Micro Comprehensive Exam 2022

Part A

(Please answer BOTH questions from this part.)

1. ***Utility representation and demand*** (after Rubinstein, Problem B3).

Consider a consumer with a rational preference relation in a world with two goods, X (an aggregated consumption good) and M ("membership in a club," for example), which can be consumed or not. In other words, the consumption of X can be any nonnegative real number, while the consumption of M must be either 0 or 1. A consumption bundle is given by (x, m) where $x \geq 0$ is the amount of aggregate consumption good and $m \in \{0, 1\}$ is the consumption of M. Assume that the consumer's preferences over the space of consumption bundles are strongly monotone, continuous and satisfy the following property:

Property E: For every x there is y such that $(y, 0) \succ (x, 1)$ (that is, there is always some amount of the aggregated consumption good that can compensate for the loss of membership).

a) Show that the consumer's preference relation can be represented by a utility function of the type:

$$u(x, m) = \begin{cases} x & if \quad m = 0 \\ h(x) & if \quad m = 1 \end{cases}$$

where $h(x) > x$ for all x.

[*Hint: First, construct such a function u with the purpose of representing the preferences and second, verify that the preferences are indeed represented by the constructed function.*]

b) Explain why continuity and strong monotonicity (without property E) are not sufficient for (a).

[*Hint: For this, you can use a lexicographic preference relation over the space of consumption bundles which gives priority to M.*]

c) Determine the consumer's Walrasian demand, that is, his optimal consumption bundle as a function of his wealth, the price of the aggregate consumption good and the price of membership.

2. ***Exchange economy with uncertainty and a risk-neutral insurer.***

The economy consists of a set of individuals $N = \{I\} \cup M$: one insurer I and m members in M. Today, individuals face uncertainty about the realization of income tomorrow. With probability π_1, state 1 realizes and with probability $\pi_2 = 1 - \pi_1$, state 2 realizes. The insurer has the same endowment of future income, α, in each state, that is, $\omega^I = (\alpha, \alpha)$. Each $i \in M$ has a risky endowment of future income, namely 0 in state 1 and 1 in state 2, that is, $\omega^i = (0, 1)$ for every $i \in M$. Today, all individuals $i \in N$ can trade claims on future income on which they deliver tomorrow. Units of income in state s are traded at price p_s. Given trades today, individual $i \in N$ reaches a final allocation of income tomorrow, denoted by $x^i = (x_1^i, x_2^i)$ where $x_s^i \geq 0$ is the final income of individual i in state $s = 1, 2$. Each individual $i \in N$ has preferences over her personal final allocations of income tomorrow that are represented by expected utility with strictly increasing vNM utility function v^i. The insurer is risk-neutral. Each $i \in M$

is strictly risk-averse, and v^i is twice differentiable, strictly concave, and the **same** function for all $i \in M$. Every individual $i \in N$ demands a trade which maximizes her expected utility from her final income allocation subject to being affordable, that is, such that x^i solves $\max_{x_1^i, x_2^i} \pi_1 v^i(x_1^i) + \pi_2 v^i(x_2^i)$ subject to $p_1 x_1^i + p_2 x_2^i \leq p_1 \omega_1^i + p_2 \omega_2^i$. An equilibrium of the economy is given by a price vector (p_1, p_2) and final allocations $x = (x^i)_{i \in N}$ of income tomorrow such that

i) for every $i \in N$,

x^i solves $\max_{x_1^i, x_2^i} \pi_1 v^i(x_1^i) + \pi_2 v^i(x_2^i)$ such that $p_1 x_1^i + p_2 x_2^i \leq p_1 \omega_1^i + p_2 \omega_2^i$, and

ii) $\sum_{i \in N} x^i \leq \sum_{i \in N} \omega^i$.

a) Suppose $\alpha \geq m\pi_2$ (where m is the number of individuals in M).

Determine the unique equilibrium of the economy (uniqueness of the equilibrium price vector in terms of relative prices, of course). Also show that there is no other equilibrium.

b) Suppose now that $\alpha < m\pi_2$ (where m is the number of individuals in M).

Show that any equilibrium (p, x) is such that $p_1/p_2 > \pi_1/\pi_2$, $x^I = (0, \alpha(1 + p_1/p_2))$, and $x^i = (\alpha/m, 1 - \alpha p_1/(m p_2))$ for all $i \in M$.

c) Briefly interpret the equilibria of a) and b) and their differences.

Part B

(Please answer BOTH questions from this part.)

1. *Finitely Repeated Games*

Consider the following stage game played between Benoit and Krishna with common discount factor $\delta = 1$.

	L	M	R
U	$1, 7$	$3, 2$	$3, 8$
C	$2, 0$	$1, 1$	$4, 2$
D	$3, 4$	$3, 3$	$1, 2$

a) Suppose the game is repeated once (therefore played twice). List *all* action profiles that can be played in the first period of a pure strategy subgame perfect equilibrium.

b) Suppose the game is repeated twice (therefore played thrice). Is there a pure strategy SPE of this game in which the action profile played in the first period can never be played in the first period of any pure strategy SPE in the 2 period model of part (a)? If so, state the profile. Otherwise, explain why no such profile exists.

2. ***Bargaining***

Consider a random proposer version of the Rubinstein bargaining game played between Ann and Bob with an exogenous risk of breakdown. In each period Ann is selected as the proposer with probability 1/3 and Bob with probability 1/2. With probability 1/6 the game ends with zero payoff to both players. The proposer (if and when there is one) makes an offer (x_A, x_B) such that $x_A + x_B \leq 1$. The responder may then accept the offer or reject it. Accepting the offer ends the game and Ann and Bob receive x_A and x_B respectively, in that period. Rejecting the offer leads the bargaining to continue to the next period. Ann and Bob have the same discount factor, δ. You can assume δ to be sufficiently large.

Write down a subgame perfect equilibrium of this game.