Control of a Flexible-Link Manipulator with Zero Steady-State Tracking Error

Dylan Caverly, supervised by Professor James Richard Forbes
Department of Mechanical Engineering, McGill University, Montreal, QC, Canada

Introduction

- All robot manipulators are flexible to some degree due to gear box deflection, or gear deflection, for example.
- Space manipulators, such as the Canadarm, are particularly flexible.
- The objective of this project is to make the payload velocity of a flexible-link manipulator track a sinusoidal signal with zero steady-state tracking error.

Definitions

Passivity: The mapping $\mathbf{u} \mapsto \mathbf{y}$ associated withe the operator $\mathcal{G} : L_{2e} \to L_{2e}$, where $\mathbf{y} = \mathcal{G}\mathbf{u}$, is passive if there exists a constant β such that [1]

$$\int_0^T \mathbf{y}^\mathsf{T}(t)\mathbf{u}(t)\mathrm{d}t \geq \beta \ \forall \mathbf{u} \in L_{2e}, \quad T \in \mathbb{R}^+.$$

SPR System: The linear time-invariant system $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}, \quad \mathbf{y} = \mathbf{C}\mathbf{x}$, is strictly positive real (SPR) if there exists a $\mathbf{P} = \mathbf{P}^\mathsf{T} > 0$ and $\mathbf{Q} = \mathbf{Q}^\mathsf{T} > 0$ such that [1]

$$\mathbf{P}\mathbf{A} + \mathbf{A}^\mathsf{T}\mathbf{P} = -\mathbf{Q}, \quad \mathbf{P}\mathbf{B} = \mathbf{C}^\mathsf{T}.$$

μ-Tip Rate: An approximation of the tip rate of a flexible appendage. A factor $0 \leq \mu < 1$ is used to scale the elastic coordinates on the tip rate. The μ-tip rate is defined as

$$\dot{\rho}_\mu = \mathbf{J}_\theta \dot{\theta} + \mu \mathbf{J}_e \dot{\mathbf{q}}_e.$$

If $\mu = 0$, the μ-tip rate represents the tip rate assuming a rigid beam, whereas as μ approaches 1, $\dot{\rho}_\mu$ approaches the true tip rate $\dot{\rho}$.

Internal Model Principle: Consider a negative feedback interconnection involving two systems, $g_1(s)$ and $g_2(s)$. If $g_1(s)$ or $g_2(s)$ (the plant or the controller) contains a model of the reference signal $r_2(s)$, the output $y_1(s)$ tracks $r_1(s)$ with zero steady-state error. [2]

Passivity Theorem: Consider a negative feedback interconnection involving two systems, $\mathcal{G}_1$ and $\mathcal{G}_2$. If $\mathcal{G}_1$ is passive and $\mathcal{G}_2$ is very strictly passive, the closed-loop system is L_2 stable, that is, $r_1, r_2 \in L_2$, then $y_1, y_2 \in L_2$. [1]

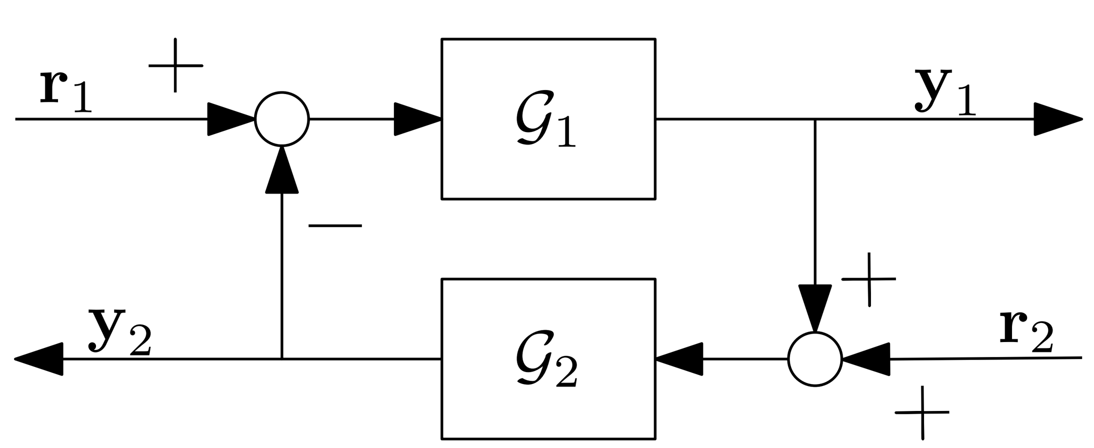

Figure: Negative feedback interconnection.

Dynamic Model

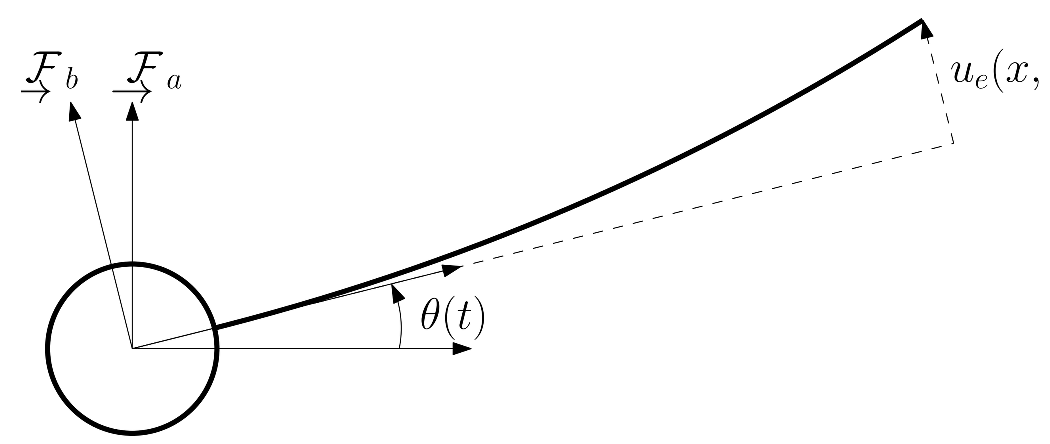

Figure: Diagram of flexible-link manipulator with tip mass.

Rayleigh-Ritz Method: The elastic deformation of a structure can be approximated as a linear combination of a set of basis functions

$$u_e(x,t) = \sum_{i=1}^{n} \Psi_i(x) q_{e_i}(t).$$

Equations of Motion: In order to satisfy all geometric boundary conditions, the chosen basis functions are of the form

$$\Psi_i(x) = x^{i+1}, \quad n = 3.$$

The elastic deformation of the flexible appendage is approximated using the Raleigh-Ritz method. Using Lagrange's equation, the dynamics of the system are described by

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \hat{\mathbf{b}}\tau$$

Controller Synthesis Method

Step 1: Add internal model to the passive plant.
Step 2: Design linear quadratic Gaussian (LQG) controller for augmented plant.
Step 3: Remove internal model from plant, augment LQG controller with internal model. This creates the controller $h_c(s)$ with state-space realization $(\mathbf{A}_c, \mathbf{B}_c, \mathbf{C}_c, \mathbf{D}_c)$.
Step 4: Find a solution $\mathbf{P} = \mathbf{P}^\mathsf{T} > 0$ to the Lyapunov equation

$$\mathbf{P}\mathbf{A}_c + \mathbf{A}_c^\mathsf{T}\mathbf{P} < \mathbf{0}.$$

Step 5: Create a new controller output matrix $\mathbf{C}_{c_2}$ such that $\mathbf{C}_{c_2} = \mathbf{B}_c^\mathsf{T}\mathbf{P}$.
Step 6: This controller, with state-space realization $(\mathbf{A}_c, \mathbf{B}_c, \mathbf{C}_{c_2}, \mathbf{D}_c)$ now contains the internal model and is Strictly Positive Real.

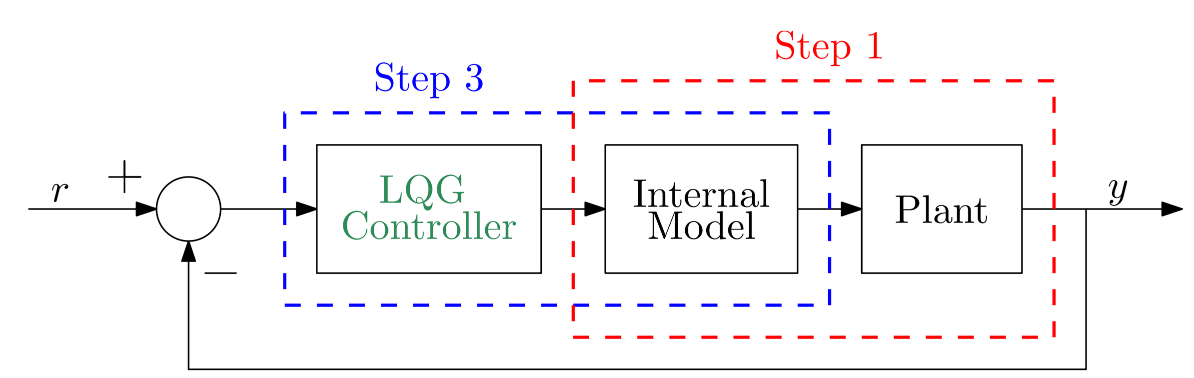

Figure: Controller Synthesis Method, Steps 1-3

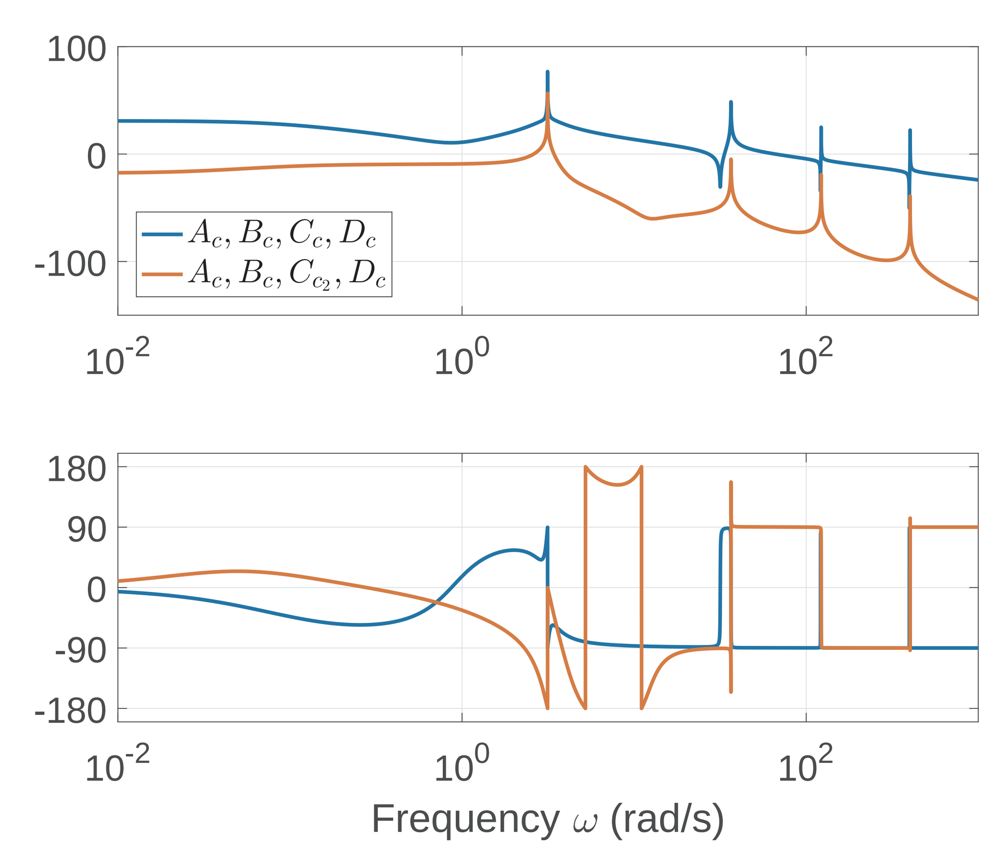

Figure: Bode diagram of controller before and after step 5.

Results

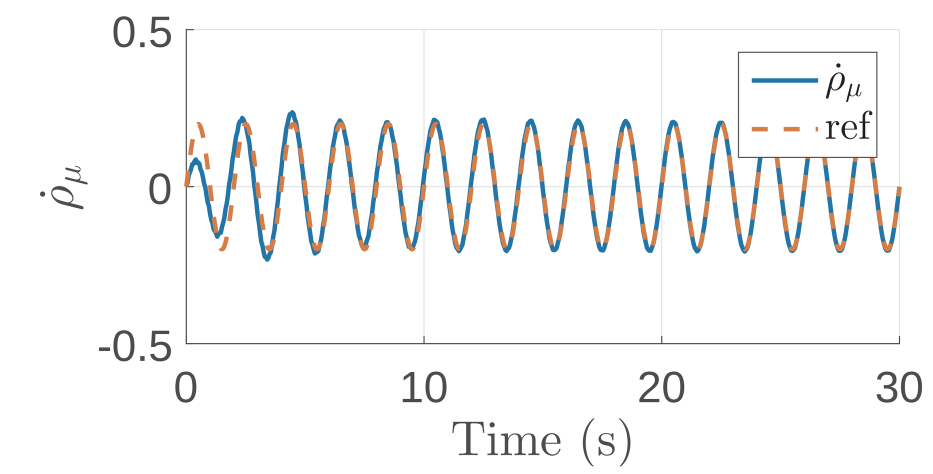

Figure: Desired and actual μ-tip rate vs. time.

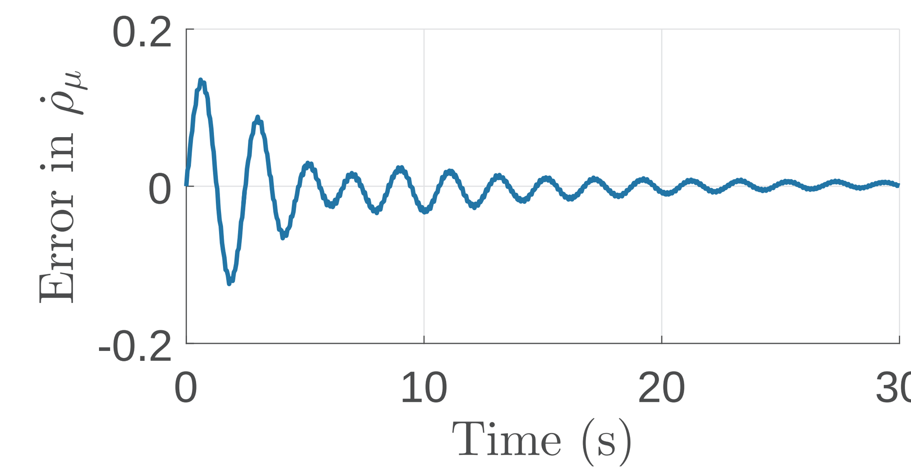

Figure: μ-tip rate error vs. time.

The system was simulated to track a sinusoidal reference signal, with $\mu = 0.5$.

Conclusion and Future Work

- The μ-tip velocity successfully tracks the reference signal with zero steady-state error.
- Use gain-scheduled controllers to track more complex signals.
- Test controller experimentally.

Acknowledgements

Special thanks to the SURE program and NSERC, as well as Professor Forbes and all graduate students that assisted me for their guidance throughout this project.

References

[1] H. Marquez, *Nonlinear Control Systems: Analysis and Design*. Hoboken, NJ: John Wiley, 2003.
[2] B. Francis and W. Wonham, "The internal model principle of control theory", *Automatica*, vol.12, no. 5, pp. 457-465, 1976.

e-mail: dylan.caverly@mail.mcgill.ca