

Development of a runway surface de-icing model to predict the melting time

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Abstract— Runway de-icing plays an important role in guaranteeing the safety of the aviation industry. Large quantities of de-icing products are applied to airport runways. After their applications, the chemicals, which didn't disperse in the environment, cause the waste of de-icing agents and environmental pollution. Any reduction in the use of chemicals would help to reduce the risk of environmental contamination in areas surrounding transportation infrastructure. In this article, we propose a mathematical model for runway de-icing. We start from a test case for the two-phase Stefan problem and verify it with Neumann's solution. Our runway model considers the time, also temperature changes inside the material along the normal direction. The runway model predicts the active time of the de-icer agents and the temperature evolution on an iced runway. We use a front tracking scheme with the enthalpy method to solve the Stefan problem. The temperature in the solid, liquid, and interface is obtained from the enthalpy. In the end, the parametric studies are done. We investigate the effect of the heat transfer coefficient. Also, a mesh study is done on the results and is presented in the last section.

Keywords— *de-icing, heat transfer, runway de-icer, melting model, enthalpy method*

I. INTRODUCTION

Runway de-icing plays an important role in guaranteeing the safety of the aviation industry. In practice, the airports use as much de-icer agents as possible to clear the runway surface of snow/ice. Large quantities of de-icing products are applied to airport runways to ensure safe take-off and landing in adverse weather conditions. After their applications, the chemicals, which didn't disperse in the environment, cause the waste of de-icing agents and environmental pollution. Limited attention has been paid to the assessment and determination of the optimal quantities of de-icing products to be spread on the runways. Any reduction in the use of chemicals would help to reduce the risk of environmental contamination in areas surrounding transportation infrastructure.

The majority of the models in this field have been developed for the roads. The article "A coupled model to simulate snow behavior on roads" can be mentioned as one of the first researches done in this field [1]. There are some models to predict how much de-icer is needed. They also predict how long it takes to melt the snow on the road. Almost all the models calculate the temperature of the pavement. BPRT (Bare Pavement Regain Time) is one kind of model that predicts the amount of de-icer needed with considering the time that will take the pavement clear from ice([2], [3]). The WIS (Water, Ice, and Salt) model predicts the amount of salt applied on the frozen road by calculating the time change of the salt solution temperature [4]. RSF_SV (Road Surface

Freezing_Salt, Vehicle) model combines the effects of salt application with a conventional road surface freezing prediction model. This model depends on the heat and water balance on the road surface [5].

There is a model for the runway [6] based on the road models. It is a case study for Oslo airport in Norway. This model gives the pavement surface temperatures. It predicts pavement surface temperature based on the heat fluxes at the pavement surface of which some are affected by the mass fluxes and surface conditions [6].

Still, a model for runway de-icing is needed to consider the specificity of airport operations. In this article, we propose a mathematical model for runway de-icing. We start from a test case for the two-phase Stefan problem [7] and verify it with Neumann's solution. Our runway model considers the time and the temperature changes inside the material along the normal direction. The runway model predicts the active time of the de-icer agents and the temperature evolution on an iced runway. The model solves the heat and mass transfer between the solid and liquid states with a melting front at the interface. We use a front tracking scheme with the enthalpy method to solve the Stefan problem. The temperature in the solid, liquid, and interface is obtained from the enthalpy.

After the numerical method presentation, the parametric studies are done. We investigate the effect of the heat transfer coefficient. Also, a mesh study is done on the results and is presented in the last section.

II. MODEL AND NUMERICAL METHOD

The model can calculate the melting front location, the ice temperature, and the solution concentration with the passage of time. As illustrated in Figure 1 (the scheme of the de-icing model), there are three phases between the runway surface and the air. This is the simultaneous phase transition of the solid phase (ice film), the mixture of solid and liquid (mushy zone), and the liquid phase (solution of de-icer and water).

A. Assumption

The assumption of the model is in the following:

- The heat conduction in the y-axis or z-axis is not taken into account, and the problem is investigated in the vertical direction (x-axis). Thus, the model is in one spatial dimension.
- The deicer solution is in the system from the beginning.
- The substance of solute in the liquid is constant per unit area and there is a solution in which the solute is evenly distributed.

- The ice film has a uniform thickness.
- The runway pavement under the ice is adiabatic and there is no heat flux.
- The mushy zone (mixture phase) is the sharp interface and is considered as a line between the solid and liquid phases.
- The thermophysical parameters are constant (k_s (solid thermal conductivity), c_s (specific heat of solid), k_L (liquid thermal conductivity), c_L (specific heat of liquid), ρ (density)). Thus, the diffusivities in both the solid and the liquid phases are constant ($\alpha_L = k_L/\rho c_L$, $\alpha_s = k_s/\rho c_s$)

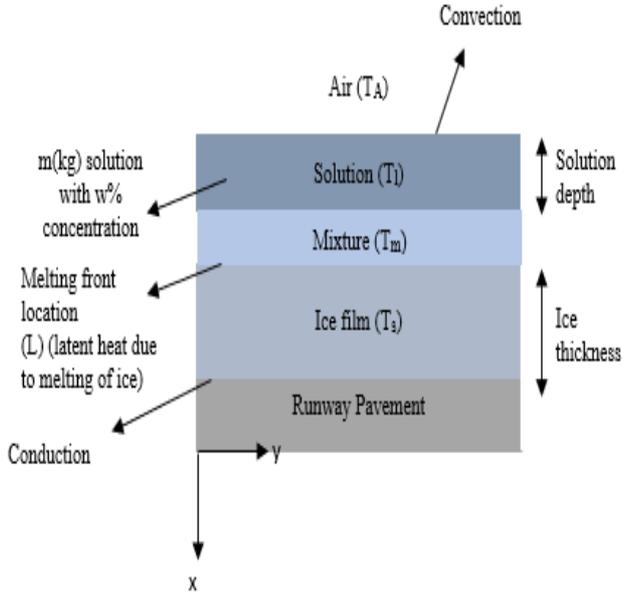


Fig. 1 Scheme of the de-icing model

B. Governing Equation

A fundamental quantity that plays a role in heat conduction is the temperature which is measured in degree Kelvin (K) or Celsius (°C). Enthalpy is the thermodynamic quantity equivalent to the total heat absorbed by material in constant pressure. The enthalpy per unit mass is denoted by e (which is measured in kJ/kg) and per unit volume by E (which is measured in kJ/m^3). The heat absorbed for pure material under constant pressure and volume is related to temperature by

$$de = c dT \quad (1)$$

where, c is specific heat define by

$$cT = de/dT \quad (2)$$

The heat flux denote by q is the amount of heat crossing a unit area per time and given by

$$q = -k\nabla T \quad (3)$$

This is Fourier's Law where; k is the thermal conductivity measured in $kJ/m K$. The first law of conductive heat transfer is the energy conservation law given by

$$\frac{\partial}{\partial t} \rho e + \nabla \cdot q = f \quad (4)$$

where f is a sink or source heat. With the equations (1), (3), and (4), we have a heat conduction equation:

$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot (k\nabla T) + f \quad (5)$$

This is a well-posedness partial differential equation satisfied by temperature $T(\vec{x}, t)$, at the location \vec{x} at time t . In the heat conduction equation, ρ is density, k is thermal conductivity, and c is specific heat. To have a well-posed problem in a spatial domain Ω for $t > 0$, we should have the following information:

$$\text{Initial condition } T(x, 0) = T_{init}(x), \quad \forall x \in \Omega \quad (6)$$

and

$$\text{Boundary condition for } T(x, 0), \quad \forall x \in \Omega \quad (7)$$

Since we considered k (thermal conductivity) constant, then we can introduce the thermal diffusivity

$$\alpha = k/\rho c, \quad (m^2/s) \quad (8)$$

so, we have the heat conduction equation:

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T + f \quad (9)$$

which is a linear parabolic partial differential equation. We assumed that we have a one-dimension problem and we do not have a heat source or sink ($f = 0$). Then we have

$$T_t = \alpha T_{xx} \quad (10)$$

In this article, the classical Stefan problem is used as a mathematical model [7]. The liquid and solid phases are separated with a sharp interface $X(t)$. We consider a one-dimension problem with $0 < x < l$ domain. From $0 < x < X(t)$ is located the liquid part and from $X(t) < x < l$ is located the solid part. The temperatures of the liquid and solid are T_L and T_S respectively. The melting temperature is T_m . This is also the interface temperature between the two phases. There is a convective flux at the face $x = 0$ and the back face ($x = l$) is insulated.

Thus, there is a heat equation for the phases:

$$T_t = \alpha_L T_{xx} \quad \text{for } 0 < x < X(t) \text{ (liquid phase)} \quad (11)$$

$$T_t = \alpha_s T_{xx} \quad \text{for } X(t) < x < l \text{ (solid phase)} \quad (12)$$

We have interface conditions:

$$T(X(t), t) = T_m, \quad t > 0 \quad (13)$$

$$\rho L X'(t) = -k_L T_x(X(t)^-, t) + k_s T_x(X(t)^+, t), \quad t > 0 \quad (14)$$

The initial conditions are:

$$X(0) = X_0 \quad (15)$$

$$T(x, 0) = T_{init(solid)} < T_m, \text{ in solid region} \quad (16)$$

$$T(x, 0) = T_{init(liquid)} > T_m, \text{ in liquid region} \quad (17)$$

The boundary conditions are: at $x=l$:

$$-k_s T_x(l, t) = 0, \quad t > 0 \quad (18)$$

(insulated boundary)

at $x=0$

If we have convective flux:

$$-kT(x, 0) = h[T_\infty(0, t) - T(0, t)], \quad t > 0 \quad (19)$$

and, if we have imposed temperature:

$$T(0, t) = T_0(t), \quad t > 0 \quad (20)$$

where, L is the latent heat of fusion, ρ is the density, c_s and c_L are the specific heat capacity of solid and liquid respectively, k_L is the liquid thermal conductivity and k_s is the solid thermal conductivity.

we have a dimensionless parameter that is useful in analysing a Stefan problem which is Stefan number (St). The Stefan number is the ratio of the sensible heat to latent heat, which is defined as following [8]:

$$St = c\Delta t/L \quad (21)$$

C. Numerical method

In this article, the enthalpy method is used to solve the phase change problem. The enthalpy approach is flexible and simple [7]. In this method, the Stefan condition (jump condition) is not imposed on the solution and is automatically obeyed (as a natural condition).

For the problem discretization, the finite difference method is used, and the problem is solved explicitly. As Figure 2 illustrates, we partition the region into a finite number (M) of control volumes (V_j) and integrate the heat balance over V_j and time interval. We have a node x_j inside each control volume (midpoints) and $x_{j-\frac{1}{2}}$, $x_{j+\frac{1}{2}}$ are the end points of the j -th subinterval. [7]

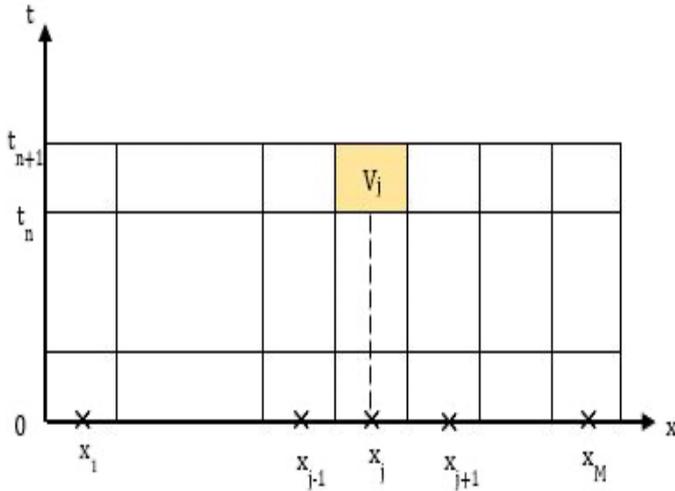


Fig. 2 Space-time grid

For the initial interface location

$$X^0 = X_{init} \quad (22)$$

At first, we know the phase and temperature of each control volume

$$T_j^n = \begin{cases} T_l, & j\Delta x \leq X^0 \quad (\text{liquid}) \\ T_m, & j\Delta x < X^0 \leq (j-1)\Delta x \quad (\text{interface}) \\ T_s, & (j-1)\Delta x > X^0 \quad (\text{solid}) \end{cases} \quad (23)$$

we can determine the initial liquid fraction with:

$$\lambda_j^n = \begin{cases} 1, & T_j^n = T_l \quad (\text{liquid}) \\ (X^0 - (j-1)\Delta x)/\Delta x, & T_j^n = T_m \quad (\text{interface}) \\ 0, & T_j^n = T_s \quad (\text{solid}) \end{cases} \quad (24)$$

we calculate the initial enthalpies with:

$$E_j^0 = \begin{cases} \rho L + \rho c_l [T - T_m], & \lambda_j^n = 1 \\ \rho c_s [T - T_m], & \lambda_j^n = 0 \\ \lambda_j^n (\rho L), & 0 < \lambda_j^n < 1 \end{cases} \quad (25)$$

and we compute conductivity from:

$$\frac{1}{k_j^n} = \frac{\lambda_j^n}{k_L} + \frac{1 - \lambda_j^n}{k_s} \quad (26)$$

for the resistances and fluxes, we have:

$$q_{j-1/2}^n = -\frac{T_j^n - T_{j-1}^n}{R_{j-1/2}}$$

$$\text{with } R_{j-1/2} = \frac{1}{2} \frac{\Delta x_{j-1}}{k_{j-1}} + \frac{1}{2} \frac{\Delta x_j}{k_j}$$

$$, j = 2, \dots, M \quad (27)$$

we have boundary condition at $x=0$

$$q_{1/2}^n = -\frac{T_1^n - T_0^n}{\frac{1}{h} + R_{1/2}}, \quad R_{1/2} = \frac{1}{2} \frac{\Delta x}{k_1} \quad (28)$$

and at $x=l$

$$q_{M+1/2}^n = 0 \quad (29)$$

An explicit determination of the enthalpy approximation E_j^{n+1} of E at the time t_{n+1} is:

$$E_j^{n+1} = E_j^n + \frac{\Delta t_n}{\Delta x_j} [q_{j-1/2}^n - q_{j+1/2}^n], \quad j = 1, \dots, M \quad (30)$$

and for updating the temperature we use:

$$T_j^n = \begin{cases} T_m + \frac{E_j^n}{\rho c_s}, & E_j^n \leq 0 \quad (\text{solid}) \\ T_m, & 0 \leq E_j^n \leq \rho L \quad (\text{interface}) \\ T_m + \frac{E_j^n - \rho L}{\rho c_L}, & E_j^n \geq \rho L \quad (\text{liquid}) \end{cases} \quad (31)$$

If the mushy node is the m -th node at time t_n , the interface location can be calculated with the following equation:

$$X^n = x_{m-\frac{1}{2}} + \lambda_m^n \Delta x_m \quad (32)$$

The melting point temperature depends on the concentration of the solution. When the melting phenomenon occurs, the solution is diluted with the passage of time. As a result, as the concentration of the solution decreases, the melting temperature increases. The melting point depression for the solvent is proportional to the molality m_B of the solute:

$$\Delta T_f = T_{f(\text{pure solvent})} - T_{f(\text{solution})} \quad (33)$$

with $\Delta T_f = K_f m_B$ and with the Van't Hoff factor (i) we have $m_B = i m_{solute}$ for molality of the solution, so:

$$\Delta T_f = T_{f(pure\ solvent)} - K_f i m_{solute} \quad (34)$$

which, K_f (cryoscopic constant) is 1.86 (K kg/mol) for water, and m_{solute} is the molality of solute in the solution. It equals the amount of substance of solute (n_{solute}) in the mass of solution ($m_{solution}$).

$$m_{solute} = \frac{n_{solute}}{m_{solution}} \quad (35)$$

In section 2-A, we assumed that the substance of solute in the liquid is constant per unit area (n_A) and there is a solution in which the solute is evenly distributed. The solution mass is related to the volume of the solution region, $m_{solution}(t) = \rho v_{solution}(t)$ and $v_{solution}(t) = A_{solution} X(t)$, so we have [9]:

$$m_{solute} = \frac{n_{solute}}{m_{solution}} = \frac{n_A A_{solution}}{\rho v_{solution}(t)} = \frac{n_A}{\rho X(t)} \quad (36)$$

thus, we have an explicit equation for the melting temperature:

$$T_{m(solution)} = T_{m(pure\ solvent)} - \frac{K_f i n_A}{\rho X(t)} \quad (37)$$

III. VERIFICATION RESULTS

In this section, we propose to present the results of the verification of the model. It contains two steps. The first step is to verify the two-phase Stefan problem suggested by V. Alexiades and A. D. Solomon. The second step is doing the parametric study for our runway model.

A. Test Case Verification

For verification, firstly we verify the two-phase Stefan problem suggested by V. Alexiades and A. D. Solomon [7] with the Neumann solution. We will explain the problem in the following.

Problem definition:

It is the two-phase Stefan problem with constant imposed temperature at $x=0$. We have a slab $0 < x < 1$ m that is initially solid with temperature $T_s = -1$ °C. The melting temperature is constant for this problem. The simulation was done until the interface location reaches half of the slab. The final time is 3.5 seconds based on the definition of the test case.

The inputs which are used for this test case are

$$T_m = 0 \text{ °C}, T_L = 1 \text{ °C}, T_s = -1 \text{ °C}, \rho = 1 \frac{kg}{m^3}, \quad (38)$$

$$c_L = 1 \frac{j}{kg} \text{ °C}, k_L = 1 \text{ w/mK}$$

and the imposed temperature is $T(\text{air})=1$ °C. These values are chosen to make identical to the dimensionless formulation.

Based on the thermophysical parameters in equation 38, they are all equal to one, so the latent heat $L=1/St$. We also have $\alpha = k/\rho c = 1$. The simulation is done with the model explicitly for $St=0.1$ (Stefan number), $M=100$ (the number of nodes) and the stability condition based on [10] is $\Delta t_{exp} < \Delta x^2/2 \max(\alpha)$. So, the time step is $\Delta t_{exp} = \Delta x^2/2 \max(\alpha)$. The results are verified with the Neumann's exact solution.

The results of the interface location as a function of time for time span 1.8 (s) in Figure 3. The vertical axis is the

location of the melting front, and the horizontal axis is time. As we mentioned before, we simulate the problem until the interface reaches the middle of the slab. the results have a good agreement until the melting process reaches half of the slab. The small error that we have is because the analytical solution is considered a semi-infinite problem, but in the numerical method, we used the finite slab $0 < x < 1$.

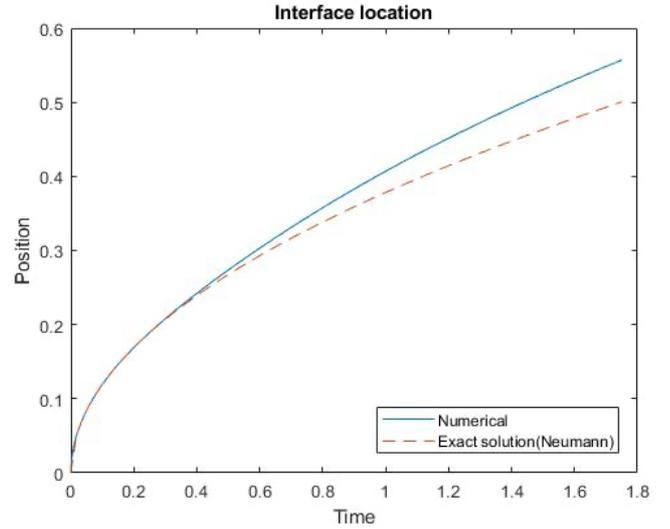


Fig. 3 Interface location (m) as a function of time, for a time span of 1.8 seconds

Figure 4 shows the temperature profile from $x=0$ to half of the slab. The dash line is for the exact solution (Neumann's solution) which are written as $x=0, 0.1, 0.2, 0.3, 0.4$ (m). The solid line is for numerical results which are written as $x=0n, 0.1n, 0.2n, 0.3n, 0.4n$ (m). In the figure, the vertical axis belongs to temperature from -1 °C (solid temperature) to 1 °C (liquid temperature), and the x -axis is the time. This figure illustrates that in the positions near the boundary ($0.1, 0.2$), the temperature as a result of the imposed temperature ($T=1$ °C) goes up sharply and reaches the melting temperature ($T_m = 0$ °C). Then it starts to melt and becomes liquid, and the temperature increase.

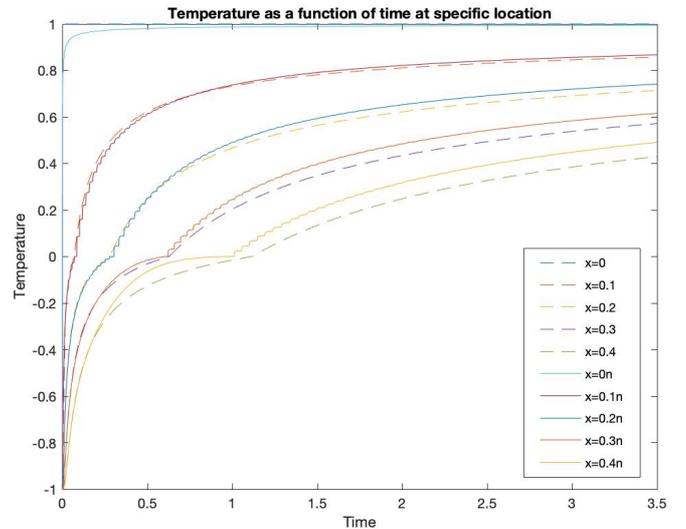


Fig. 4 Temperature profile (°C) until half of the slab in 3.5 seconds

B. Parametric Verification of the Model

After verifying the code against the two-phase Stefan problem solution (with non-dimensional parameters) with Neumann's solution, we consider having a de-icer solution as the liquid part. As a result of the existing de-icer, there is a variable melting temperature that depends on the concentration of de-icer in the solution (Eq37).

The experimental tests are done with the de-icer agents for the runway at Anti-Icing Materials International Laboratory (AMIL) in Chicoutimi. The experimental set-up uses the Petri dishes that have ice samples in them. The experimentalist applies a specific amount of de-icer on the ice sample and monitors the temperature evolution in space and time with a camera. The camera is fixed above the sample. It is done in a room with a specific temperature. They used SAE Aerospace Standard AIR6170A [11] document for the ice melting test.

The experimental set-up is modeled as a two-phase Stefan problem with the following values:

for the liquid part:

- 5 g KAC (potassium acetate) solution (50% concentration)
- Initial temperature $-2\text{ }^{\circ}\text{C}$

for the solid part

- 60g of ice with temperature $-10\text{ }^{\circ}\text{C}$
- The thickness of ice is $34 \times 10^{-4}\text{ m}$

for air:

- The air temperature $-2\text{ }^{\circ}\text{C}$

and the other values we used for the simulation:

- Latent heat $L=333 \times 10^3\text{ j/kg}$
- $M= 40$ (number of nodes)
- $h=5\text{ W/m}^2\text{K}$ (heat transfer coefficient)
- $\rho = 1500\text{ kg/m}^3$, $c_L = 3300\text{ j/kg }^{\circ}\text{C}$, $k_L = 0.48\text{ W/mK}$
- $c_s = 2093\text{ j/kg }^{\circ}\text{C}$, $k_s = 2.30\text{ W/mK}$
- $\Delta t_{exp} = \Delta x^2/2 \max(\alpha)$ (Time step as we mentioned in 3_A)

with these values, we obtained the results for the temperature profile which is shown in Figure5. You can see the results for the temperature profile at location $x= 4.3 \times 10^{-5}\text{m}$, $1.28 \times 10^{-4}\text{m}$, $2.13 \times 10^{-4}\text{m}$, $2.98 \times 10^{-4}\text{m}$, $3.83 \times 10^{-4}\text{m}$. The x-axis of the figure corresponds to time and the y-axis is the temperature in $^{\circ}\text{C}$. The temperature results show that in the solution part, the temperature drops sharply at first because the concentration of the solution is maximum. In the model, we consider the interface temperature is at the melting temperature. So, if we are in the mushy zone, it means that the temperature is equal to the melting temperature. We see a decrease in temperature to $-32\text{ }^{\circ}\text{C}$ in the first position at the first time. After that, because the solution becomes diluted, we have a rise in the temperature. In the other positions, also we have a drop in the temperature at the first time then it goes up, and before time 0.5s the temperatures reach $-11\text{ }^{\circ}\text{C}$.

Figure6 illustrates the amount of melted mass of ice. We have the amount of melted ice in kg on the y-axis and time on the x-axis. the melting phenomenon occurs from the beginning. It happens because of the existing de-icer. Due to

the presence of KAC, the melting temperature comes down to -32 and melting occurs. For this reason, we are seeing a decrease in temperature in this area. We have 8.10^{-6} kg melted ice until $t=1\text{ s}$.

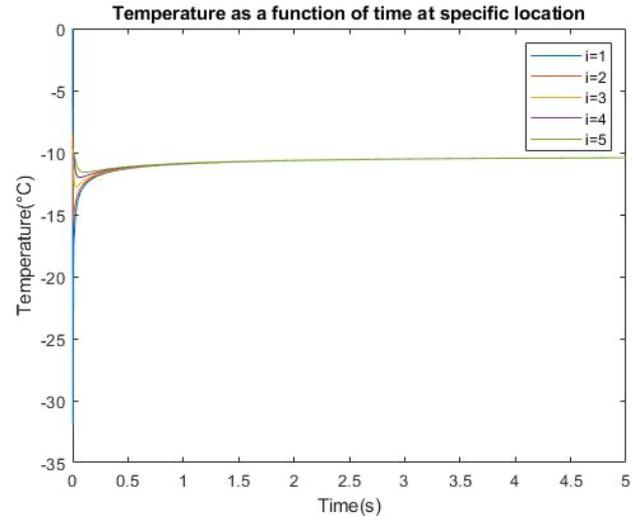


Fig. 5 Temperature profile after 5 seconds

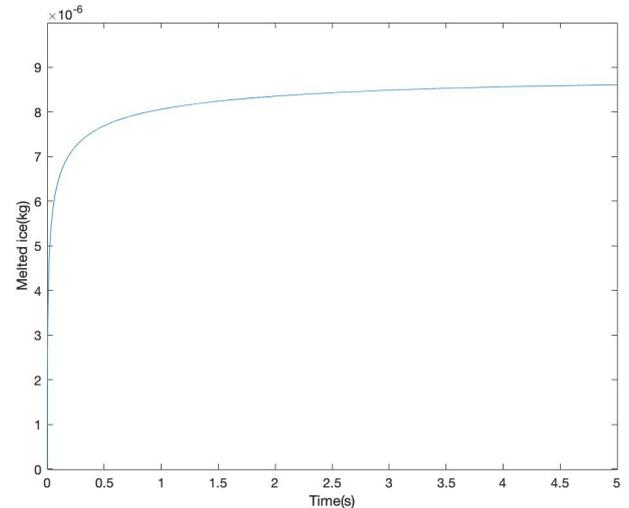


Fig. 6 Mass of ice melted in 5 seconds

1) Mesh Study on the Results

In this section, we study the effect of different meshes on the results to conclude that the model is independent of the mesh size.

The results of the amount of ice melted illustrate in Figure7 with three different node numbers 40,60, and 100. On the y-axis we have melted ice and on the x-axis we have time. You can see the results of the temperature profile at the location $x=1.659 \times 10^{-3}\text{ m}$ for the different numbers of mesh in Figure8.

As shown in Figure7, the difference of the results for the mass of ice melted is very small with increasing the mesh number. We have a small difference in the temperature results (Figure8) when the number of nodes is increasing. Hence, a mesh study verifies that the model's results are not dependent on the mesh size.

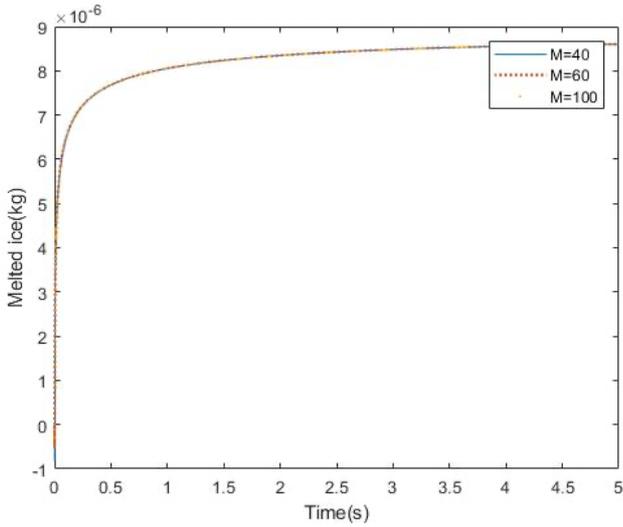


Fig. 7 Melted ice with meshes= 40, 60, 100

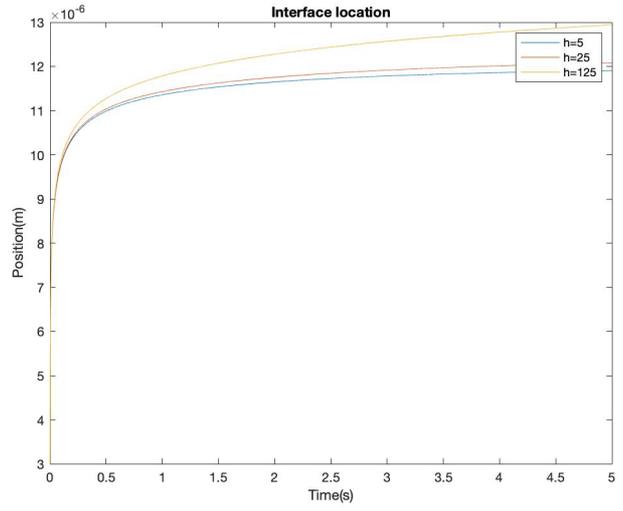


Fig. 9 Interface location in 5 sec with $h= 5, 25, 125 \text{ w/m}^2\text{K}$

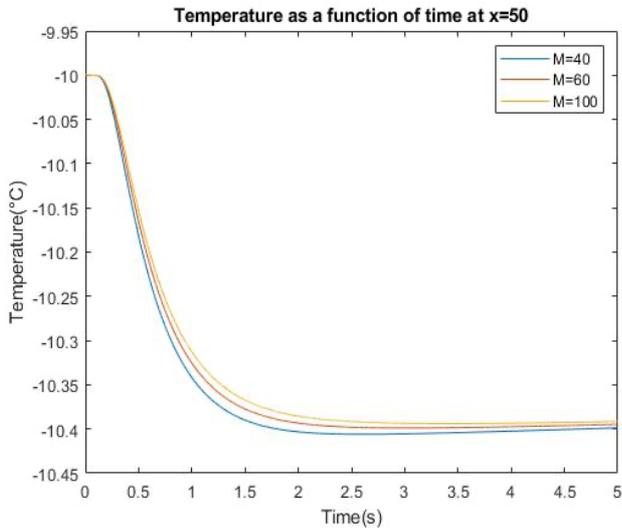


Fig. 8 Temperature in the middle with meshes=40, 60, 100

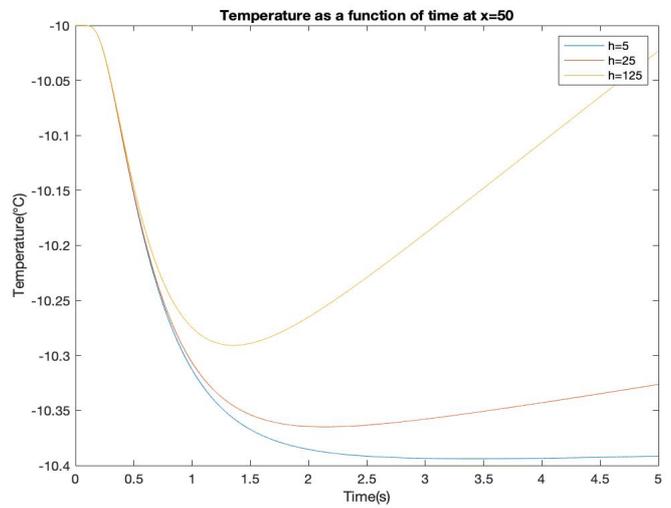


Fig. 10 Temperature profile in the middle with $h= 5, 25, 125 \text{ W/m}^2\text{K}$

2) *Effect of Heat Transfer Coefficient:*

For the parametric verification, we used three values for h (heat transfer coefficient) and analyzed the results. We chose h for parametric study because most probably this value has a major effect. The results obtained for the melting front location present in Figure9, and Figure10 illustrate the temperature in the middle ($x= 1,659 \times 10^{-3} \text{ m}$) as a function of time. You can see the mass of ice melted with three values $h=5, 25,$ and $125 \text{ (w/m}^2\text{K)}$ in Figure11.

As is shown in Figure9 and Figure10, the results of interface location and the temperature for the free convection heat transfer coefficient $h=5, 25, 125 \text{ (w/m}^2\text{K)}$ are shown.

As in Figures9 and Figure11 are illustrated, the major mass of melting happens before $t= 0.5$ at the initial time. With $h=125 \text{ w/m}^2\text{K}$ we still have the melting, and the amount of melted ice increased 10^{-6} kg .

In Figure10, you can see the temperature drops until $t=1.5\text{s}$. Decreasing in temperature with $h=125$ is less than $h=25$, also the same thing with $h=25$ decreasing in temperature is less than $h=5$.

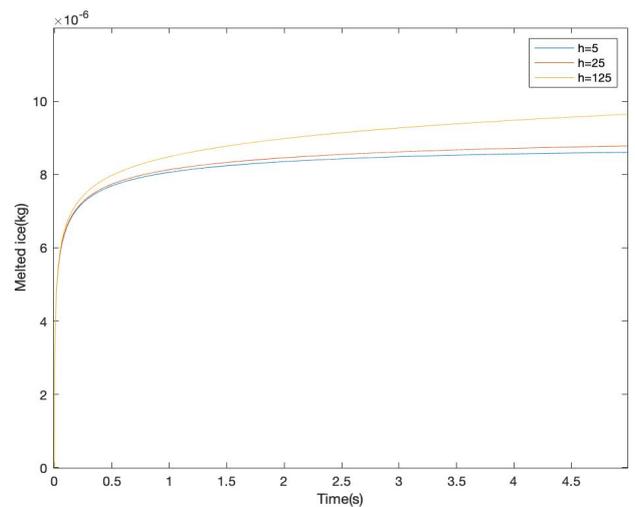


Fig. 11 amount of melted ice with $h=5, 25, 125 \text{ W/m}^2\text{K}$

IV. CONCLUSION

The objective of this paper is to develop a model for deciding the runway. Based on the literature reviewed, a two-

phase Stefan problem was selected. At first, we presented the model and numerical method. Then we verified the two-phase Stefan problem with dimensionless parameters to solve the ice melting problem. The enthalpy method was chosen for the numerical part, and it solves explicitly with the finite difference method. It was compared with the exact solution (Neumann's solution) of the two-phase Stefan problem to verify the code.

Thereafter, we implement the physical conditions regarding de-icing problems to build a runway model. In our model, we tried to mimic the standard document. The intended melting temperature depends on the concentration of KAC (potassium acetate) in the solution.

The results of the temperature profile, the melting front location as well as the mass of the melted ice are presented. In the end, the mesh study was performed to verify the model is independent from the mesh size. A parametric study was done to verify the model. We investigated the effect of the heat transfer coefficient on the results.

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