

# A Combined Analytical-CFD Approach for Wind Turbine Icing Prediction

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**Abstract**— Icing is a complex problem facing wind turbines operating in Nordic climates. It is affected by various instantaneously fluctuating parameters. Due to the deformation of blades' airfoil on account of icing, a significant drop in aerodynamic performance brings turbines to lose much of their productivity. Modelling and simulation became indispensable tools to estimate the effect of icing on the operation of wind turbines. However, the analysis of the iced airfoils via simulation is not practical for real-time ice prediction. This paper uses a combined analytical-CFD simulation approach to rapidly estimate the aerodynamic parameters of iced airfoils. The method is used to create a database of aerodynamic losses for several scenarios of weather conditions. The results have been processed by a neural network optimization algorithm to predict the aerodynamic losses of iced airfoils under varying scenarios of icing conditions. The ANN analysis results demonstrated consistency between the different scenarios of the database. They also highlighted the important influence of certain simulation parameters, such as the liquid water content and the angle of attack. Further investigation is recommended to determine correlations between the relevant parameters.

**Keywords**— *wind turbine icing; modelling of ice accretion; icing simulation; CFD; aerodynamic performance*

## I. INTRODUCTION

Wind turbines are important alternative sources of energy to combat climate change. Very high wind potential is available in northern regions, especially in winter. The most challenging problem of wind turbine exploitation in very cold environments is icing.

The ice accretion phenomenon is treated in this paper in terms of its main impact on wind turbines, i.e., the estimation of production loss due to the geometric deformation of the blades' airfoils. The problem of icing is addressed in this study being a complex phenomenon whose resolution calls on several areas of knowledge [1]. At the same time, the metrological and wind turbines' operational parameters vary considerably according to the wind farm site. Hence, the need to optimize icing protection methods to adapt wind turbines to local icing conditions.

The literature review showed that modelling and simulation are essential tools to estimate the production loss in order to optimize wind turbines' operation under specific icing conditions. These tools depend on Computational Fluid Dynamics (CFD) to estimate the aerodynamic coefficients for the iced airfoils [2]. Then, these coefficients can be extrapolated to generate the power curves for both clean and iced wind turbines in order to estimate the production loss. The overall process has been discussed in a review of studies on the CFD-BEM approach for estimating power losses of iced-up wind turbines [2]. The modelling approaches and the

simulation techniques adopted for wind turbine icing are also discussed and synthesized in another review article [3]. These approaches and techniques have been adopted in this research study, having been reviewed and recommended in the literature.

Estimating the aerodynamic losses due to icing via simulation becomes unfeasible when it comes to modelling several scenarios of icing conditions or for the real-time prediction of icing. Alternatively, a combined analytic-CFD proposed in this paper can help rapidly estimate the aerodynamic losses of iced airfoils under multiple icing conditions.

The objective of this research study consists in its finality to achieve an intelligent system with reduced parameters, which makes it possible to identify, in real-time, the meteorological conditions favourable to the occurrence of icing events, as well as to predict the form, type and severity of ice accretion and its impact on wind turbines in order to optimize wind turbine operation scenarios in icing conditions. In this vision, the availability of a method for estimating aerodynamic losses for a combination of geometric design, meteorological and operational parameters of the wind turbine is crucial to achieving this final objective. This work brings together several research studies to recommend a strategy to quickly create a database of aerodynamic losses in several scenarios of metrological conditions without the need to analyze icing airfoils using costly simulations.

The proposed method depends on an analytical model referenced in the ISO 12494 standard for ice accretion on a cylinder [4]. In a previous study submitted for publication, this model was used with the CFD simulations to generate a conversion factor between the mass of ice collected on a cylinder and an airfoil. The selected airfoil is the NACA 64-618, located at a section of 97% of the blade span of the NREL 5MW wind turbine [5]. This blade's section has the smallest chord and thickness to chord ratio and the highest speed of the blade airfoils; Therefore, more ice accumulation is found in this section [6, 7]. The averaged conversion factor has been used in this paper to create a database of aerodynamic losses under several icing conditions.

The scenarios created in the database using the proposed method have been analyzed by an artificial neural network (ANN) with the aim of developing correlations between icing conditions and production losses. The ANN analysis of the databases showed the importance of specific parameters in estimating aerodynamic losses via simulation.

The results examined with the artificial neural network demonstrated consistency between the different scenarios. They also demonstrated the important influence of specific

parameters. The proposed method could help find correlations between the meteorological conditions and the aerodynamic losses of the iced airfoils. The remainder of the paper is organized in the following sections:

- Section II presents the combined analytical-CFD simulation approach used to calculate the averaged conversion factor. This approach can rapidly estimate the aerodynamic losses of iced airfoils for several scenarios of weather conditions.
- Section III presents the proposed method for the analytical estimation of aerodynamic losses. An extract from the database created for iced airfoils is also presented.
- Section IV presents the use of a neural network optimization algorithm to predict the aerodynamic losses of wind turbine blade airfoil under varying scenarios of icing conditions.
- Section V discussed the results of the ANN analysis on the database.
- Section VI presents the conclusion and recommendations to enhance the prediction and make use of it.

## II. COMBINED ANALYTICAL-CFD SIMULATION APPROACH

A method validated and incorporated in another paper submitted for publication has been used in this study to rapidly estimate the aerodynamic characteristics of the NACA 64-618 iced airfoil. The method depends on a combined analytical-CFD approach used to create a conversion factor between the ice accreted on the airfoil and that accreted on a reference collector calculated analytically. This factor has been calculated considering the same icing conditions applied for both the cylinder and the airfoil. The factor has been averaged and validated using CFD simulations for several scenarios of icing conditions.

Based on the review study of modelling and simulation approaches by Martini, et al. [2], the CFD simulation has been carried out using the “Multi-Shot” simulation scheme available in ANSYS FENSAP-ICE software [8]. The study focused on the dry regime of accretion. The temperature zone in which the database was elaborated is shown in Figure 1. The simulation scenarios have been chosen for the dry zone of ice accretion, for temperatures between  $-10^{\circ}\text{C}$  and  $-5^{\circ}\text{C}$  corresponding to 10 m/s wind speed. As described in ISO12494. [4], the hard rime is generally granular, white, or translucent with a density of  $600\text{--}900\text{ kg/m}^3$ , while soft rime is white or opaque of density:  $100\text{--}600\text{ kg/m}^3$  [9]

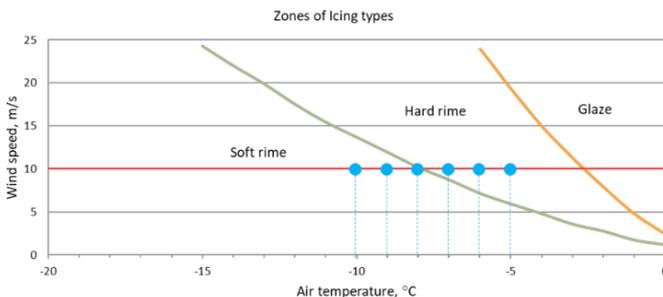


Figure 1. Empirical relationship between ice type, wind speed and temperature as described in ISO 12494 standard [4]

The Makkonen model [10] forms the theoretical basis for ice accretion calculations, ultimately determining the ice accretion rate on a structure [11]. It is based on three ratios: collision efficiency, collection efficiency and accretion efficiency. This model has recently been updated to include a more detailed treatment of wet snow growth. This flexibility allows the model to be used for a wide range of icing problems [12]. As simplified accretion models can be run quickly, these models have often been coupled with Numerical Weather Prediction (NWP) models to estimate risk under different weather conditions [12]. A VTT study [13] used a conversion factor between ice accretion on a cylinder and ice accretion on a rotating wind turbine blade to estimate the effects of icing on energy production under typical meteorological conditions of the Finnish climate. The method for calculating the averaged k-factor with the considered icing conditions is described in Figure 2. In this part of the study, the Makkonen model was used to create a database of aerodynamic losses under several scenarios of weather conditions. A standard cylinder of 30 mm in diameter was chosen for the modelling based on the standard of ISO12494 [14]. The slowly rotating cylinder encountering the same conditions that lead to ice accretion on a wind turbine blade is used as a reference collector [15]. The details of the theoretical basis of the modelling are presented in the ISO12494. [4] and in Makkonen and Poots [10].

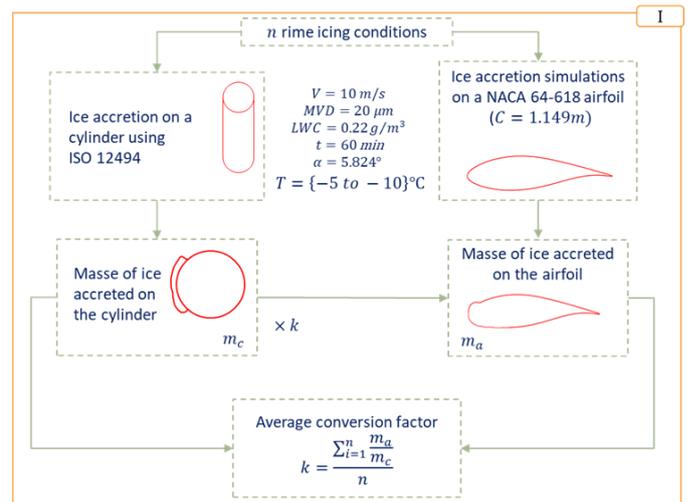


Figure 2. A relation between ice accretion on a cylinder and an airfoil is determined (k-factor)

## III. ANALYTICAL ESTIMATION OF AERODYNAMIC LOSSES

The above-mentioned analytical-CFD study has been used to rapidly estimate the aerodynamic characteristics of the NACA 64-618 iced airfoil for several scenarios of icing conditions. The proposed method and the formulas developed, the steps and the calculations necessary to carry out this method are illustrated in Figure 3. The steps of the methodology adopted to create the database of the aerodynamic losses as a function of icing conditions are described in the following steps: The first step is to calculate the averaged k-factor in several scenarios of icing conditions. The second step considers the estimation via a CFD simulation of the ice accretion mass on the airfoil and the aerodynamic characteristics of the airfoil for a representative case. The third

step is to estimate the airfoil aerodynamic characteristics for different scenarios of icing conditions using the averaged k-factor and analytical calculations of icing around the cylinder. The fourth step consists in creating a database of the aerodynamic losses of the iced airfoils for several scenarios of icing conditions. This database was quickly generated by applying the explained method.

The key parameters affecting the simulation to calculate the aerodynamic losses are limited in this study to the liquid water content (LWC), the median volume diameter of water droplets (MVD), the wind speed (V), the air temperature (T), and the angle of attack (AOA). The study is limited to the NACA 64-618 airfoil of chord length ( $C=1.149$  m) since the estimate of the average k-factor depends on the airfoil geometry [16]. The accretion time is taken as one hour (3600 sec) for all scenarios. Therefore, the influence of time was not considered in this study in order to reduce the number of parameters to a minimum. However, if the accretion time changes, one must create a different database for each period or introduce this parameter as a key factor. The roughness factor is crucial to the accuracy of ice accretion simulations [17]. This factor was not among the key parameters in our database. When using the Shin, et al. [18] model of roughness, this parameter is implicitly considered, being dependent on the other key parameters.

parameters (V, T, LWC, MVD, AOA). An extract from the database created for iced airfoils is presented in Figure 4.

#### IV. USING THE ANN FOR THE PREDICTION OF AERODYNAMIC LOSSES

This section presents a neural network optimization algorithm to directly predict the aerodynamic losses of wind turbine blade airfoil under varying scenarios of icing conditions. The database resulting from the analytical-CFD simulation method is analyzed with the Artificial Neural Network (ANN) to examine the consistency between the different scenarios and find out possible correlations between the affecting parameters to investigate the possibility of reducing the order of the model. The ANN is a powerful tool that can help for a real-time prediction of ice that keeps learning and enhancing.

To model a neural network, one needs to present the independent factors at the input to get a MISO (Multiple-Input, Single-Output) or a MIMO style (Multiple-Input, Multiple-Output) to give a specific or multifaceted result. Several independent factors and several outputs dependent on these factors refer to a MIMO style as “multiple-input, multiple-output.”

The results obtained in the database created for iced airfoils (see Figure 4) are presented by scenarios of four angles of attack for every group of icing parameters. These data were reorganized and presented by scenarios of five input parameters to be consistent with the MIMO style (Multiple-Input, Multiple-Output), as shown in Figure 5. This configuration has been processed, examined, and analyzed by the artificial neural network (ANN). This representation of data gives five factors or five independent variables that will be presented at the network's input (V, T, LWC, MVD, AOA) to give a function with two dependent variables ( $\%C_L$  and  $\%C_D$ ). We also investigated with five independent variables and one dependent variable for the  $\%C_L$  case and then for the  $\%C_D$  case in order to compare the efficiency of the method. The data in Figure 5 has been transformed into a text file to be read by the ANN program. The application chooses an optimal configuration of the neural network, which will allow, according to the five factors, to build a function that will make it possible to predict the values of  $\%C_L$  and  $\%C_D$ .

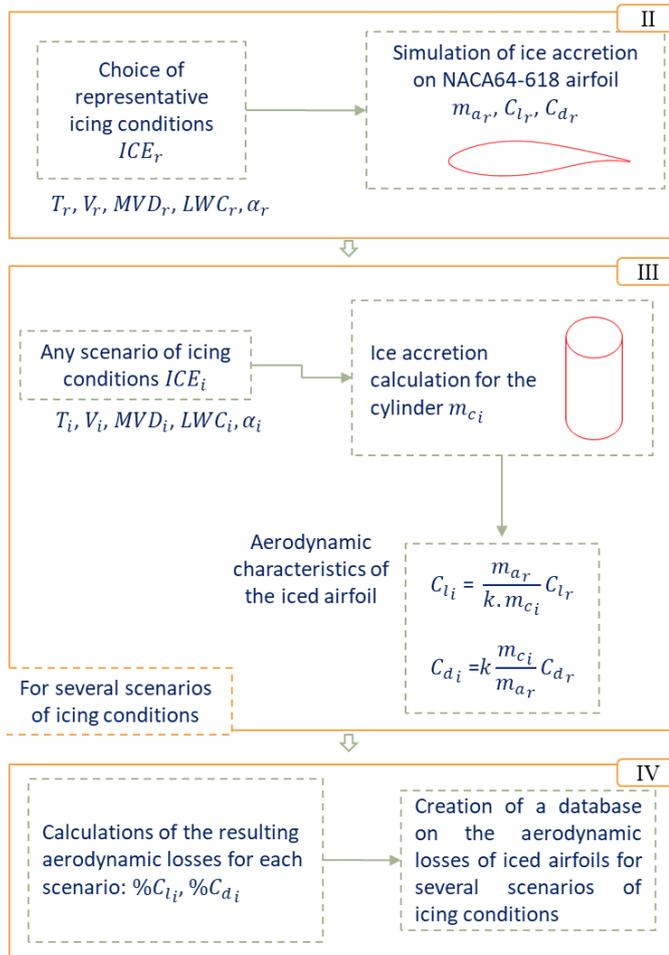


Figure 3. Method for rapid calculation of aerodynamic parameters of iced airfoils

The database was prepared with percentage values of aerodynamic losses  $\%C_L$  and  $\%C_D$  according to the five input

Vair	T	LWC	MVD	AoA = 0 degrees				AoA = 5 degrees				AoA = 10 degrees				AoA = 15 degrees			
				CL	CD	CL%	CD%	CL	CD	CL%	CD%	CL	CD	CL%	CD%	CL	CD	CL%	CD%
10	-7,5	0,2	25	0,35	0,0123	-0,22	101%	0,76	0,0192	-0,23	76%	1,01	0,0454	-0,25	69%	1,03	0,1217	-0,29	783%
20	-10	0,2	19	0,17	0,0259	-0,63	322%	0,36	0,0403	-0,63	270%	0,48	0,0954	-0,64	256%	0,49	0,2556	-0,66	2963%
15	-9	0,6	23	0,07	0,0629	-0,85	924%	0,15	0,0979	-0,85	799%	0,20	0,2317	-0,85	763%	0,20	0,6205	-0,86	8905%
13	-8	0,2	25	0,23	0,0186	-0,49	202%	0,51	0,0289	-0,49	166%	0,67	0,0684	-0,50	155%	0,68	0,1832	-0,53	1785%
8	-7	0,2	30	0,40	0,0110	-0,13	78%	0,86	0,0170	-0,13	57%	1,14	0,0404	-0,16	50%	1,16	0,1081	-0,20	562%
20	-6	0,2	18	0,18	0,0240	-0,60	291%	0,39	0,0374	-0,60	244%	0,52	0,0885	-0,62	230%	0,53	0,2371	-0,64	2662%
15	-5	0,6	23	0,07	0,0626	-0,85	920%	0,15	0,0975	-0,85	795%	0,20	0,2307	-0,85	760%	0,20	0,6179	-0,86	8863%
16	-5,6	0,7	20	0,06	0,0678	-0,86	1004%	0,14	0,1055	-0,86	869%	0,18	0,2497	-0,86	831%	0,19	0,6688	-0,87	9691%
9	-5,7	0,2	30	0,33	0,0131	-0,27	113%	0,72	0,0204	-0,27	87%	0,95	0,0482	-0,30	80%	0,97	0,1292	-0,33	906%
8	-5,8	0,2	30	0,40	0,0109	-0,13	78%	0,86	0,0170	-0,13	56%	1,14	0,0403	-0,16	50%	1,16	0,1079	-0,20	560%
8	-6,2	0,48	33	0,15	0,0293	-0,67	377%	0,32	0,0456	-0,67	319%	0,43	0,1079	-0,69	302%	0,43	0,2890	-0,70	3508%
12	-7,2	0,48	33	0,08	0,0521	-0,82	748%	0,18	0,0811	-0,82	645%	0,24	0,1919	-0,82	615%	0,24	0,5139	-0,83	7170%

Figure 4. An extract from the database created for iced airfoils.

Vair	T	LWC	MVD	AoA	CL%	CD%
10	-7,5	0,2	25	0	-0,22	101%
20	-10	0,2	19	0	-0,63	322%
15	-9	0,6	23	0	-0,85	924%
13	-8	0,2	25	0	-0,49	202%
8	-7	0,2	30	0	-0,13	78%
20	-6	0,2	18	0	-0,60	291%
15	-5	0,6	23	0	-0,85	920%
16	-5,6	0,7	20	0	-0,86	1004%
9	-5,7	0,2	30	0	-0,27	113%
8	-5,8	0,2	30	0	-0,13	78%
8	-6,2	0,48	33	0	-0,67	377%
12	-7,2	0,48	33	0	-0,82	748%
10	-7,5	0,2	25	5	-0,23	76%
20	-10	0,2	19	5	-0,63	270%
15	-9	0,6	23	5	-0,85	799%
13	-8	0,2	25	5	-0,49	166%
8	-7	0,2	30	5	-0,13	57%
20	-6	0,2	18	5	-0,60	244%
15	-5	0,6	23	5	-0,85	795%
16	-5,6	0,7	20	5	-0,86	869%
9	-5,7	0,2	30	5	-0,27	87%
8	-5,8	0,2	30	5	-0,13	56%
8	-6,2	0,48	33	5	-0,67	319%
12	-7,2	0,48	33	5	-0,82	645%

Figure 5. The MIMO style of the database.

The ANN application provides a network of 3 hidden layers [2, 1, 5] (of two neurons on layer 1, one neuron on layer 2 and five neurons on layer 3). The input consists of 5 linearly independent factors for an output of 2 dependent factors. Figure 6 presents the configuration of the network produced by the ANN application.

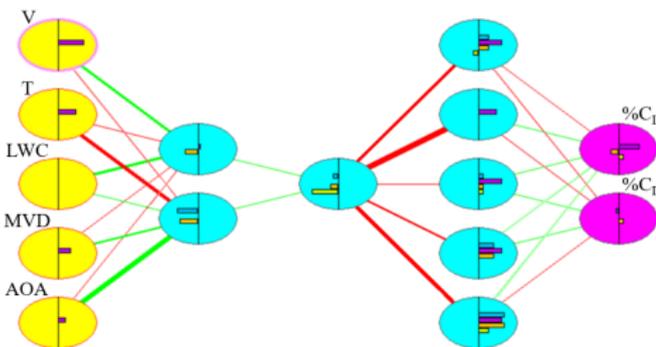


Figure 6. The neural network configuration

## V. RESULTS AND DISCUSSION

Once the configuration has been chosen, the neural network builds the function by supervised learning according to a percentage of the samples chosen randomly (the network will train about 33% of the valid data). The network then works on the validation data (about 60% of the complete database). The rest of the data is eliminated since it was outside the permitted range calculated by the application. Figure 7 gives information about the number of learning cycles needed to find the adjustment function. Note that the learning curve quickly reaches the expected error, which is smaller than 0.01. The score is 113 for data validation of 80.14% within 10%. This learning speed is dependent on the validation percentage expected when learning the database. If we want better precision during a real-time application, it is important to find a compromise that will allow us to predict the accumulation of ice that will form on the blades with good precision. It is very likely that the alert will come too late and that the accumulation of ice cannot be removed in time to restore the wind turbine to optimal operation. The learning time found in this example is on the scale of a second for a microcontroller operating at 150 MHz.

We also note that there are 22 lines of training data and 141 lines of validation data out of a total of 195 lines of data. This results in 32 rows of data that have been eliminated from the validation examples because the values of one or other of the factors were outside the range allowed by the configuration of the control. The prediction will therefore be within a range of 10%. As mentioned above, it is possible to obtain a smaller range for a prediction close to reality, but there will be a delay to be taken into consideration which may exceed the time taken to warn the defrosting system during the app in real-time.

The results of the ANN analysis over the database for iced airfoils have demonstrated consistency between the different scenarios of the database. They also highlighted the important influence of certain parameters, such as the liquid water content and the angle of attack. Figure 8 is provided by the ANN tool to present the relative importance of the factors that will form the MIMO system from 5 independent factors to 2 dependent outputs. It demonstrates the relative importance of the icing parameters on the resulting aerodynamic losses. The liquid water content (LWC) and angle of attack (AOA) were the most important parameters that affect the estimation of the aerodynamic losses due to icing. The effect of temperature T and wind speed V showed less importance. In fact, as our analysis is limited to the rime ice formation, the importance of the temperature has less effect when accretion stays in the dry zone of ice.

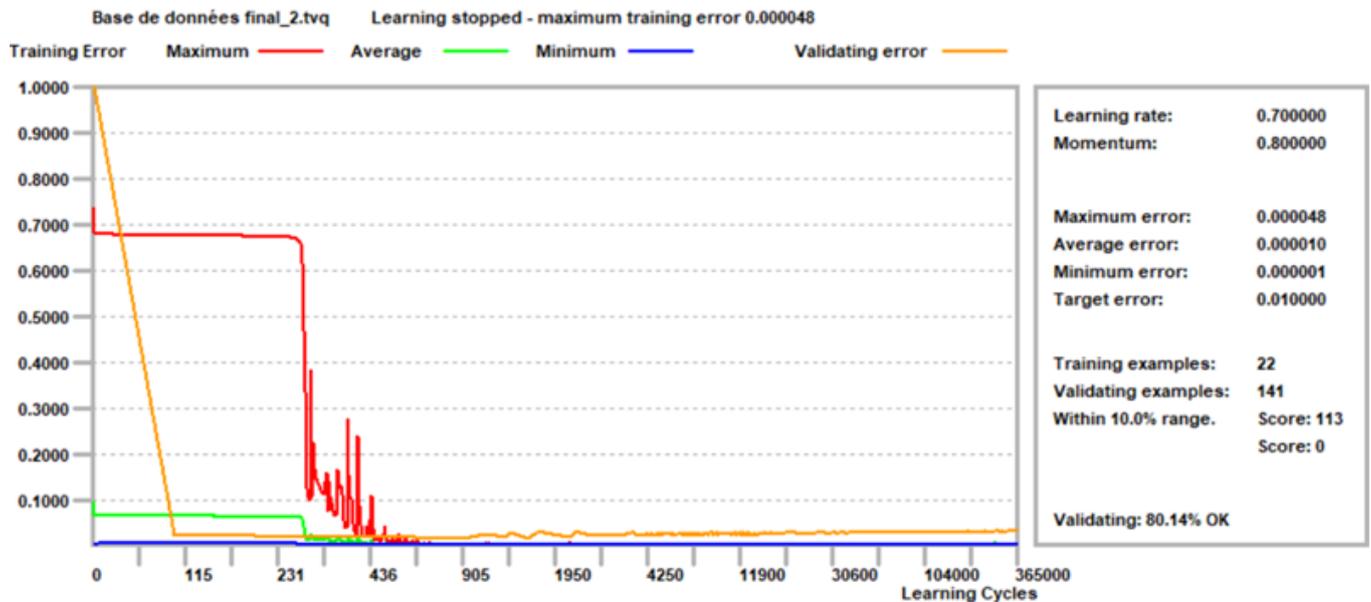


Figure 7. Training curve of the neural network

We note that the median volume diameter (MVD) is of less importance in ANN analysis, explained by the fact that this factor is probably found in the LWC factor, or it is implicitly correlated with the rest of the parameters. Further investigation is needed to confirm this analysis and investigate the possibility of dropping the MVD of the input parameters to reduce the modelling order. Indeed, one of the main objectives of the reduced-order modelling in this field is to be able to predict the aerodynamic losses by omitting the LWC and MVD parameters. These two parameters are difficult to obtain for wind turbine farms. They are correlated with the meteorological conditions by complex empirical correlations according to the type of precipitation and the atmospheric conditions of icing [16]. For the moment, these two parameters are present in this database because we are unable to obtain results without these two factors.



Figure 8. The relative importance of the independent factors

In order to improve learning, enormous computing power is required. Deep learning would certainly allow more accurate results than what we get with the ANN tool. Moreover, the activation functions of the neurons are unknown to the network established by the used ANN tool. If we had Gaussian activation functions, we would have much better prediction and faster learning. Tests on five inputs and one output at a time, either for  $\%C_L$  or  $\%C_D$ , respectively, are also recommended.

## VI. CONCLUSION

As mentioned in the introduction, the objective of this research study consists of its finality to realize an intelligent real-time icing prediction system. The system, with reduced parameters, should be able to identify the meteorological conditions favourable to the occurrence of icing events and

predict the form, type and severity of ice accretion and its impact on wind turbines to optimize wind turbine operation in icing conditions.

This study proposed a method for rapid estimation of the aerodynamic losses of wind turbine blade airfoils in different scenarios of meteorological conditions. The method made it possible to estimate production losses directly from weather conditions without having to analyze the iced airfoil with simulations. The generated database has been analyzed using a neural network optimization algorithm for predicting aerodynamic losses. The results of the ANN analysis demonstrated consistency between the different scenarios of the database. It showed that the parameters most affecting the estimation of aerodynamic losses by simulation are the liquid water content LWC and the angle of attack AOA. The median volumetric diameter of water droplets MVD also affects the simulation results. However, there is little evidence from the latest analysis that the neural network could dispense with this parameter to predict aerodynamic losses. Additional studies for a wide range of scenarios are recommended to confirm the possibility of dispensing with the MVD to predict the aerodynamic losses of the iced airfoils. Also, deep learning would certainly allow more accurate results to improve learning.

The use of the ANN in a real-time prediction of ice depending on the weather conditions is a powerful tool that keeps learning and enhancing to improve the energy efficiency of wind turbines in Nordic countries.

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