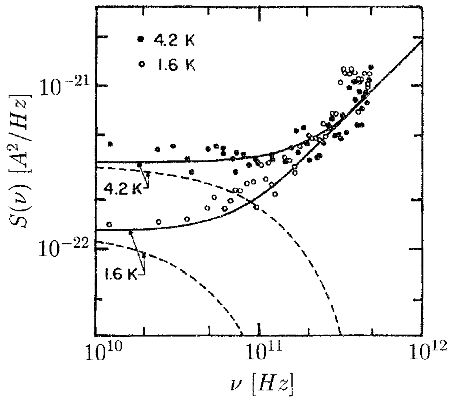

Fig. 1. Spectral density of current noise as measured in Koch et al.'s experiment [13] for two different temperatures. The solid line is the prediction of Eq. (13), whereas the dashed line is given by $(4h\nu/R)(\exp(h\nu/kT) - 1)^{-1}$.

Zero-point fluctuations thus have theoretically predicted and experimentally measured effects in Josephson junctions. We therefore expect that the energy density associated with these fluctuations has physical meaning as well: it is a prime candidate for dark energy, being isotropically distributed and temperature independent. Note that the experimentally measured fluctuations in Fig. 1 are physical reality and have to be distinguished from "theoretical" zero-point fluctuations that just formally enter into QED calculations without any cutoff. The vacuum energy associated with the measured data in Fig. 1 cannot be easily discussed away.

Assuming that the vacuum energy associated with the measured fluctuations in Fig. 1 is physically relevant, we predict that the measured spectrum in Josephson junction experiments must exhibit a cutoff at the critical frequency ν_c. If not, the corresponding vacuum energy density would exceed the currently measured dark energy density of the universe. In future experimental measurements that may reach higher frequencies one would have to carefully distinguish between intrinsic cutoffs (due to experimental constraints) and fundamental cutoffs (due to new physics).

4. Implications for dark energy from present and future experiments

4.1. Lower bound on dark energy density

The largest frequency reached in the Koch et al. [13] experiment was $\nu_{\max} \simeq 6 \times 10^{11}$ Hz $\approx \frac{1}{3}\nu_c$. From (7) this implies a minimum value of dark energy density in the universe:

$$\rho_{\text{dark}} \geqslant \frac{\pi h}{c^3} \nu_{\max}^4 = 0.062\ \text{GeV/m}^3. \tag{14}$$

If larger frequencies $\nu_{\max}$ could be reached in a similar experiment, they would provide a better lower bound.

4.2. $h\nu_c$ and neutrino masses

The energy associated with the computed cutoff frequency ν_c

$$E_c = h\nu_c = (7.00 \pm 0.17) \times 10^{-3}\ \text{eV} \tag{15}$$

coincides with current experimental estimates of neutrino masses. The LMA (large-mixing angle) solution of the solar neutrino problem yields a mass square difference of roughly $\Delta m^2_{\text{sun}} \simeq 7 \times 10^{-5}\ \text{eV}^2$ between two neutrino species [26]. Assuming a hierarchy of neutrino masses, this gives a neutrino mass of the order of magnitude $m_\nu \simeq 8 \times 10^{-3}$ eV.

If this coincidence is confirmed in future experiments, one might try to develop a theory that links the cutoff frequency of the zero-point fluctuations to an as yet unknown property of the neutrino sector of the standard model. For previous work that relates the dark energy scale to the mass of neutrinos, see [27]. Generally, in quantum field theory bosons are associated with positive vacuum energy and fermions with negative energies [28]. In supersymmetric models both contributions cancel exactly. To explain a coincidence of the type $h\nu_c \simeq m_\nu c^2$, a possible idea would be that negative vacuum energy associated with neutrinos (or neutrino-like particles) might cancel positive vacuum energy associated with photons as soon as the energy $E = h\nu$ exceeds the neutrino rest mass. A toy model of this type is worked out in Section 5.

4.3. Effective degrees of freedom contributing to dark energy

Photons and other particles contribute to the total vacuum energy density of the universe. General quantum field theoretical considerations imply that a particle of mass m and spin j makes a contribution [28]

$$\rho_{\text{vac}} = \frac{1}{2}(-1)^{2j}(2j+1)\int \frac{d^3k}{(2\pi)^3}\sqrt{\mathbf{k}^2 + m^2} \tag{16}$$

in units where $\hbar = c = 1$. Here $\mathbf{k}$ represents the momentum and the energy is given by $E = \sqrt{\mathbf{k}^2 + m^2}$. The integral is divergent and the actual contribution depends on the regularization scheme chosen.

It is likely that the Josephson junction experiment only measures vacuum fluctuations that couple to electric charge, since this experiment is purely based on electromagnetic interaction (see also [29] on possible interactions of mesoscopic quantum systems with gravity). Thus this experiment is likely to see only a fraction $\kappa < 1$ of the total dark energy of the universe. This would modify the expected cutoff frequency as

$$\nu_c = \left(\kappa \rho_{\text{dark}}^{\text{total}}\right)^{1/4}\left(\frac{c^3}{\pi h}\right)^{1/4}. \tag{17}$$

In particular, a small κ can significantly lower the cutoff frequency. A measurement of κ would thus give information on the effective number of degrees of freedom that produce the entire dark energy density of the universe.

5. Dark energy and $1/f$ noise

In the following we consider a simple model where a bosonic contribution to vacuum energy is suppressed by a fermionic contribution as soon as the energy exceeds $h\nu_c = mc^2$, where m is the mass of the fermion under consideration.

Assume $j = 1/2$. From Eq. (16) we obtain the fermionic contribution to the vacuum energy as

$$\rho_{\text{vac}}^{\text{ferm}} = -\int \frac{d^3k}{(2\pi)^3}\sqrt{\mathbf{k}^2 + m^2}$$
$$= -\frac{1}{2\pi^2}\int_0^{k_{\max}} k^2\sqrt{k^2 + m^2}\, dk, \tag{18}$$

where $k = |\mathbf{k}|$ and k_{max} is a suitable upper cutoff. Transforming from k to $E = \sqrt{k^2 + m^2}$ this can be written as

$$\rho_{vac}^{ferm} = -\frac{1}{2\pi^2} \int_m^{E_{max}} \sqrt{E^2 - m^2} E^2 \, dE. \tag{19}$$

Additionally the massless boson contributes with

$$\rho_{vac}^{bos} = +\frac{1}{2\pi^2} \int_0^{E_{max}} E^3 \, dE, \tag{20}$$

in agreement with Eq. (7), setting $E = h\nu$ and $\hbar = c = 1$. Adding the two contributions, one obtains

$$\rho_{vac} = \frac{1}{2\pi^2} \int_0^{E_{max}} \left(E^3 - \sqrt{E^2 - m^2} E^2 \theta(E - m)\right) dE, \tag{21}$$

where the θ-function is defined by

$$\theta(x) = \begin{cases} 1, & x \geqslant 0, \\ 0, & x < 0. \end{cases} \tag{22}$$

The integrand in Eq. (21), divided by E^2/π^2, represents the effective zero-point energy of this problem. Correlated vacuum fluctuations of this type would thus produce in Josephson junctions the power spectrum

$$S(\nu) = \frac{4}{R} \begin{cases} \frac{1}{2} h\nu, & h\nu \leqslant mc^2, \\ \frac{1}{2}\left(h\nu - \sqrt{h^2\nu^2 - m^2c^4}\right), & h\nu > mc^2. \end{cases} \tag{23}$$

There is a rapid decrease of spectral power above the critical frequency $h\nu_c = mc^2$. For frequencies $h\nu > mc^2$ Eq. (23) implies

$$S(\nu) = \frac{2}{R} h\nu \left(1 - \sqrt{1 - \frac{m^2c^4}{h^2\nu^2}}\right). \tag{24}$$

For large $h\nu$

$$\sqrt{1 - \frac{m^2c^4}{h^2\nu^2}} \approx 1 - \frac{m^2c^4}{2h^2\nu^2}, \tag{25}$$

and we have

$$S(\nu) = \frac{1}{R} m^2c^4 \frac{1}{h\nu}. \tag{26}$$

Thus, asymptotically, the vacuum fluctuation spectrum is inversely proportional to ν so suppressed vacuum fluctuations produce $1/f$ noise. $1/f$ noise is commonly observed in many electric circuits, and was also observed in Koch et al.'s experiment but was subtracted from the data [13]. Our simple theoretical considerations show that high-frequency $1/f$ noise can arise naturally if bosonic vacuum fluctuations are suppressed by fermionic ones. If the coefficient multiplying $1/\nu$ in Eq. (26) is measured in the experiment, then it can be used to determine the cutoff scale $h\nu_c = mc^2$.

6. Conclusion

We propose a repeat of the experiments of Koch et al. with new generations of Josephson junctions at higher frequencies. If it is possible to increase the maximum frequency by a factor of about 3, then this experiment could provide valuable information on the nature of dark energy. If the vacuum energy associated with the fluctuations measured in Fig. 1 is physically relevant, then we predict that a deviation from linear growth of $S(\nu)$ will be seen at higher frequencies, and in fact a rapid decrease of zero-point power near the critical frequency ν_c is expected. If this is not seen in the experiment, then we must conclude that the dark energy of the universe probably has nothing to do with vacuum fluctuations at all but is purely classical. Alternately, if this decrease is not observed another interpretation would be that the Josephson junction experiment is insensitive to the process which cancels the photonic vacuum energy at large frequencies. If the frequency cutoff is observed, it could be used to determine the fraction κ of dark energy density that is produced by electromagnetic processes. Moreover, if the Josephson junction experiment is repeated at different temperatures, then a possible temperature dependence of ν_c could provide information on whether the dark energy density is really independent of the expansion of the universe (i.e., its temperature) or whether it changes slightly with the expansion (as in the models [30]). Finally, we think that it could be interesting to analyze experimentally observed high-frequency $1/f$ noise in electrical circuits under the hypothesis that it could be a possible manifestation of suppressed zero-point fluctuations.

References

[1] M. Planck, The Theory of Heat Radiation, Blakistons, 1914.
[2] M. Planck, The Theory of Heat Radiation, American Institute of Physics, New York, 1988.
[3] W. Nernst, Verh. Dtsch. Phys. Ges. 18 (1916) 83.
[4] P. Debye, Ann. Phys. 43 (1914) 49.
[5] T. Welton, Phys. Rev. 74 (1948) 1157.
[6] E. Power, Am. J. Phys. 34 (1966) 516.
[7] H. Casimir, Kon. Ned. Akad. Wetensch. B 51 (1948) 793.
[8] H. Casimir, D. Polder, Phys. Rev. 73 (1948) 360.
[9] G. Bressi, et al., Phys. Rev. Lett. 88 (2002) 041804.
[10] T. Boyer, Phys. Rev. D 8 (1973) 1679.
[11] T. Boyer, Phys. Rev. A 36 (1987) 5083.
[12] R.H. Koch, D. van Harlingen, J. Clarke, Phys. Rev. Lett. 45 (1980) 2132.
[13] R.H. Koch, D. van Harlingen, J. Clarke, Phys. Rev. B 26 (1982) 74.
[14] J. Weber, Phys. Rev. 90 (1953) 977.
[15] I. Senitzky, Phys. Rev. 119 (1960) 670.
[16] C.L. Bennett, et al., Astrophys. J. Supp. Ser. 148 (2003) 1, astro-ph/0302207.
[17] D.N. Spergel, et al., Astrophys. J. Supp. Ser. 148 (2003) 148, astro-ph/0302209.
[18] P.J.E. Peebles, B. Ratra, Rev. Mod. Phys. 75 (2003) 559, astro-ph/0207347;
S. Weinberg, astro-ph/0005265;
M. Trodden, S. Caroll, astro-ph/0401547;
A.D. Dolgov, hep-ph/0405089;
P.H. Frampton, astro-ph/0409166.
[19] C. Beck, Phys. Rev. D 69 (2004) 123515, astro-ph/0310479.
[20] H.B. Wang, P.H. Wu, T. Yamashita, Phys. Rev. Lett. 87 (2001) 107002.
[21] Y.Y. Divin, et al., Physica C 372–376 (2002) 416.
[22] P.W. Milonni, The Quantum Vacuum: An Introduction to Quantum Electrodynamics, Academic Press, Boston, 1994.
[23] H. Callen, T. Welton, Phys. Rev. 83 (1951) 34.
[24] C.W. Gardiner, Quantum Noise, Springer, Berlin, 1991.
[25] Y. Levinson, Phys. Rev. B 67 (2003) 184505.
[26] P. Aliani, et al., hep-ph/0309156.
[27] P.Q. Hung, hep-ph/0010126;
P.Q. Hung, H. Paes, astro-ph/0311131;
R. Fardon, A.E. Nelson, N. Weiner, astro-ph/0309800;
D.B. Kaplan, A.E. Nelson, N. Weiner, hep-ph/0401099.
[28] P. Wess, Introduction to Supersymmetry and Supergravity, World Scientific, Singapore, 1990.
[29] C. Kiefer, C. Weber, gr-qc/0408010.
[30] P.J.E. Peebles, B. Ratra, Astrophys. J. 325 (1988) L17;
M.S. Turner, M. White, Phys. Rev. D 56 (1997) 4439;
R.R. Caldwell, R. Dave, P.J. Steinhardt, Phys. Rev. Lett. 80 (1998) 1582.