Micro Comprehensive Exam 2023

Part A

(Please answer BOTH questions from this part.)

1. *Consumer preferences and demand*

Let there be two goods ($X = \mathbb{R}^2_+$). A consumer has the following rational preference relation $\succsim$ over X. The consumer cares only about increasing the quantity of good 1 as long as that quantity is less than 10, and when the quantity of good 1 is at least 10, she cares only about increasing the quantity of good 2. Precisely, for any $x, y \in X$,

$(x_1, x_2) \succsim (y_1, y_2)$ if and only if (i) $y_1 < 10$ and $x_1 \geq y_1$ or (ii) $x_1 \geq 10$, $y_1 \geq 10$, and $x_2 \geq y_2$.

a) Give a utility function that represents the preferences.

b) Draw the four indifference sets that contain bundles $(5, 2)$, $(8, 4)$, $(10, 1)$ and $(12, 4)$ in an $(x_1, x_2)-$diagram. Also indicate the direction(s) towards which utility is increasing.

c) Is the preference relation monotone? Is it strongly monotone? If so, prove it; if not give a counter-example.

d) Is the preference relation convex? Is it strictly convex? If so, prove it; if not give a counter-example.

e) Is the preference relation continuous? If so, prove it; if not give a counter-example.

f) Derive the Walrasian demand for this preference relation, as a function of income $w > 0$ and prices $p_1, p_2 > 0$.

g) How does your answer to f) change if the preference relation was instead defined as follows: $(x_1, x_2) \succsim (y_1, y_2)$ if and only if (i) $y_1 \leq 10$ and $x_1 \geq y_1$ or (ii) $x_1 > 10$, $y_1 > 10$, and $x_2 \geq y_2$.

Give reason.

2. *General Equilibrium.*

Consider a pure exchange economy composed of $2I+1$ consumers. Of these, I consumers each initially own one right shoe (and nothing else) and $I+1$ consumers each initially own one left shoe (and nothing else). Shoes are indivisible goods. Everyone has the same utility function, which is $\min\{R, L\}$ where R and L are, respectively, the quantities of right and left shoes consumed.

a) Show that a feasible allocation of shoes is strongly Pareto efficient if and only if every individual consumes the same number of left and right shoes, except for one individual who consumes one more left shoes than right shoes.

b) Characterize the core of this economy.

(This time, in the definition of the core allow for weak dominance in blocking. That is, a feasible allocation x is in the core if there is no coalition S and allocation x' such that $\sum_{k \in S} x'_k = \sum_{k \in S} \omega_k$

(where ω_k is coalition member k's initial endowment) and $x'_k \succsim x_k$ for all $k \in S$ and $x'_k \succ x_k$ for some $k \in S$. In words, a feasible allocation x is in the core if there is no coalition that can distribute its members' endowments within the coalition such that all members of the coalition are weakly better off and at least one strictly, relative to allocation x.)

c) Let p_R and p_L be the respective prices of the two kinds of shoes. Determine the Walrasian equilibria of the economy *in which markets clear.*

Part B

(Please answer BOTH questions from this part.)

3. *Finitely Repeated Games*

Consider the following stage game played between Benoit and Krishna with common discount factor $\delta = 1$.

	L	M	R
U	$3, 2$	$2, 5$	$9, 4$
C	$6, 3$	$1, 2$	$1, 3$
D	$5, 4$	$0, 9$	$4, 7$

a) Suppose the game is repeated once (therefore played twice). List *all* action profiles that can be played in the first period of a pure strategy subgame perfect equilibrium.

b) Suppose the game is repeated twice (therefore played thrice). Is there a pure strategy SPE of this game in which the action profile played in the first period can never be played in the first period of any pure strategy SPE in the 2 period model of part (a)? If so, state the profile. Otherwise, explain why no such profile exists.

4. *Bilateral Trade*

A single indivisible good is to be sold. There is a single buyer and a single seller. The buyer could either value the good at 100 or at 15. $\Theta_b = \{h, l\}$. The seller could value it either at 80 or at 0. $\Theta_s = \{h, l\}$. The buyer can choose any non negative number as her bid. The seller can choose any non negative number as her ask price. The players choose their actions simultaneously. The uncertainty regarding the types is captured by the following probability function, which is commonly known.

$$Pr(h, h) = 1/6, Pr(h, l) = 1/12, Pr(l, h) = 1/2, Pr(l, l) = 1/4$$

where the type profiles are of the form (θ_b, θ_s).

If the buyer's bid is no less than the seller's ask price the good is transferred to the buyer and the buyer pays the seller the price equal to *the average of the two prices.* In such a scenario trade is said to have taken place. For instance, if a buyer who values the good at a 100 bids 50 and the seller bids 30 then the buyer's payoff is $100 - (\frac{50+30}{2})$. If the seller in this case valued the good at 80, her payoff following such a bid profile would be $\frac{50+30}{2}$. If instead the high type buyer had bid 10 then no trade would have occurred. The high type buyer would have received a payoff of 0, and the seller a payoff of 80.

a) Is there a Bayes Nash equilibrium in which both types of the buyer, high and low, trade with positive probability? If so, state the equilibrium. Otherwise explain why not.

b) Is there a Bayes Nash equilibrium such that the buyers low type bids at least as high as her value *and* the only scenario in which trade takes place is when the buyer is of the high type (value=100) and the seller is of the low type (value=0)? If so, state the equilibrium. Otherwise explain why not.