A Mixed Bentham-Rawls Criterion for Intergenerational Equity: Theory and Implications

Francisco Alvarez-Cuadrado and Ngo Van Long

McGill University*

25 May 2007

Abstract

Ranking development programs using integrals of discounted utilities can yield drastic consequences that offend our sense of justice. New alternative social welfare criteria should be considered. A reaction to discounted utilitarianism is to moderate its effects by adding to the social welfare function a second term that takes seriously the welfare of the generations that live in the far distant future. Chichilnisky proposes a social welfare function that has two desirable properties: (i) non-dictatorship of the present, and (ii) non-dictatorship of the future. However, in many economic models, there exists no optimal path under the Chichilnisky criterion. We introduce a third desideratum: "non-dictatorship of the least advantaged," and propose a new welfare criterion that is morally compelling. It is a weighted average of two terms: (a) the sum of discounted utilities, and (b) the utility level of the least advantaged generation. We derive necessary conditions to characterize growth paths that satisfy our criterion, and show that in some models with familiar dynamic specifications, an optimal path exists and displays appealing characteristics.

JEL-Classifications: D63, H43, O21, Q20. **Keywords**: Intergenerational equity, maximin, sustainable development

*Address for correspondence: Ngo Van Long, Department of Economics, McGill University. 855 Sherbrooke St West, Montréal, Québec, Canada H3A-2T7. Email: ngo.long@mcgill.ca.

1 Introduction

When performing evaluations of projects that involve future costs and benefits, economists typically use a positive rate of discount. This procedure is rather non-controversial for short-term projects, especially those of a marginal nature. However, for long-term projects that have significant implications for future generations, discounting the future is a controversial issue, both at the philosophical level, and at the level of practical implications. Sidgwick (1907, p. 414) argued against discounting, on the philosophical ground that "the time at which a man exists cannot affect the value of his happiness from a universal point of view;...the interests of posterity must concern a Utilitarian as much as those of his contemporaries"[1]. Perhaps Ramsey (1928) was the first economist to have articulated this problem in an infinite horizon framework. Like Sidgwick, Ramsey considered it unethical to discount the utilities of future generations. Various utilitarian welfare criteria that avoid discounting have been proposed. Among these are Ramsey's "minimum distance from Bliss" criterion, and the "overtaking criterion" suggested by von Weizsäcker (1965) and Gale (1967)[2]. As pointed out by Chichilnisky (1996), these criteria fail to rank all possible welfare streams, i.e., they fail the "completeness" test.

The practical implications of ranking time-paths of utilities on the sole basis of comparing the values of the associated integrals of discounted utilities can be quite drastic. For example, in the Solow-Dasgupta-Heal model[3] with a man-made capital stock and an exhaustible resource,

[1] Sidgwick's argument against discounting has led to the "equity principle à la Sidgwick", which is embodied in the form of the "anonymity" condition: a stream of utility $s = \{x, y, z, ...\}$ should be judged as equal to a permuted stream $s^p = \{y, x, z, ...\}$. Diamond (1965) shows that if one requires a social welfare function $W(.)$ to satisfy the strict Paretian property, a weak form of anonymity and some kind of continuity, then $W(.)$ does not exist. Basu and Mitra (2003) confirm Diamond's result even without requiring continuity. Svensson (1980) however shows that if, instead of seeking a (real-valued) function, we merely look for the ability to rank infinite streams of utilities, then existence of a social welfare relation, or ordering, is ensured. Unfortunately, Svensson did not offer a constructive proof, so almost nothing is known about such ordering.

[2] Unaware of the contributions by economists, the philosopher Krister Segerberg (1976, p. 226) poses the following problem in ethics, in his article titled "A Neglected Family of Aggregation Problems in Ethics", published in Noûs (1976):

"Pascal believes that eternity consists of infinitely many days [and] that when his body is dead his soul will spend each following day in Heaven or Hell...Outcomes can be represented by infinite sequences x_0 x_1... x_n..., where each x_n is either 1(Heaven) or 0 (Hell)....Problems arise when he wants to compare prospects containing both 1's and 0's. Particularly difficult is it to deal with with prospects containing infinitely many 1's and also infinitely many 0's."

[3] Solow (1974), Dasgupta and Heal (1979, pp. 288-300.)

while it is feasible to maintain a constant positive level of consumption for ever, utilitarianism with discounting would prescribe a path with vanishing consumption in the long run, no matter how small the discount rate is. This consequence offends our sense of justice. It seems, therefore, that some alternative social welfare criteria should be considered when we make choices among alternative courses of actions that significantly affect future generations. An extreme form of egalitarianism has been proposed by some philosophers and economists: the maximin criterion, according to which one stream of utilities is better than another if and only if the utility level of the least advantaged person in the former is higher than that of the least advantaged person in the latter. The maximin criterion has also been called the "Rawlsian criterion" even though Rawls (1971, 1999) had expressed strong reservations about the use of maximin as a principle for intergenerational equity[4]. The fascination with the maximin criterion has spawned a stream of theoretical literature that seeks to characterize development paths that ensure a constant level of consumption, or constant utility, for all generations. (See, for example, Solow (1974), Hartwick (1977), Dasgupta and Heal (1979), Asheim (1988), Asheim, Buchholtz and Withagen (2003).) The insistence on constant consumption, however, can yield consequences that are unpalatable. As Rawls (1999, p.254) pointed out, the unmodified maximin principle would entail "either no saving at all or not enough saving to improve social circumstances," which is totally unacceptable to him, especially for the case of very poor countries with a low stock of capital[5].

Another type of reaction to discounted utilitarianism is to moderate the effects of discounting by adding to the social welfare function a second term that takes seriously the welfare of the generations that live in the far distant future. This second term does not contain a discount rate. This approach was proposed by Chichilnisky (1996), who coined the term "dictatorship of the present" to describe welfare criteria (such as comparing integrals of discounted utility streams) that give practically a weight of zero to the utility levels of far-away generations. She does not object to discounting *per se*, and does not insist that the social welfare function must have the anynomity property. To her, as long as the interest

[4] See Long (2007) for a discussion of Rawls's reservations, and a review of the related literature.

[5] See also Rawls (1971, p.291).

of future generations are adequately protected, in the sense that the social welfare function does not display "dictatorship of the present"[6], some form of discounting is acceptable. She proposes a social welfare function that has three desirable properties: (i) non-dictatorship of the present, (ii) non-dictatorship of the future[7], and (iii) strong Pareto. The Chichilnisky social welfare function takes a simple and intuitively appealing form: social welfare is a weighted sum of two terms, the first being the conventional sum of discounted utilities, and the second term takes on a value which depends only on the limiting behavior[8] of the utility sequence under consideration. The weight given to each term must be strictly positive. It is interesting to observe that a weighted average of two criteria, one displaying dictatorship of the present, the other displaying dictatorship of the future, is a criterion that does not display these undesirable properties.

While Chichilnisky's social welfare function is well defined and can rank all utility sequences, in many economic models of interest, there does not exist a utility stream that is optimal under that criterion[9]. To illustrate, take the standard Solow neoclassical growth model, and choose a time path of saving rate to maximize social welfare under the Chichilnisky criterion. Any path that approaches the golden rule level of capital stock, k_g, will maximize the second term of the weighted sum. It pays therefore to delay the approach to k_g as much as possible, and stay near the modified golden rule level k_m as long as possible, because doing so would increase the value of the first term, and would not affect the second term. It follows that among all paths that approach k_g asymptotically, any feasible path is inferior (according to the Chichilnisky social welfare function) to some other feasible path.

In this paper, we complement Chichilnisky's non-dictatorship of the present and non-dictatorship

[6] A social welfare function $W(.)$ is said the display "dictatorship of the present" if for any two sequences of utilities, say $s = \{u_t\}$ and $s' = \{v_t\}$, and $W(.)$ ranks s higher than s', there exists some time $T > 0$ such that no modification of the tail-ends (beyond T) of s and s' could reverse the ranking. The conventional utilitarian criterion with discounting implies dictatorship of the present.

[7] A social welfare function $W(.)$ is said the display "dictatorship of the future" if whenever $W(.)$ ranks s higher than s', all modifications of s and s' that do not affect their limiting behavior would preserve the original ranking.

[8] For example, with the utility sequence $\{u_t\}$the second term could be $\limsup_{t\to\infty} u_t$, or $\liminf_{t\to\infty} u_t$, or some weighted average of these two limiting values.

[9] There exists a simple model of non-renewable resource where the Chichilnisky's criterion does identify an optimal path. See Chichilnisky (1997), and Figuiere and Tidball (2007).

of the future by introducing a third desideratum: "non-dictatorship of the least advantaged"[10], and propose a new welfare criterion which, as we argue below, is morally compelling. Our criterion is a weighted average (with strictly positive weights) of two terms. The first term is the conventional sum of discounted utilities, and the second term is the utility level of the worse-off generation. We call this new criterion the Mixed Bentham-Rawls criterion (MBR). We use the word "Bentham" because our criterion is utilitarian, in the sense that it permits trade-offs of utilities of different individuals, and the word "Rawls" because of its special (but not exclusive) emphasis on the wellbeing of the least advantaged[11].

In the next section, we will present arguments that justify this criterion. In a later section, we will develop a set of necessary conditions to characterize growth paths that satisfy the MBR criterion, and show that in some models with familiar dynamic specifications, an optimal path under MBR exists and displays appealing characteristics.

2 The mixed Bentham-Rawls criterion

Consider an economy with infinitely many generations. Since we wish to focus on the question of distributive justice among generations, we make the simplifying assumption that within each generation, all individuals receive the same income and have the same tastes. Thus, by assumption, the question of equity within each generation does not arise. This framework has been used in, for examples, Solow (1974), Hartwick (1977), Dasgupta and Heal (1979, Chapters 9-10), Dixit et al. (1980), Mitra (1983), and Chichilnisky (1996).

Let c_t denote the vector of consumption (of various goods and services) allocated to the representative individual of generation t. Let $u_t \equiv u(c_t)$ be the life-time utility of this individual (u_t is a real number, and $u(.)$ is a real-valued function). We interpret "utility" as "standard of living"of individuals, rather than some kind of happiness they get when consuming and/or contemplating their childrens' and grand-childrens' life prospects. To fix ideas, it is convenient to assume that each individual lives for just one period. Consider for

[10] Rawls's argument, cited above, against the use of the unmodified maximum principle, may be considered as a refusal to accept "dictatorship of the least advantaged", though he did not use this term.

[11] In the context of intergenerational equity, Rawls's emphasis on the least advantaged is not exclusive: if the least fortunate generation is the first generation, he still wants savings to take place (1971, p. 291).

the moment two alternative projects, denoted by 1 and 2. Project i (where $i = 1, 2$) yields an infinite stream of utilities denoted by

$$\{u_t^i\}_{t=1,2,...,} \equiv \{u_1^i, u_2^i, ..., u_t^i, u_{t+1}^i, ...\}$$

where u_t^i stands for $u_t(c_t^i)$.

We assume that while an individual of generation t might care about the consumption vector of his/her son or daughter, c_{t+1}, and that of his/her[12] grand-son or grand-daughter, c_{t+2}, these vectors have no impact on the "utility" level u_t. Thus it might be preferable to refer to u_t as the "standard of living" rather than "utility" of generation t.

For simplicity of notation, we use the symbol $\mathbf{u}^i$ to denote the utility stream $\{u_t^i\}_{t=1,2,...,}$. Roughly speaking, a welfare criterion is a way of ranking all possible utility streams. Let S be the set of all possible utility streams. A social welfare function, denoted by $W(.)$, is a function that maps elements of S to the real number line[13].

To simplify matters, we assume that the function $u(.)$ is bounded.

Assumption 1: (Boundedness) *Utility is bounded*

$$A \leq u(c) \leq B$$

Remark: *The number B is the highest possible level of utility. We shall refer to B as the "Bliss Utility Level".*

In what follows, we consider only welfare functions $W(.)$ that are non-decreasing in u_t. That is, if the utility level of one generation increases, the social welfare cannot decrease. This is the well known Paretian property[14].

---

[12] To avoid repetitive uses of his/her etc., in all that follows, when referring to hypothetical persons, we use the masculin gender, on the understanding that it embraces the feminin gender.

[13] This definition of "social welfare function" is quite common, see, for example, Chichilnisky (1996, p. 240), Basu and Mitra (2003). This is to be distinguished from Arrow's use of the term "social welfare function" which is a mapping from the space of all possible individual preference orderings (of social states) to the space of social orderings.

[14] The Paretian Property can be strengthened to the "Strict Paretian Property" by replacing the word "non-decreasing" by "increasing".

Property P:(Paretian Property) *Welfare is non-decreasing in u_t.*

Utilitarian social welfare functions permit comparing (and trading-off) an increment in the utility level of an individual (or group of individuals) with a 'decrement' (negative change) in the utility level of another individual (or group). A familiar example is the "utilitarian criterion with discounting". Non-utilitarian social welfare functions (such as maximin and sufficientarianism[15]) admit interpersonal comparison of utility levels, but do not permit trading off.

Under the "utilitarian criterion with discounting" (at a positive rate $\delta_t > 0$), social welfare is denoted by W^d and is defined as follows:

$$W^d(\mathbf{u}^i) = \frac{u_1^i}{(1+\delta_1)} + \frac{u_2^i}{(1+\delta_1)(1+\delta_2)} + \frac{u_3^i}{(1+\delta_1)(1+\delta_2)(1+\delta_3)} + ... + ...$$

According to this criterion, a utility stream $\mathbf{u}^j$ is ranked higher than a utility stream $\mathbf{u}^i$ if and only if $W^d(\mathbf{u}^j) > W^d(\mathbf{u}^i)$. Thus, a small decrease in the utility level of an individual (no matter how disadvantaged he already is) can be justified by some increase in the utility level of some other individuals.

The Maximin Criterion is denoted by W^m. According to this criterion, a utility stream $\mathbf{u}^i$ is ranked higher than utility stream $\mathbf{u}^j$ if and only if the utility level of the worst off generation in stream $\mathbf{u}^i$ is higher than the utility level of the worst off generation in stream $\mathbf{u}^j$, that is, if and only if,

$$\inf \left\{u_t^i\right\}_{t=1,2,...,} > \inf \left\{u_t^j\right\}_{t=1,2,...,}$$

The utilitarian criterion with discounting has been attacked by many economists, from Ramsey (1928) to Chichilnisky (1996). To quote a forceful example from Chichilnisky (1996, page 235):

"...Discounting future utility is generally inconsistent with sustainable development. It can produce outcomes which seem patently unjust to later generations. Indeed, under any positive discount rate, the long-run future is deemed irrelevant. For example, at a standard 5%

[15] For explanations and discussions of the sufficientarianism criterion, see Chichilnisky (1977), Frankfurt (1988), Anderson (1999), Arneson (2002), and Roemer (2003).

discount rate, the present value of the earth's aggregate output discounted 200 years from now, is a few hundred thousand dollars. A simple computation shows that if one tried to decide how much it is worth investing in preventing the destruction of the earth 200 years from now, the answer would be no more than one is willing to invest in an apartment."
Chichilnisky (1996) argues that all utilitarian criteria with discounting place too much emphasis on the present. In fact these criteria display insensitivity to the utility of distant generations. To formalize this idea, let us follow Chichilnisky and define $({}_T\mathbf{s}^i, \mathbf{a}_T)$ to be a utility sequence obtained from $\mathbf{s}^i$ by replacing all elements of $\mathbf{s}^i$ except the first T elements by the tail of the utility sequence $\mathbf{a}$, where

$$\mathbf{a}_T \equiv \{a_{T+1}, a_{T+2}, ...\}$$

$${}_T\mathbf{s}^i \equiv \left\{s_1^i, s_2^i, ..., s_T^i\right\}$$

$$({}_T\mathbf{s}^i, \mathbf{a}_T) \equiv \left\{s_1^i, s_2^i, ..., s_T^i, a_{T+1}, a_{T+2}, ...\right\}$$

Consider the following definition:

Definition 1: (dictatorship of the present; Chichilnisky 1996)

A welfare criterion $W(.)$ is said to display "dictatorship of the present" if the following condition holds:

For every pair $(\mathbf{s}^i, \mathbf{s}^j)$, $W(\mathbf{s}^i)$ is greater than $W(\mathbf{s}^j)$ if and only if, for all T sufficiently large[16]*, $W({}_T\mathbf{s}^i, \mathbf{a}_T) > W({}_T\mathbf{s}^j, \mathbf{b}_T)$ for* ***all*** *pairs of utility sequences $(\mathbf{a}, \mathbf{b})$, where $({}_T\mathbf{s}^i, \mathbf{a}_T)$ means that all elements of $\mathbf{s}^i$ except the first T elements are replaced by the tail of the sequence $\mathbf{a}$, and $({}_T\mathbf{s}^j, \mathbf{b}_T)$ means that all elements of $\mathbf{s}^j$ except the first T elements are replaced by the tail of the utility sequence $\mathbf{b}$.*

In other words, dictatorship of the present means that any modification of utility levels of generations far away in the future would not be able to reverse the welfare ranking of two utility streams. Given Assumption 1, the utilitarian criterion with positive discounting

[16] *More precisely, for all $T > \widehat{T}$ for some $\widehat{T}$ that may depend on $\mathbf{s}^i$ and $\mathbf{s}^j$.*

displays dictatorship of the present.

A welfare function is said to display "non-dictatorship of the present" if for any pair $(\mathbf{s}^i, \mathbf{s}^j)$ such that $W(\mathbf{s}^i) > W(\mathbf{s}^j)$, there exists **some** modifications to utilities of individuals in the distant future that reverse the ranking.

Let us turn to the other extreme, and consider some welfare criteria that pay no attention to the utility levels of generations that are living at the present or in the "near future". Given a utility sequence $\{u_t\}_{t=1,2,...,} \equiv \{u_1, u_2, ..., u_t, u_{t+1}, ..., ..., ...\}$, let us consider the tail beginning at t, $\{u_t, u_{\,t+1}, ...\}$, and define the number z_t and y_t to be respectively the greatest lower bound and least upper bound of this tail

$$z_t \equiv \inf_t \{u_t, u_{t+1}, ...\}$$

$$y_t \equiv \sup_t \{u_t, u_{t+1}, ...\}$$

The resulting sequence $\{z_t, z_{t+1}, ...\}$ is by construction a non-decreasing sequence, and hence must converge to a limit $\overline{z}$:

$$\lim_{t\to\infty} z_t = \overline{z}$$

i.e.

$$\lim \inf_{t\to\infty} \{u_t, u_{t+1}, ...\} = \overline{z}$$

Similarly, the sequence $\{y_t, y_{t+1}, ...\}$ is by construction a non-increasing sequence, and hence must converge to a limit $\overline{y}$:

$$\lim_{t\to\infty} y_t = \overline{y}$$

i.e.

$$\lim \sup_{t\to\infty} \{u_t, u_{t+1}, ...\} = \overline{y}$$

Clearly, lim inf and lim sup are both well defined social welfare functions. These functions are entirely insensitive to the utility levels of the generations that are living at the present or in the "near future". Welfare comparisons using either of these criteria depend only on the utility levels of generations born in the distant future. Chichilnisky (1996) pointed out that such criteria give a "dictatorial role" to the future. Formally, a welfare criterion $W(.)$ is said to display "dictatorship of the future" if it has the following property:

Definition 2: (dictatorship of the future; Chichilnisky 1996)

A welfare criterion $W(.)$ is said to display "dictatorship of the future" if the following condition holds:

For every pair $(\mathbf{s}^i, \mathbf{s}^j)$, $W(\mathbf{s}^i)$ is greater than *$W(\mathbf{s}^j)$ if and only if, for all T sufficiently large, $W({}_T\mathbf{a}, \mathbf{s}^i_T) > W({}_T\mathbf{b}, \mathbf{s}^j_T)$ for* ***all*** *pairs of sequences $(\mathbf{a}, \mathbf{b})$, where $({}_T\mathbf{a}, \mathbf{s}^i_T)$ means that the first T elements of $\mathbf{s}^i$ are replaced by the vector ${}_T\mathbf{a} \equiv (a_1, a_2, ..., a_T)$, and $({}_T\mathbf{b}, \mathbf{s}^j_T)$ means that the first T elements of $\mathbf{s}^j$ are replaced by the vector ${}_T\mathbf{b} \equiv (b_1, b_2, .., b_T)$.*

Both the lim inf and the lim sup social welfare functions display dictatorship of the future. A welfare function is said to display "non-dictatorship of the future" if for any pair $(\mathbf{s}^i, \mathbf{s}^j)$ such that $W(\mathbf{s}^i) > W(\mathbf{s}^j)$ there exists **some** modifications to utilities of individuals in the early generations that reverse the ranking.

Chichilnisky argued that both dictatorship of the present and dictatorship of the future are undesirable. She proposed a criterion that rules out both forms of dictatorship.

The welfare function proposed by Chichilnisky[17] is a weighted sum of two terms, the first term being the usual discounted stream of utilities, while the second term is defined in a way that its value depends only on the limiting behavior of the utility sequence. Formally,

$$W^C(\mathbf{u}^i) = (1-\theta)\sum_{t=1}^{\infty} \lambda_t u^i_t + \theta\phi(\mathbf{u}^i)$$

where $0 < \theta < 1$, $0 < \lambda_t < 1$, $\sum_{t=1}^{\infty} \lambda_t < \infty$ and, by definition,

$$\phi(\mathbf{u}^i) \equiv \lim_{t\to\infty} u^i_t$$

Here, the limit can be defined to be lim sup, or lim inf, or some weighted average of the two. The social welfare function $W^C(.)$ clearly has the properties of "non-dictatorship of the

[17] This welfare function has the strict Paretian property, and satisfies the axioms of "non-dictatorship of the present" and "non-dictatorship of the future." If two more axioms are added, "continuity" and "independence" (in the sense of linearity in u_t), then this is the only form the welfare function can take.

present" and "non-dictatorship of the future".

It is interesting to observe that W^C is a weighted average (a convex combination) of two functions that are themselves based on rejected welfare criteria. The second function, $\phi(\mathbf{u}^i) = \lim_{t\to\infty} u_t^i$, implies dictatorship of the future, while the first function, $\sum_{t=1}^{\infty} \lambda_t u_t^i$, implies dictatorship of the present. A convex combination that gives strictly positive weights to two "undesirable" welfare functions is free from their associated undesirable properties.

A major problem with the Chilchinisky welfare function $W^C(.)$ is that for many growth models, including the familiar one-sector growth model, there does not exist an optimal path under this objective function. The intuition behind this non-existence is as follows. The function $\phi(\mathbf{u}^i) = \lim_{t\to\infty} u_t^i$ would insist on reaching, in the long run, the golden rule capital stock, but does not care how soon or how early. The second function, $\sum_{t=1}^{\infty} \lambda_t u_t^i$, would insist on reaching, instead, the modified golden rule capital stock. Any path $\mathbf{u}^i$ that goes near the modified golden rule capital stock and eventually veers to the golden rule capital stock at some time T_i will be beaten by another path $\mathbf{u}^j$ that does a similar thing but at a later date $T_j > T_i$. The latter path $\mathbf{u}^j$ in turn will be beaten by another path $\mathbf{u}^h$ with $T_h > T_j$ and so on. So an optimal path does not exist.

We propose to modify the Chichilnisky criterion by discarding the second term, $\theta\phi(\mathbf{u}^i)$, and replacing it with the maximin utility[18]. For this purpose, we define the following concept:

Definition 3: (dictatorship of the least advantaged)

A welfare criterion $W(.)$ is said to display "dictatorship of the least advantaged" if the following condition holds:

For any pair $(\mathbf{s}^i, \mathbf{s}^j)$, $W(\mathbf{s}^i)$ is greater than $W(\mathbf{s}^j)$ if and only if

$$\inf \left\{ s_1^i, s_2^i, ..., s_n^i, ... \right\} > \inf \left\{ s_1^j, s_2^j, ..., s_n^j, ... \right\}$$

[18] The maximin utility is inf, not lim inf.

Now, for any number θ, where $0 < \theta < 1$, let us consider the following social welfare function

$$W^{mbr}(\mathbf{u}^i) = (1-\theta)\sum_{t=1}^{\infty}\lambda_t u_t^i + \theta \inf\left\{u_1^i, u_2^i, ..., u_n^i, ...\right\} \tag{1}$$

This social welfare function is a weighted average of two functions, one displaying dictatorship of the present, the other displaying dictatorship of the least advantaged, with a strictly positive weight given to each[19]. As a result, it displays "non-dictatorship of the present" and "non-dictatorship of the least advantaged". The superscript *mbr* is an abreviation for "Mixed Bentham-Rawls". We use the word "Bentham" because our criterion is utilitarian, in the sense that it permits trade-offs of utilities of different individuals, and the word "Rawls" because of its special (but not exclusive) emphasis on the wellbeing of the least advantaged[20]. Notice that the function $W^{mbr}(.)$ also implies that the welfare of the present generation matters, i.e., it displays non-dictatorship of the future. To see this, consider any pair $(\mathbf{s}^i, \mathbf{s}^j)$ such that $W(\mathbf{s}^i) > W(\mathbf{s}^j)$. Then, we can destroy this strict ranking by replacing s_1^i with the lowest possible utility; i.e., modifications of present utilities do modify the ranking of two utility streams.

Let us offer a brief justification for our proposed social welfare function. Consider a Rawlsian hypothetical original position, under the assumption that the contracting parties are family lines[21]. A family line is at the same time "one" and "many". Being "one", it is like a single individual. There are no valid reasons to object to an individual's discounting of his future consumption. But a family line is also "many." The least advantaged individuals have special claims *like* those accorded to the "contemporaneous individuals" of the simpler atemporal Rawlsian model. It is therefore arguable that each contracting party would (i) place a special weight on the utility level of the least advantaged generation of the family line, and (ii) care about the sum of weighted utilities of all generations. It seems also sensible

[19] Unlike Chichilnisky's social welfare function, our social welfare function is not linear in utilities, because $\inf\left(\alpha\mathbf{u}^i + \beta\mathbf{u}^j\right)$ is not equal to $\alpha\inf(\mathbf{u}^i) + \beta\inf(\mathbf{u}^j)$. For example, consider $\mathbf{u}^i = \{0, 1, 01, 0, 1, ...\}$ and $\mathbf{u}^j = \{1, 0, 1, 0, 1, 0, ...\}$, with $\alpha = \beta = 1/2$. Then $\inf\left(\alpha\mathbf{u}^i + \beta\mathbf{u}^j\right) = 1/2$ while $\inf(\mathbf{u}^i) = 0 = \inf(\mathbf{u}^j)$.

[20] See Rawls (1971, p. 287-291).

[21] In the context of intergenerational equity, Rawls postulated that the contracting parties represent "family lines" (1971, p. 292).

to allow a trade-off between (i) and (ii) above, because each party represents a family line. Our proposed mixed Bentham-Rawls criterion is in sharp contrast to the standard utilitarian tradition (e.g. see any graduate macro-economic textbook) which would treat a family line as an infinitely-lived individual. Such a textbook position could result in requiring great sacrifices of early generations who are typically poor. In contrast, our proposed approach avoids imposing very high rates of savings at the earlier stages of accumulation.[22]

3 Finding the social optimum under the mixed Bentham-Rawls criterion

In this section we derive necessary conditions for an optimal path under the mixed Bentham-Rawls criterion. It is more convenient to work with a continuous time model. Let x be a vector of n state variables, and c a vector of m control variables. Denote the instantaneous utility function by $u(x(t), c(t), t)$.

The transition equations are $\dot{x}_i(t) = g_i(x(t), c(t), t)$, for $i = 1, 2, ..., n$. Given the values of the state variables, the control variables at time t must belong to a feasibility set $A(x(t), t)$ which is characterized by a set of s inequality constraints:

$$h_i(x(t), c(t), t) \geq 0,\ i = 1, 2, ..., s.$$

Consider first the case of a finite horizon T. The initial stocks $x_i(0)$, $i = 1, 2, ..., n$, are given. For a given time path $\widehat{c}(.)$ and the associated time path $\widehat{x}(.)$, let $\underline{u}$ be the greatest lower bound of the resulting time path of utility:

$$\underline{u} = \inf_t \left\{ u\left(\widehat{x}(t), \widehat{c}(t), t\right) \right\}$$

This implies that

$$u\left(\widehat{x}(t), \widehat{c}(t), t\right) \geq \underline{u}$$

[22] Rawls also complained about the standard utilitarian approach, as it "may direct us to demand heavy sacrifices of the poorer generations for the sake of greater advantages for the later ones that are far better off" (1971, p. 287). He instead advocated that "when people are poor and savings are difficult, a lower rate of savings should be required; whereas in a wealthier society greater savings may reasonably be expected since the real burden is less." (1971, p. 287).

Assume a constant rate of discount $\rho \geq 0$. The social welfare generated by the time path $\widehat{c}(.)$ under the mixed Bentham-Rawls criterion is then the continuous time counterpart of (1):

$$\int_0^T (1-\theta)e^{-\rho t}u\left(\widehat{x}(t), \widehat{c}(t), t\right) dt \ + \theta \underline{u} \tag{2}$$

To maximize social welfare given the vector of initial stocks $x_0 \equiv (x_{10}, x_{20}, ..., x_{n0})$, the planner chooses the number $\underline{u}$ and the time path $c(t)$ to maximize the above welfare function, subject to

$$u(x(t), c(t), t) \geq \underline{u} \tag{3}$$

$$h(x(t), c(t), t) \geq 0 \tag{4}$$

$$\dot{x}(t) = g(x(t), c(t), t) \tag{5}$$

and

$$x_i(T) \geq 0 \tag{6}$$

Note that $\underline{u}$ must belong to a feasible set $Z(x_0)$, which is defined as follows:

$$Z(x_0) \equiv \left\{ \begin{array}{c} y : \exists c(t) \in A(x(t), t), u(x(t), c(t), t) \geq y, \\ \dot{x}(t) = g(x(t), c(t), t), x(T) \geq 0, x(0) = x_0 \end{array} \right\} \tag{7}$$

We define

$$b = \sup \{y : y \in Z(x_0)\} \tag{8}$$

Here b is the highest feasible minimum living standard that can be imposed as a constraint, given the initial stock x_0.

3.1 The necessary conditions

Since $\underline{u}$ is a constant to be chosen optimally, the optimization problem (2) is an optimal control problem with $\underline{u}$ treated as a control parameter. The necessary conditions for such problems can be derived from Hestenes's Theorem[23]. They are as follows.

Let $\pi(t)$ be the vector of costate variables, $\lambda(t)$ be the vector of multipliers associated with the inequality constraints (4) and $\omega(t)$ the multiplier associated with the constraint (3). The Hamiltonian for this problem is

$$H(t,x(t),c(t),\pi(t)) \equiv (1-\theta)e^{-\rho t}u(x(t),c(t),t) + \pi(t)g(x(t),c(t),t)$$

and the Lagrangian is

$$L(t,x(t),c(t),\pi(t),\lambda(t),\omega(t),\underline{u}) = H + \lambda(t)h(x(t),c(t),t))$$

$$+\omega(t)\left[u(x(t),c(t),t) - \underline{u}\right]$$

An optimal path must satisfy the following conditions:

(i) The maximum condition: The control variables maximize the Hamiltonian subject to the inequality constraints (3) and (4),

(ii) The adjoint equations:

$$\dot{\pi} = -\frac{\partial L}{\partial x}$$

(iii) The transition equations:

$$\dot{x} = \frac{\partial L}{\partial \pi}$$

(iv) The transversality condition with respect to the control parameter $\underline{u}$ is

$$\theta + \int_0^T \frac{\partial L}{\partial \underline{u}} dt \geq 0 \ (= 0 \text{ if } \underline{u} < b)$$

(where b is defined by (8)), and that with respect to the final stocks is

$$x(T) \geq 0,\ \pi(T) \geq 0,\ \pi(T)x(T) = 0$$

[23] See Leonard and Long (1991, Theorem 7.11.1) for an exposition of Hestenes' Theorem which deals with optimal control problems involving control parameters and various constraints.

(v) The Hamiltonian and the Lagrangian are continuous functions of time, and

$$\frac{d}{dt}H(t, x(t), c(t), \pi(t)) = \frac{d}{dt}L(t, x(t), c(t), \pi(t), \lambda(t), \omega(t), \underline{u}) = \frac{\partial L}{\partial t}$$

3.2 Implications for genuine savings

Since one of the fundamental questions concerning intergenerational equity is the "how much a generation ought to save" given its current circumstances, let us derive the implications of our mixed Bentham-Rawls criterion for saving rates. Following Hamilton and Hartwick (2005) and Hamilton and Withagen (2006), let us define "present-value genuine savings" by

$$S(t) \equiv \pi(t)g(x(t), c(t), t)$$

and "current-value genuine savings" by

$$S^c(t) \equiv e^{\rho t}\pi(t)g(x(t), c(t), t)$$

Then, by definition of the Hamitonian H and of genuine savings S,

$$\frac{d}{dt}H = -\rho(1-\theta)e^{-\rho t}u(t) + (1-\theta)e^{-\rho t}\dot{u} + \dot{S} \tag{9}$$

On the other hand,

$$\frac{\partial L}{\partial t} = -\rho(1-\theta)e^{-\rho t}u(t) + \pi(t)g_t + \lambda(t)h_t + \left[(1-\theta)e^{-\rho t} + \omega\right]u_t \tag{10}$$

Using (v), it follows that along the optimal path, utility is rising at time t if and only if the *rate of change* in present-value genuine savings, adjusted for technological progress impact (the term inside the curly brackets in the equation below), is negative:

$$\dot{S} - \left\{\pi g_t + \lambda h_t + \left[(1-\theta)e^{-\rho t} + \omega\right]u_t\right\} = -(1-\theta)e^{-\rho t}\dot{u} \tag{11}$$

i.e.

$$\dot{S} + (1-\theta)e^{-\rho t}\dot{u} = \left\{\pi g_t + \lambda h_t + \left[(1-\theta)e^{-\rho t} + \omega\right] u_t\right\} \tag{12}$$

Thus, the constancy of present-value genuine savings ($\dot{S} = 0$) is consistent with growing utility if technological progress impact is positive. In particular, suppose the technological progress impact is zero. Then, as is clear from (11), if utility is constant over some time interval $[t_1, t_2]$,then the present-value genuine savings must be constant as well.

Now, by definition,

$$S(t) = e^{-\rho t} S^c(t)$$

Hence

$$\frac{\dot{S}}{S} = -\rho + \frac{\dot{S}^c}{S^c}$$

Thus, under our objective function, along the time interval $[t_1, t_2]$ *when utility is constant, the current-value genuine savings must be rising*:

$$\frac{\dot{S}^c}{S^c} = \rho \tag{13}$$

We will see that this result is confirmed in our numerical simulations below.

Remark: Our result, equation (12), is a generalization of the proposition of Hamilton and Withagen (2006), and Hamilton and Hartwick (2005). Those papers were concerned only with the standard utilitarian objective, and thus had no place for the multiplier ω.

3.3 Infinite horizon optimization under the mixed Rawls-Bentham criterion

Suppose the time horizon is infinite and the rate of discount ρ is a positive constant. Then the social planner chooses $\underline{u}$ and $c(.)$ to maximize the mixed Rawls-Bentham (MBR) objective function:

$$\theta \underline{u} + \int_0^\infty (1-\theta) u(x, c, t) e^{-\rho t} dt$$

It will be convenient to re-write this objective function as an integral:

$$\int_0^\infty \{\theta \underline{u}\rho + (1-\theta)u(x,c,t)\} e^{-\rho t} dt \tag{14}$$

Let $\psi(t) = e^{\rho t}\pi(t)$, $\mu(t) = e^{\rho t}\lambda(t)$ and $w(t) = e^{\rho t}\omega(t)$. The current value Hamiltonian of this infinite horizon problem is

$$H^c = \theta\underline{u}\rho + (1-\theta)u(x,c,t) + \psi g(x,c,t)$$

and the current-value Lagrangian is

$$L^c = H^c + \mu h(x,c,t) + w\left[u(x,c,t) - \underline{u}\right]$$

Then we obtain the following conditions:

$$\frac{\partial L^c}{\partial c} = (1-\theta)u_c + \psi g_c + \mu h_c + w u_c = 0$$

$$\mu \geq 0, g(x,c,t) \geq 0, \mu g(x,c,t) = 0$$

$$w \geq 0, u(x,c,t) - \underline{u} \geq 0, w\left[u(x,c,t) - \underline{u}\right] = 0$$

$$\dot{\psi} = \rho\psi - \frac{\partial L^c}{\partial x}$$

$$\dot{x} = \frac{\partial L^c}{\partial \psi}$$

The optimality condition with respect to the control parameter is

$$\int_0^\infty e^{-\rho t}\frac{\partial L^c}{\partial \underline{u}} dt \geq 0 \ (= 0 \text{ if } \underline{u} < \underline{u}_H)$$

where $\underline{u}_H$ is the infinite-horizon counterpart of b in equation (8). Finally, the transversality conditions with respect to the stocks are

$$\lim_{t\to\infty} e^{-\rho t}\psi(t) \geq 0, \text{ and } \lim_{t\to\infty} e^{-\rho t}\psi(t)x(t) = 0.$$

4 An Example: Optimal Renewable Resource Use under the MBR Criterion

Let us illustrate the implications of the MBR criterion for a model of renewable resource exploitation. The resource stock is a scalar $x(t)$. Its growth function is

$$\dot{x} = G(x) - c$$

where $G(x)$ is a strictly concave function which reaches a maximum at some $x_M > 0$. We call x_M the "maximum sustainable yield" stock level. Assume $G(0) = 0$ and $G'(0) > 0$. The variable c denotes the harvest rate.

The utility function is assumed to be dependent on both the stock (which provides amenity services) and the consumption:

$$u = u(x, c)$$

We assume u to be homothetic, strictly quasi-concave and increasing, with $u_{cx} \geq 0$, $u_c(x, 0) = \infty$ and $u_x(0, c) = \infty$. This means the indifference curves have the usual convex shape in the space (x, c).

We define the "golden rule stock level", denoted by x_g, as the stock level that maximizes long-run sustainable utility:

$$\max_x u(x, G(x))$$

This level is uniquely determined by the following equation, which equalizes the marginal rate of substitution with the marginal rate of transformation:

$$\frac{u_x(x_g, G(x_g))}{u_c(x_g, G(x_g))} = -G'(x_g)$$

Clearly, $x_g > x_M$, because $G'(x_g) < 0 = G'(x_M)$.

By the "modified golden rule stock level", we mean the stock level $\overline{x}_u$ which is defined by the equation

$$\frac{u_x(\overline{x}_u, G(\overline{x}_u))}{u_c(\overline{x}_u, G(\overline{x}_u))} = -G'(\overline{x}_u) + \rho$$

Then, since $G(x)$ is concave,

$$\overline{x}_u < x_g$$

This is because as we move along the curve $c = G(x)$ toward greater values of x, the ratio $G(x)/x$ falls, so the marginal rate of substitution $u_x(x, G(x))/u_c(x, G(x))$ falls, and thus $[u_x(x, G(x))/u_c(x, G(x))] + G'(x)$ falls, i.e. $[u_x(x, G(x))/u_c(x, G(x))] + G'(x)$ is a decreasing function of x. In particular, for any $\widetilde{x}$ that lies between $\overline{x}_u$ and x_g,

$$\rho = \frac{u_x(\overline{x}_u, G(\overline{x}_u))}{u_c(\overline{x}_u, G(\overline{x}_u))} + G'(\overline{x}_u) > \frac{u_x(\widetilde{x}, G(\widetilde{x}))}{u_c(\widetilde{x}, G(\widetilde{x}))} + G'(\widetilde{x}) > \frac{u_x(x_g, G(x_g))}{u_c(x_g, G(x_g))} + G'(x_g) = 0 \qquad (15)$$

Now consider the optimal growth program under the mixed Bentham-Rawls objective function.

$$\max \theta \underline{u} \; + (1-\theta) \int_0^{\infty} e^{-\rho t} u(x, c) dt$$

subject to

$$\dot{x} = G(x) - c$$

$$u(x, c) \geq \underline{u}$$

where $x(0) = x_0 > 0$.

An interesting question is: under the MBR criterion, does the optimal path approach a steady state that is somewhere between the modified golden rule stock level, $\overline{x}_u$, and the golden rule stock level, x_g? Proposition 1 below gives the answer.

Proposition 1: Under the MBR criterion, the steady state depends on whether the initial stock, x_0, is smaller or greater than x_ρ.

(i) If $x_0 > \overline{x}_u$, the optimal path consists of two phases. Phase I begins at $t = 0$ and ends at some finite $T > 0$. During Phase I, the utility level and the resource stock are both falling. Genuine saving is negative and rises toward zero. At time T, the pair (x, c) reaches a mixed

Bentham-Rawls steady-state pair $(\overline{x}_{mbr}, \overline{c}_{mbr})$ where

$$\overline{x}_u < \overline{x}_{mbr} < x_g$$

During Phase II, the system stays at the mixed Bentham-Rawls steady state $(\overline{x}_{mbr}, \overline{c}_{mbr})$. Genuine saving is constant and equal to zero.

(ii) If $x_0 < \overline{x}_u$, the optimal path consists of two phases. Phase I begins at $t = 0$ and ends at some finite $T > 0$. During Phase I, utility is constant, which implies a time path of falling harvest rate, and rising stock. Genuine saving in this phase is positive. In Phase II, the economy follows the standard *utilitarian path approaching asymptotically the utilitarian steady state given by modified golden rule stock level*, $\overline{x}_u$. Genuine saving is positive and falls steadily toward zero.

Proof: Please see the Appendix.

5 Sensitivity Analysis of Optimal Paths under the MBR criterion

5.1 A Renewable Resource Model with Amenity Values

We assume logarithmic preferences, $u(c, x) = \ln c + \ln x$, and a logistic specification for the reproduction function of the resource:

$$\dot{x} = rx\left(1 - \frac{x}{K}\right) - c$$

where K is the carrying capacity and r the intrinsic regeneration rate. We follow Brander and Taylor (1998), setting $r = 0.04$ and $k = 12,000$. We set the rate of time preference, $\rho = 0.05$, and we assign equal weights to the rawlsian and utilitarian components in our objective function, so $\theta = 0.5$. Let us use subscripts u, r and mbr to denote the solutions under the discounted utilitarian, maximin and mixed Bentham-Rawls criteria respectively and denote steady states by upper-bar variables.

As already pointed out, the dynamic adjustment and the steady state of our economy depend crucially on whether the initial resource stock, x_0, is below or above the modified golden rule stock of resources, $\bar{x}_u$. We shall examine both cases in detail.

Figure 1 illustrates the transitional dynamics of an economy that begins with a stock of resources equal to four times the modified golden rule stock. The Rawlsian steady state is given by the golden rule stock of resource, and since the initial stock exceeds this level by 1.5 times we set a time path for Rawlsian consumption, $c_r(t) = \bar{c}_r$, that coincides with the long-run level of consumption under this criteria. With an initial stock of resource equal to the carrying capacity, the rate of consumption $c_r(t)$ at $\bar{c}_r$ leads to a negative saving rate and a falling stock of resource that, in finite time, reaches the golden rule stock of resource[24]. In steady state, rawlsian consumption exceeds the modified golden rule one by 18.5% guaranteeing that each future generation exactly inherits the golden rule stock of resources On the other hand the utilitarian dynamics are driven by the trade-off between the marginal rate of growth of the resource and the rate of time preference. Since with the stock close to the carrying capacity its rate of reproduction is negligible this trade-off is dominated by impatience and the initial generation enjoys a level of consumption that exceeds by almost 4.5 times the modified golden rule one. At this level, consumption is substantially larger than the capacity of regeneration of the resource and therefore the resource stock begins to fall. The rest of the transition is characterized by subsequent decreases in consumption and the stock of resource, with genuine savings increasing monotonically towards zero as the growth rate of the stock increases.

The mixed Bentham-Rawls criterion, which is a compromise between the previous two, leads to an optimal path with two clearly distinctive phases. During Phase I, that begins at $t = 0$ and ends at some finite time, $t_\theta > 0$, this path follows the unstable dynamics of the utilitarian solution. The initial level of consumption lies between the rawlsian choice and the utilitarian one, exceeding the modified golden rule level of consumption by more than 3.5 times. As in the utilitarian case, this high level of consumption leads to an initial phase characterized

[24] Notice that in our model with amenity values, the golden rule stock of resource, which is the stock that maximizes long-run sustainable utility, exceeds the stock of resource that maximizes long-run sustainable consumption, i.e. the "maximum sustainable yield."

by decreases in the resource stock and consumption, with the saving rate monotonically increasing towards zero. This process of stock depletion continues for 120 generations[25], $t_\theta = 120.3$. At this point in time the mixed Bentham-Rawls solution reaches its steady state characterized by a stock of resource that exceeds the modified golden rule stock by 80% representing around 68% of the Rawlsian steady state stock of resource. Panel (e) reproduces the evolution of welfare under our three criteria. Since the stock of resource is initially high, the utilitarian solution yields an initial level of welfare that exceeds the modified golden rule level of welfare by 23%. As a result of the high rate of utilitarian consumption the first 100 generations are better off under the utilitarian solution than under any of the other two. On the other hand the Rawlsian solution yields a path of welfare that falls mildly as the stock of resources converges to the golden rule stock from above. In the long run Rawlsian welfare exceeds by 9% that of the utilitarian steady state. The mixed Bentham-Rawls solution takes advantage of the initial abundance of resources to provide early generations with a level of welfare that exceeds the rawlsian one by more than 12%, but at the same time uses this abundance to guarantee all future generations a level of welfare that exceeds the utilitarian steady state one by 7.5%.

The last panel of **Figure 1** and the first three columns of **Table 1** explore the sensitivity of the optimal adjustment path under the mixed Bentham-Rawls criteria to θ. As the weight placed in the Rawlsian component of our objective function, θ, increases the mixed Bentham-Rawls steady-state stock of resource increases towards the Rawlsian one. This reduces the level of consumption during the initial generations and shortens the length of the transition. As the initial stock of resources increases, the mixed Bentham-Rawls steady-state stock of resource increases relative the utilitarian and Rawlsian ones, which still reach the modified golden rule and the golden rule stock of resource respectively. The length of the initial phase of the transition also increases. In a sense the higher initial stock is fairly distributed across generations, on one hand increasing the number of generations in Phase I and on the other hand increasing the steady-state levels of consumption and stock to allow all future

[25] Notice that under our benchmark calibration the speed of convergence exhibited by our model is very low, with the half-life of a deviation close to 50 generations.

generations to enjoy higher levels of welfare.

Figure 2 and **Table 2** summarize our results when the initial stock of resources is below the modified golden rule stock, specifically in our benchmark calibration we consider the case when $x_0 = 0.5 * \bar{x}_u$. The maximin solution chooses a level of consumption below 60% of the modified golden rule level of consumption, consistent with a stock of resource that stays constant and equal to its initial level. The utilitarian solution with its emphasis on intertemporal trade-offs chooses a level of consumption for the initial generation that is only 43% of the modified golden rule level. This low level of initial consumption is associated with substantial increases in the stock of resources. The utilitarian transition is characterized by a monotonic increase in consumption and the stock of resources and a monotonic decrease in genuine saving. Our proposed criterion leads to an initial level of consumption close to 50% of the modified golden rule level, that allows for a rate of saving above 14%. During Phase I of the mixed Bentham-Rawls transition consumption falls as the other source of utility, the stock of resource, accumulates. With consumption falling and the stock of resource increasing the saving rate increases during this first phase of the transition. After 18 generations, $t_\theta = 17.71$, the saving rate peaks and Phase II of the transition begins with consumption growing at a rate that still allows for positive saving and therefore increases in the stock of resources that monotonically converges to the modified golden rule stock. As we increase the weight placed on the rawlsian component of our objective function, θ, the initial level of consumption, c_θ, and the length of Phase I of the transition, t_θ, increase. Decreases in the initial stock of resources lead to simultaneous decreases in the initial level of consumption and in the length of Phase I of the transition under our proposed criterion.

5.2 A Capital Accumulation Model

We now consider a simple model of economic growth. Assume an economy with a single capital stock, $k(t)$. The production function, $F(k(t))$, exhibits positive and diminishing marginal product in its only argument. Let $\delta > 0$ be the rate of depreciation and $c(t)$ the level of consumption. The path of the capital stock is given by,

$$\dot{k} = F(k) - c - \delta k$$

The utility function is

$$u = u(c)$$

which is strictly concave and increasing $u'(0) = \infty$.

Now consider the optimal growth program under the Mixed Bentham-Rawls objective function. Assume $0 < \theta < 1$.

$$\max \theta \underline{u} + (1 - \theta) \int_0^\infty e^{-\rho t} u(c) dt$$

subject to

$$\dot{k} = F(k) - c - \delta k \tag{16}$$

$$u(c) \geq \underline{u}$$

where $k(0) = k_0 > 0$.

We assume logarithmic preferences, $\ln(c)$, and a standard Cobb-Douglas production specification, $F(k) = Ak^\alpha$, with an elasticity of output to capital, $\alpha = 0.3$, and a constant level of technology, $A = 1.5$. In our bechmark calibration we set the rate of time preference, $\rho = 0.05$, and the depreciation rate, $\delta = 0.1$. As before we assign equal weights to the rawlsian and utilitarian components in our objective function, $\theta = 0.5$, and continue with our notational convention.

Figure 3 illustrates the transitional dynamics of an economy that begins with 50% of the modified golden rule level of capital. The phase diagram in panel (a) highligths the main differences between the three criteria considered. The maximin solution chooses levels of consumption and saving that guarantee that each future generation exactly inherits the intial capital stock. The utilitarian solution, with its disregard for intergenerational equity, places all the burden of accumulation on the initial generations leading to a transition characterized by increasing capital, consumption and therefore welfare. The mixed Bentham-Rawls criterion, which is a compromise between the previous two, leads to an optimal path with two clearly distinctive phases. Phase I, that reflects rawlsian considerations, begins at t_0 and ends at some finite time, $t_\theta > 0$. The level of consumption during this phase, c_θ, is constant at some level that lies between the initial utilitarian level of consumption and the maximin level of consumption, $\bar{c}_r$. In Phase II, that reflects utilitarian considerations, the economy increases capital and consumption, approaching asymptotically the modified golden rule level of capital, $\bar{k}_u$.

Panels (b), (c) and (d) reproduce the time paths of consumption, saving and capital under our benchmark calibration. The maximin solution chooses an initial level of consumption close to 90% of the modified golden rule level. Given the low capital stock, this high level of consumption is associated with a saving rate of 12% just enough to replace the depreciated capital. This condemns all future generations to the same initial level of consumption. The utilitarian adjustment path is driven by the difference between the marginal product of capital, the return to saving, and the rate of time preference, the cost of giving up and saving an additional unit of consumption. With low levels of capital the return to investment exceeds its cost in terms of utility and therefore the first generation enjoys a level of consumption that is barely 70% of the steady state level of consumption. This low level of consumption is associated with an initial investment rate close to 30% which leads to a transition characterized by increasing levels of capital. As capital accumulates the incentives to defer consumption fall and as a result the saving rate monotonically decreases as consumption increases. The mixed Bentham-Rawls criteria chooses an initial level of consumption that exceeds its utilitarian counterpart by almost 17%. This level of consumption, although high by utilitarian standards, allows for a saving rate that exceeds its rawlsian counterpart by more than 50% and therefore capital begins to accumulate. With consumption constant at c_θ and output growing, the saving rate increases at this early stage of the transition. At the beginning of the the sixth generation, $t_\theta = 5.1$, the capital stock has increased to around 75% of its steady state level and Phase II of the mixed Bentham-Rawls path begins. At this point consumption begins to grow and the saving rate reaches its maximum. The dynamics of this phase are purely driven by utilitarian concerns and therefore as capital accumulates consumption grows, and saving falls, converging monotonically to the steady states that coincides with the utilitarian steady state.

Finally the path of welfare mimics the path of consumption. The utilitarian solution requires large sacrifices on early generations, the initial welfare is barely 50% of the modified golden rule level of welfare, achieving the constant Rawlsian level of welfare only after 5 generations. Thereafter utilitarian welfare overtakes its rawlsian counterpart. Our compromise criteria guarantees a minimum level of welfare for the initial generations and as a result the economy must wait around 8 generations to reach the Rawlsian level of welfare, overtaking it after this point. On average the first three generations enjoy around 22% more welfare under the mixed Bentham-Rawls criteria than under the utilitarian solution.

The last panel of **Figure 3** compares the level of consumption, c_θ, and the number of generations, t_θ, that the mixed Bentham-Rawls solution remains in Phase I for different values of θ. As we increase θ the mixed Bentham-Rawls solution converges to the maximin solution and therefore c_θ converges to $\bar{c}_r$ and Phase I lasts forever. **Table 3** explores the effects of changes in the rate of time preference and the initial stock of capital on the optimal path of our proposed welfare criteria. Our results are not very sensitive to the rate of time preference but as the initial stock of capital increases towards the modified golden rule level the three welfare criteria converge to the same optimal path and as a result the mixed Bentham-Rawls criteria chooses initial levels of consumption closer to the rawlsian criteria lengthening the initial phase of the transition.

Now we turn to the case when the initial capital stock is above the modified golden rule stock, especifically $k_0 = 1.5 * \bar{k}_u$. **Figure 4** reproduces the phase diagram and the transitional path of the relevant economic indicators for our three welfare criteria. Since k_0 is below the golden rule capital stock, the maximin solution chooses a level of consumption, and therefore a saving rate, consistent with mantaining the capital stock permanently at its initial level. As a result of the diminishing returns to capital the utilitarian solution initially chooses very high levels of consumption, exceeding by as much as 29% its steady state level. This high consumption can only be maintained at the expense of the capital stock, and with a saving rate below 10% capital begins to fall. As capital falls, so does consumption and output, that monotonically converge to the modified golden rule levels while the saving rate increases as the marginal product of capital net of depreciation increases towards the rate of time preference, ρ.

The mixed Bentham-Rawls optimal path consists again of two phases. During Phase I this solution follows the unstable dynamics of the utilitarian solution. The initial level of consumption lies between the utilitarian choice and the Rawlsian one, exceeding the modified golden rule level of consumption by 28%. As in the utilitarian case, this high level of consumption leads to an initial phase characterized by decreases in capital, output and consumption with associated increases in the saving rate. The economy remains in this phase for almost sixteen generations, $t_\theta = 15.9$. At this point the mixed Bentham-Rawls solution reaches its steady state, Phase II, characterized by levels of capital and consumption that exceed by 5% and 0.5% respectively the modified golden rule levels asymptotically achieved by the utilitarian solution. **Table 4** explores the effects of changes in θ and k_0 in our solutions. Under the mixed Rawls-Bentham criteria increases in the weigth of the rawlsian component of the objective function, θ, reduce the initial level of consumption towards the maximin level, shortening the initial phase of the transition and increasing its steady state level, $\bar{c}_{mbr}$. Similarly increases in the initial capital stock allow for the Rawls-Bentham criteria to reach permanently higher steady state stocks, that in the case of $k_0 = 2 * \bar{k}_u$, exceed by almost 7% the utilitarian steady state. Notice that when the initial capital stock is above the golden rule level of capital the Rawlsian criteria converges to this point.

6 Concluding Remarks

In this paper, we proposed a new welfare criterion, called the Mixed Bentham-Rawls Criterion, that we believe does justice to the rawlsian notion of intergenerational equity. We have restricted attention to the problem of intergenerational equity, and to facilitate the analysis, we have abstracted from intra-generational equity.

We showed that optimal growth paths under the Mixed Bentham-Rawls criterion can be characterized using standard techniques. These paths seem intuitively plausible, and reflect both the Rawlsian concerns for the least advantaged, and the utilitarian principle. We also obtained a characterization of the relationship between the growth rate of genuine savings and the growth rate of utility.

Acknowledgments: This paper has benefited from the useful comments of Geir Asheim, Hassan Benchekroun, Gerard Gaudet, John M. Hartwick, Seamus Hogan, Murray C. Kemp, Pierre Lasserre, Kim Long, Andres Pandiella, Frank Stähler, Kotaro Suzumura, the late Koji Shimomura, and participants at the CIREQ conference on intergenerational allocation, and seminar partcipants at the Australian National University, University of Montreal, University of Canterbury, University of Otago, and Victoria University of Wellington.

APPENDIX

Proof of Proposition 1

Define the current-value Hamiltonian and Lagrangian:

$$H^c = \theta \underline{u} \rho + (1-\theta)u(x,c) + \psi \left[G(x) - c\right]$$

$$L^c = H^c + w \left[U(x,c) - \underline{u}\right]$$

The optimality conditions are

$$\frac{\partial L^c}{\partial c} = (1-\theta+w)u_c - \psi = 0 \tag{17}$$

$$\dot{\psi} = \psi \left[\rho - G'(x)\right] - (1-\theta+w)u_x = (1-\theta+w)u_c \left\{\rho - G'(x) - \frac{u_x}{u_c}\right\} \tag{18}$$

Let us define the "effective shadow price" p by

$$p \equiv \frac{\psi}{1-\theta+w} \tag{19}$$

Then equation (17) yields the optimal control c as a function of p and x:

$$u_c(x,c) = p$$

At any time, if the stock level x and the effectice shadow price p are known, we can determine the optimal harvesting rate

$$c = c(x,p)$$

where

$$u_{cc}\frac{\partial c}{\partial p} = 1$$

$$u_{cx} + u_{cc}\frac{\partial c}{\partial x} = 0$$

$$c_p(x,p) = -\frac{1}{u_{cc}} > 0$$

$$c_x(x,p) = -\frac{u_{cx}}{u_{cc}} \geq 0$$

Note that $\dot{\psi} = 0$ if and only if

$$\frac{U_x}{U_c} = \rho - G'(x)$$

We ask the following questions, which are particularly relevant in our mixed Bentham-Rawls optimization problem. First, is it optimal to approach the utilitarian modified golden rule stock level, $\overline{x}_u$, and the associated modified golden rule consumption, $\overline{c}_u \equiv G(\overline{x}_u)$? Second, can $\dot{\psi} > 0$ and yet (c,x) stay constant at a steady state which is different from the standard utilitarian pair $(\overline{c}_u, \overline{x}_u)$? We will show that if $x_0 > \overline{x}_u$ we should approach a steady state $\overline{x}_{mbr} > \overline{x}_u$, at which point we have a stationary effective shadow price, i.e., $\dot{p} = 0$ and yet $\dot{\psi} > 0$ because of (15) and (18). In what follows, we construct a path that satisfies all the necessary conditions, and then apply the sufficiency theorem to show that it is the optimal path.

From the definition of p

$$\dot{p} = \frac{\dot{\psi}}{1-\theta+w} - p\left(\frac{\dot{w}}{1-\theta+w}\right) \tag{20}$$

Thus

$$\dot{p} = u_c\left\{\rho - G'(x) - \frac{u_x}{u_c}\right\} - p\left(\frac{\dot{w}}{1-\theta+w}\right)$$

Now we prove part (i) of the Proposition:

Suppose $x_0 > \overline{x}_u$. It is feasible to approach the modified golden rule stock, $\overline{x}_u$, along a path with monotone non-increasing consumption, and this would yield the utility level $u(\overline{x}_u, G(\overline{x}_u))$ for the least advantage generations. But under the mixed Bentham-Rawls criterion, it is not optimal to do so, because the least advantaged generations can be made better off if the planner chooses to approach some stock level $\overline{x}_{mbr}$ such that $\overline{x}_u < \overline{x}_{mbr} < x_g$. Their indifference curve would be to the right of the curve $u(x,c) = u(\overline{x}_u, G(\overline{x}_u))$.

To see this formally, we note that if we approach $\overline{x}_u$ along the standard utilitarian saddle-point path, then it follows that $w(t) = 0$ for all finite t, which implies a violation of the transversality condition that

$$\theta = \int_0^\infty e^{-\rho t} w(t) dt$$

So the optimal path must reach, in finite time, a steady state stock level $\overline{x}_{mbr}$ where

$$\overline{x}_u < \overline{x}_{mbr} < x_g$$

At $\overline{x}_{mbr}$,

$$\frac{u_x(\overline{x}_{mbr}, G(\overline{x}_{mbr}))}{u_c(\overline{x}_{mbr}, G(\overline{x}_{mbr}))} + G'(\overline{x}_{mbr}) < \frac{u_x(\overline{x}_u, G(\overline{x}_u))}{u_c(\overline{x}_u, G(\overline{x}_u))} + G'(\overline{x}_u) = \rho$$

but $\dot{p} = 0$ as long as $\dot{w}$ satifies the condition

$$\frac{\dot{w}}{1-\theta+w} = \frac{u_c}{p^*}\left\{\rho - G'(\overline{x}_{mbr}) - \frac{u_x(\overline{x}_{mbr}, G(\overline{x}_{mbr}))}{u_c(\overline{x}_{mbr}, G(\overline{x}_{mbr}))}\right\} > 0$$

where

$$p^* = u_c(\overline{x}_{mbr}, G(\overline{x}_{mbr}))$$

At the steady state $\bar{x}_{mbr}$

$$\frac{\dot{\psi}}{\psi} = \rho - G'(\overline{x}_{mbr}) - \frac{u_x(\overline{x}_{mbr}, G(\overline{x}_{mbr}))}{u_c(\overline{x}_{mbr}, G(\overline{x}_{mbr}))} > 0$$

The transversality condition

$$\lim_{t\to\infty} e^{-\rho t}\psi(t)x(t) = 0$$

is satisfied because

$$\lim_{t\to\infty} e^{-\rho t}\psi(t) = \lim_{t\to\infty} e^{-\rho t} A \exp\left\{\left[\rho - G'(\overline{x}_{mbr}) - \frac{u_x(\overline{x}_{mbr}, G(\overline{x}_{mbr}))}{u_c(\overline{x}_{mbr}, G(\overline{x}_{mbr}))}\right] t\right\} = 0$$

since, for $\overline{x}_{mbr} < x_g$, it holds that

$$-G'(\overline{x}_{mbr}) < \frac{u_x(\overline{x}_{mbr}, G(\overline{x}_{mbr}))}{u_c(\overline{x}_{mbr}, G(\overline{x}_{mbr}))}$$

It is not possible to find a closed form expression for $\overline{x}_{mbr}$ because $\overline{x}_{mbr}$ depends on the initial stock x_0. But we can state the conditions that must be satisfied.

Starting from $x_0 > \overline{x}_u$, there are two phases.

In Phase I, utility is strictly falling, and $u > \underline{u}$, so that $w(t) = 0$. During this phase, the harvest rate satisfies the condition

$$(1-\theta)u_c - \psi = 0$$

hence

$$c = c(x, \psi/(1-\theta))$$

The evolution of ψ in Phase I is described by

$$\dot{\psi} = (1-\theta+w)u_c(x, c(x, \psi/(1-\theta))) \left\{ \rho - G'(x) - \frac{U_x(x, c(x, \psi/(1-\theta)))}{U_c(x, c(x, \psi/(1-\theta)))} \right\}$$

In Phase II, c is a constant, $\dot{\psi} > 0$ but $\dot{p} = 0$. During this phase

$$\frac{\dot{w}}{1-\theta+w} = \rho - G'(\overline{x}_{mbr}) - \frac{U_x(\overline{x}_{mbr}, \overline{c}_{mbr})}{U_c(\overline{x}_{mbr}, \overline{c}_{mbr})} \equiv q(\overline{x}_{mbr}) < \rho \text{ since } \overline{x}_{mbr} < x_g \tag{21}$$

where

$$\overline{c}_{mbr} \equiv G(\overline{x}_{mbr})$$

and

$$q'(\overline{x}_{mbr}) > 0, \lim_{\overline{x}_{mbr} \to x_\rho} q(\overline{x}_{mbr}) = 0, \lim_{\overline{x}_{mbr} \to x_g} q(\overline{x}_{mbr}) = \rho$$

Let T denote the transition time from Phase I to Phase II. The following transversality condition must be met

$$\theta = \int_T^\infty e^{-\rho t} w(t) dt \tag{22}$$

where $w(T) = 0$. Thus, from (21), and $w(T) = 0$, we get, for $t \geq T$

$$w(t) = (1-\theta)e^{q(t-T)} - (1-\theta)$$

Substituting into (22)

$$\theta = (1-\theta)e^{-qT}\int_T^\infty e^{-(\rho-q)t}dt - (1-\theta)\int_T^\infty e^{-\rho t}dt$$

Thus

$$e^{\rho T} = \left(\frac{1-\theta}{\theta}\right)\frac{q}{\rho(\rho-q)}$$

This equation requires T to be an increasing function of q and hence an increasing function of $\overline{x}_{mbr}$:

$$T = \widetilde{T}(\overline{x}_{mbr}) \tag{23}$$

Now consider Phase I. During this phase, $w(t) = 0$. We have two differential equations

$$\dot{x} = G(x) - c(x,p)$$

$$\dot{p} = U_c\left\{\rho - G'(x) - \frac{u_x}{u_c}\right\}$$

with boundary conditions, $x(0) = x_0$, $x(T) = \overline{x}_{mbr}$ and $p(T) = u_c(\overline{x}_{mbr}, G(\overline{x}_{mbr}))$. These equations yield

$$T = \widehat{T}(x_0, \overline{x}_{mbr}) \tag{24}$$

where $\frac{\partial \widehat{T}}{\partial x_0} < 0$ and $\frac{\partial \widehat{T}}{\partial \overline{x}_{mbr}} > 0$.

The two equations (23) and (24) yield

$$\widetilde{T}(x_\theta) - \widehat{T}(x_0, x_\theta) = 0$$

from which we obtain

$$\widetilde{T}'(\overline{x}_{mbr})d\overline{x}_{mbr} - \frac{\partial \widehat{T}}{\partial \overline{x}_{mbr}}d\overline{x}_{mbr} - \frac{\partial \widehat{T}}{\partial x_0}dx_0 = 0$$

thus

$$\frac{d\overline{x}_{mbr}}{dx_0} = \frac{\frac{\partial \widehat{T}}{\partial x_0}}{\left[\widetilde{T}'(\overline{x}_{mbr}) - \frac{\partial \widehat{T}}{\partial \overline{x}_{mbr}}\right]} \tag{25}$$

Then

$$\overline{x}_{mbr} = X(x_0).$$

It remains to show that $X(.)$ is an increasing function for all $x_0 > x_\rho$ (i.e., that the denominator of (25) is negative), and

$$\lim_{x_0 \to \overline{x}_u} X(x_0) = \overline{x}_u.$$

Part (ii) of the Proposition can be proved in a similar way.

References

[1] Anderson, Elizabeth (1999) 'What is the Point of Equality?' *Ethics 1*09 (2),287-337.

[2] Arneson, Richard J. (2002) ' Why justice requires Transfer to Offset Income and Wealth Inequalities,' *Social Philosophy and Policy* 19(1), 172-200.

[3] Arrow, K. J.(1973a) 'Some Ordinalist-Utilitarian Notes on Rawls' Theory of Justice', *The Journal of Philosophy*, vol. 70 (9), 245-263.

[4] Arrow, K. J. (1973b) 'Rawls' Principle of Just Savings', *Sweedish Journal of Economics*, vol. 75, 323-35.

[5] Asheim, Geir B. (1988), "Rawlsian Intergenerational Justice as a Markov-Perfect Equilibrium in a Resource Technology," *Review of Economic Studies* 55, 3, 469-483.

[6] Asheim, G. B.(2000) 'Green National Accounting: Why and How?', *Environment and Development Economics*, vol. 5 (1), pp. 25-48.

[7] Asheim, G. B., W. Buchholtz, and C. Withagen (2003) 'The Hartwick Rule: Myths and Facts', *Environmental and Resource Economics*, vol. 25 (2), 129-50.

[8] Asheim, G. B. and W. Buchholtz (2004) 'A General Approach to Welfare Measurement through National Income Accounting', *Scandinavian Journal of Economics,* vol. 160, June, pp 360-84.

[9] Basu, K. and Mitra, T. (2003) 'Aggregating Infinite Utility Streams with Intergenerational Equity: The Impossibility of Being Paretian', *Econometrica* 79, 1557-1563.

[10] Beltratti, A., G. Chichilnisky, and G. Heal (1997) 'Sustainable Use of Renewable Resources", in G. Chichilnisky, G. Heal, and Vercelli (eds), *Sustainability, Dynamics and Uncertainty*, Kluwer Academic Publisher.

[11] Brander, James A. and M. Scott Taylor (1998) 'The Simple Economics of Easter Island: A Ricardo-Malthus Model', *American Economic Review,* Vol. 88, pp. 119-138.

[12] Chichilnisky, G. (1977) 'Economic Development and Efficiency Criteria in the Satisfaction of Basic Needs', *Applied Math Modeling* 1(6), 290-297.

[13] Chichilnisky, G. (1996) 'An Axiomatic Approach to Sustainable Development', *Social Choice and Welfare,* vol. 13 (3), 231-257. (www.chichilnisky.com)

[14] Chichilnisky, G. (1997) 'What is Sustainable Development?" *Land Economics* 73(4), 467-91.

[15] Dasgupta, P. S.(1974) 'On Some Alternative Criteria for Justice Between Generations', *Journal of Public Economics*, vol. 3, 405-423.

[16] Dasgupta, P. S. and G. M. Heal (1979), *Economic Theory and Exhaustible Resources,* Cambridge Economic Handbooks, CUP.

[17] Diamond, P. (1965) 'The Evaluation of Infinite Utility Streams', *Econometrica* 33, 170-177.

[18] Dixit, A., P. Hammond, and M. Hoel (1980)'On Hartwick's Rule for Regular Maximin Paths of Capital Accumulation and Resource Depletion', *Review of Economic Studies,* vol. 47, 551-6.

[19] Figuiere, Charles and Mabel Tidball (2007) 'Sustainable Exploitation of Natural Resource: A Satisfying Use of Chichilnisky's Criterion' (INRA, UMR LAMETA, Montpellier, France)

[20] Fleurbaey, M. and P. Michel (2003) 'Intertemporal Equity and the Extension of the Ramsey Criterion,' *Journal of Mathematical Economics*, 39, 777-802.

[21] Frankfurt, H. (1987) 'Equality as a Moral Idea', *Ethics* 98, 21-42.

[22] Gale, David (1967) 'Optimal Development in a'Multi-sector Economy', *Review of Economic Studies*, 34, 11-8.

[23] Hamilton, Kirk and John M. Hartwick (2005) 'Investing Exhaustible Resource Rents and the Path of Consumption', *Canadian Journal of Economics* 38(2), 615-21.

[24] Hamilton, Kirk and Cees Withagen (2006),'Savings Growth and the Path of Utility', *Canadian Journal of Economics* (to appear).

[25] Hartwick, J. M. (1977) 'Intergenerational Equity and the Investing of Rents from Exhaustible Resources,' *American Economic Review* 69, 972-974.

[26] Heal, G. (1998) *Valuing the Future: Economic Theory and Sustainability* (New York: University of Columbia Press)

[27] Koopmans, T. C. (1965) 'On the Concept of Optimal Economic Growth', *Pontificiae Academiae Scientiarum Scripta Varia,* Vol. 28, 225-287.Reprinted in *Scientific Papers of T.C. Koopmans,* 1970 (Berlin:Springer)

[28] Leonard, D. and N. V. Long (1991) *Optimal Control Theory and Static Optimization in Economics* (Cambridge:Cambridge University Press)

[29] Long, N. V. (2007), Toward a theory of a just savings principle, in Roemer, J. and K. Suzumura, eds., *Intergenerational Equity and Sustainable Development,* Palgrave, UK. Chapter 1, 290-319.

[30] Mitra, T. (1983) 'Limits on Population Growth under Exhaustible Resource Constraint', *International Economic Review,* vol. 24, Feb., 155-68.

[31] Pezzey, J. (1994) Theoretical Essays on Sustainability and Environmental Policy, Ph. D. Thesis, University of Bristol.

[32] Pezzey, J. (1989) 'Economic Analysis of Sustainable Growth and Sustainable Development', *World Bank Environment Department*, Working Paper No. 15.

[33] Pezzey, J. and C. Withagen (1998) 'The Rise, Fall, and Sustainability of Capital-Resource Economies", *Scandinavian Journal of Economics*, Vol.100, 513-527.

[34] Pezzey, J. (2004) 'Sustainability Policy and Environmental Policy', *Scandinavian Journal of Economics,* Vol.106(2), 339-59.

[35] Ramsey, F. P. (1928)'A Mathematical Theory of Saving', *Economic Journal,* Vol. 38, 543-559

[36] Rawls, J. (1971) *A Theory of Justice* (First Edition) (Cambridge, Mass:The Belknap Press of the Harvard University Press)

[37] Rawls, J. (1999) *A Theory of Justice* (Revised Edition) (Cambridge, Mass:The Belknap Press of the Harvard University Press)

[38] Roemer, J. (2003) 'Eclectic Distributional Ethics,' *Cowles Foundation Discussion Paper* 1408.

[39] Segerberg, Krister (1976) 'A Neglected Family of Aggregation Problems in Ethics', *Noûs,* vol. 10, No.2, Symposium on Utilitarianism, pp 221-244.

[40] Sen, A. K.(1976) 'Rawls versus Bentham: An Axiomatic Examination of the Pure Distribution Problem', in N. Daniels (ed.) *Reading Rawls: Critical Studies on Rawls' A Theory of Justice*, (New York:Basic Book)

[41] Sidgwick, H. (1907), *The Methods of Ethics*, Macmillan, London.

[42] Solow, R. M. (1974) 'Intergenerational Equity and Exhaustible Resources', *Review of Economic Studies (*Symposium issue), vol. 41, pp. 29-45

[43] Solow, R. M. (1986) 'On the Intergenerational Allocation of Natural Resources', *Scandinavian Journal of Economics,* vol. 88, pp 141-9.

[44] Svensson, L.-L. (1980) 'Equity among Generations', *Econometrica* 48, 1251-1256.

[45] von Weizsäcker , C. C. (1965), 'Existence of Optimal Programmes of Accumulation for an Infinite Time Horizon', *Review of Economic Studies* Vol 32 , pp 85-104.

[46] Withagen, C. and G. B. Asheim (1998) 'Characterizing Sustainability: the Converse of Hartwick's Rule', *Journal of Economic Dynamics and Control,* vol. 323(1), 159-165.

Tables and Figures

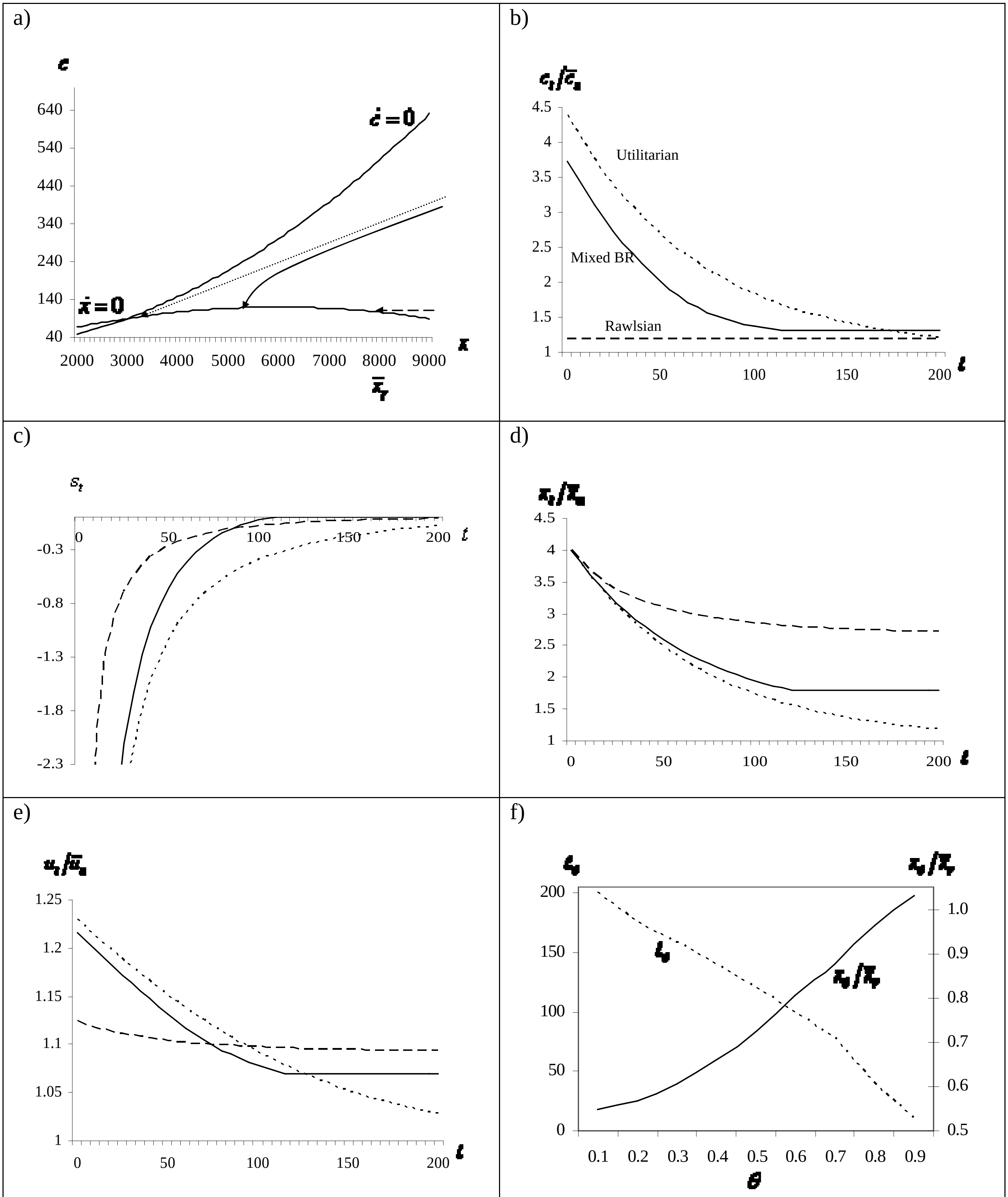

Figure 1. Optimal extraction path when $x_0 = 4 * \bar{x}_u$ and sensitivity of the mixed Bentham-Rawls transition to changes in θ.

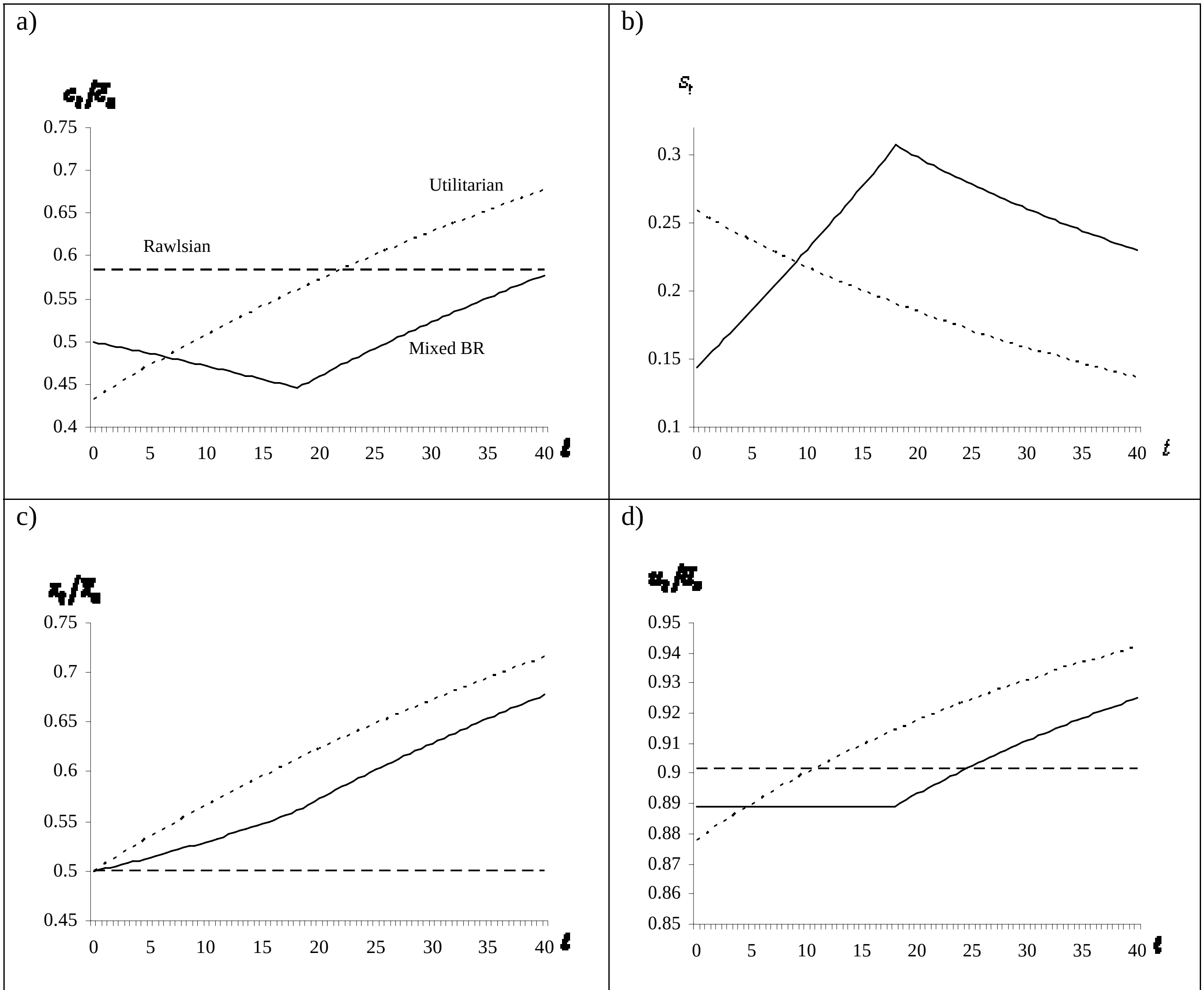

Figure 2. Optimal extraction path when $x_0 = 0.5 * \bar{x}_u$

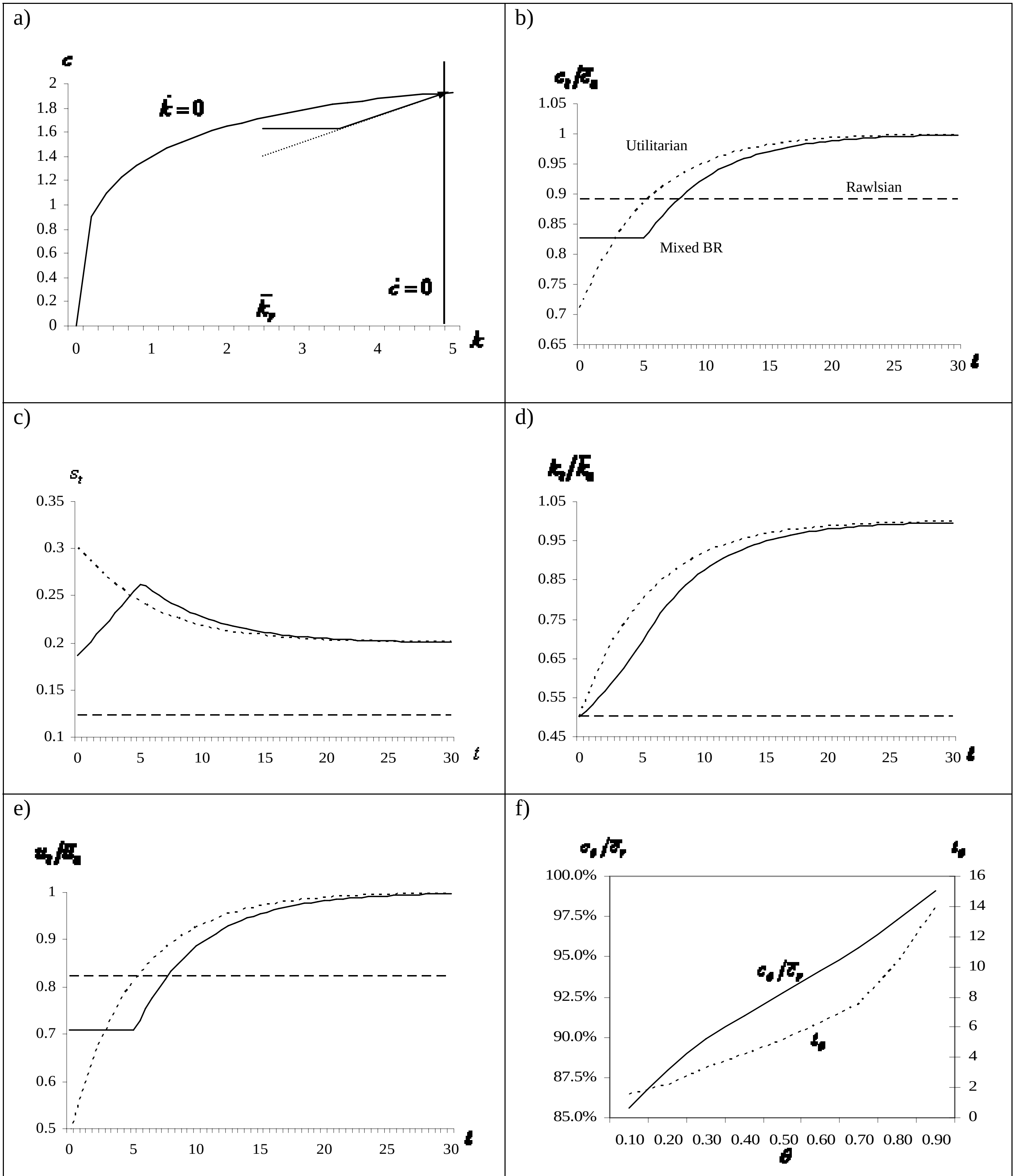

Figure 3. Optimal growth paths when $k_0 = 0.5 * \bar{k}_u$ and sensitivity of the initial phase to different values of θ.

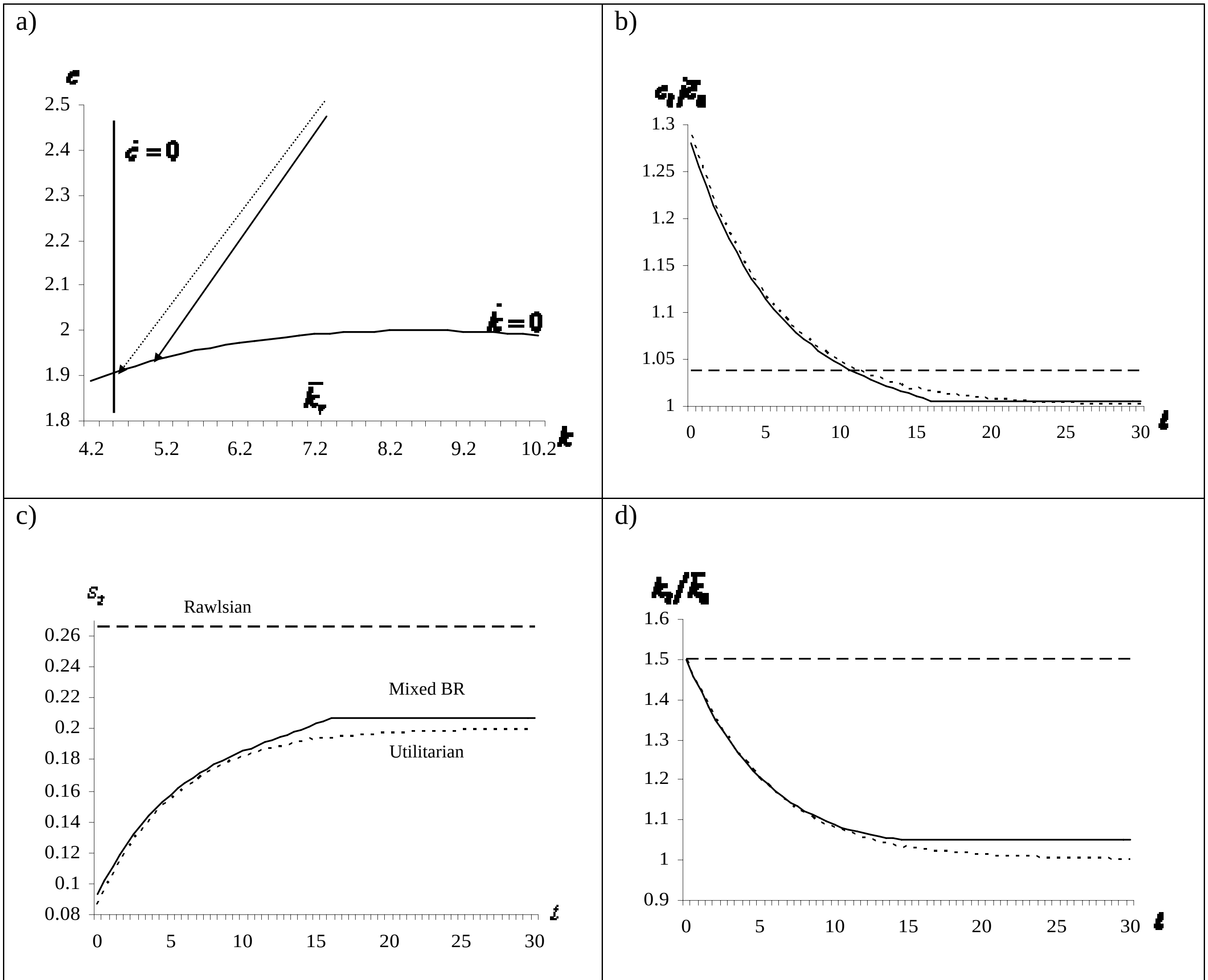

Figure 4. Optimal growth path when $k_0 = 1.5 * \overline{k_u}$

	$\theta = 0.25$	$\boldsymbol{\theta} = 0.5$	$\theta = 0.75$	$x_0 = 3 * \overline{x}_u$	$x_0 = 5 * \overline{x}_u$
$\overline{x}_u$	-	**3000**	-	-	-
$\overline{c}_u$	-	**90**	-	-	-
$c_u(0)/\overline{c}_u$	-	**4.405**	-	3.270	5.541
$\overline{x}_r/\overline{x}_u$	-	**2.667**	-	-	-
$\overline{c}_r/\overline{c}_u$	-	**1.185**	-	-	-
$\overline{x}_{mbr}/\overline{x}_u$	1.629	**1.800**	2.015	1.691	1.875
$\overline{c}_{mbr}/\overline{c}_u$	1.287	**1.320**	1.333	1.301	1.328
$c_{mbr}(0)/\overline{c}_u$	3.972	**3.856**	3.670	3.100	5.023
t_θ	140	**120.3**	98.6	109.22	121.73

Table 1. Sensitivity analysis optimal renewable resource extraction.
Benchmark: $\theta = 0.5$, $\rho = 0.05$, $r = 0.04$, $K = 12,000$, $x_0 = 4 * \overline{x}_u$.

	$\theta = 0.25$	$\boldsymbol{\theta} = 0.5$	$\theta = 0.75$	$x_0 = 0.25 * \overline{x}_u$	$x_0 = 0.75 * \overline{x}_u$
$\overline{x}_u$	-	**3000**	-	-	-
$\overline{c}_u$	-	**90**	-	-	-
$c_u(0)/\overline{c}_u$	-	**0.43**	-	0.152	0.719
$\overline{x}_r/\overline{x}_u$	-	**0.500**	-	0.250	0.750
$\overline{c}_r/\overline{c}_u$	-	**0.582**	-	0.316	0.813
$\overline{x}_{mbr}/\overline{x}_u$	-	**1.000**	-	1.000	1.000
$\overline{c}_{mbr}/\overline{c}_u$	-	**1.000**	-	1.000	1.000
$c_{mbr}(0)/\overline{c}_u$	0.482	**0.499**	0.533	0.223	0.601
t_θ	10.37	**17.71**	29.84	17.33	20.90

Table 2. Sensitivity analysis optimal renewable resource extraction.
Benchmark: $\theta = 0.5$, $\rho = 0.05$, $r = 0.04$, $K = 12,000$, $x_0 = 0.5 * \overline{x}_u$.

	$\theta = 0.25$	$\boldsymbol{\theta = 0.5}$	$\theta = 0.75$	$\rho = 0.01$	$\rho = 0.1$	$k_0 = .25 * \bar{k}_u$	$k_0 = .75 * \bar{k}_u$
$\bar{k}_u$	-	**4.803**	-	7.482	3.185	-	-
$\bar{c}_u$	-	**1.921**	-	1.999	1.805	-	-
$\bar{s}_u$	-	**0.200**	-	0.273	0.150	-	-
$c_u(0)/\bar{c}_u$	-	**0.711**	-	0.722	0.703	0.566	0.855
$\bar{k}_r/\bar{k}_u$	-	**0.500**	-	0.500	0.500	0.250	0.750
$\bar{c}_r/\bar{c}_u$	-	**0.890**	-	0.929	0.867	0.762	0.959
$\bar{s}_r$	-	**0.123**	-	0.172	0.089	0.075	0.163
$c_\theta/\bar{c}_u$	0.792	**0.826**	0.862	0.836	0.823	0.666	0.937
t_θ	2.96	**5.14**	8.67	6.05	4.43	2.825	9.823

Table 3. Sensitivity analysis optimal growth path. Benchmark: $\theta = 0.5$, $\rho = 0.05$, $\alpha = 0.3$, $k_0 = 0.5 * \bar{k}_u$.

	$\theta = 0.25$	$\boldsymbol{\theta = 0.5}$	$\theta = 0.75$	$k_0 = 1.25 * \bar{k}_u$	$k_0 = 2 * \bar{k}_u$
$\bar{k}_u$	-	**4.803**	-	-	-
$\bar{c}_u$	-	**1.921**	-	-	-
$\bar{s}_u$	-	**0.200**	-	-	-
$c_u(0)/\bar{c}_u$	-	**1.289**	-	1.145	1.579
$\bar{k}_r/\bar{k}_u$	-	**1.500**	-	1.250	1.784
$\bar{c}_r/\bar{c}_u$	-	**1.037**	-	1.024	1.041
$\bar{s}_r$	-	**0.266**	-	0.234	0.300
$\bar{k}_{mbr}/\bar{k}_u$	1.022	**1.050**	1.111	1.043	1.057
$\bar{c}_{mbr}/\bar{c}_u$	1.003	**1.006**	1.012	1.005	1.007
$\bar{s}_{mbr}$	0.203	**0.207**	0.215	0.206	0.208
$c_{mbr}(0)/\bar{c}_u$	1.285	**1.280**	1.270	1.140	1.558
t_θ	20.52	**15.98**	11.71	13.12	18.85

Table 4. Sensitivity analysis optimal growth path.
Benchmark: $\theta = 0.5$, $\rho = 0.05$, $\alpha = 0.3$, $k_0 = 1.5 * \bar{k}_u$.