Micro Comprehensive Exam 2021

Part A

(Please answer BOTH questions from this part.)

1. *Walrasian Demand*

A consumer with wealth $w = 500$ "must" hire one of three moving companies for his move. The companies provide an identical service, and might only differ in prices for the service. Denote the prices of the moving companies for doing the move by p_1, p_2, p_3, respectively. All prices are below w in the relevant range. The consumer uses the following procedure to decide on a moving company: he compares the prices of companies 1 and 2 and hires company 1 if its price is not greater than the price of company 2. If $p_1 > p_2$, he compares the prices of companies 2 and 3 and hires company 2 if its price is not greater than the price of company 3; otherwise he hires company 3. He uses the remainder of his wealth to purchase other goods (the price of "other goods" is normalized to 1).

i. What is the (Walrasian) demand function derived from this consumer's procedure?

ii. Is this demand function rationalizable?

iii. Consider the function $v(w, p_1, p_2, p_3) = w - p_{i^*}$ where i^* is the company which the consumer hires if the prices are (p_1, p_2, p_3). What does this function represent?

iv. Explain why $v(\cdot)$ is not monotonically decreasing in p_i for some i. Compare with the indirect utility function of the classic consumer model, where demand is derived from maximization of a utility function.

2. *Production Economy*

Consider the following economy: There are three goods, two consumers, and two firms. Good 3 is only used as an input to the production process, and provides no utility of consumption. Firm 1, which is owned entirely by Ann, has a technology that allows good 3 to be made into good 1, according to the simple linear technology $x_1 \leq 3x_3$. That is, if firm 1 uses x_3 units of the input good, it can make up to $3x_3$ units of the first consumption good. Firm 2, owned entirely by Bob, uses good 3 to make the consumption good 2, and its technology is described by $x_2 \leq 4x_3$. Each consumer initially owns 5 units of good 3; that is, endowments are $\omega_A = \omega_B = (0, 0, 5)$. Ann's utility function is $u_A(x_1, x_2, x_3) = x_1^{3/5} x_2^{2/5}$, while Bob's utility function is $u_B(x_1, x_2, x_3) = x_1^{1/2} x_2^{1/2}$.

i. Derive the Walrasian equilibria of this economy.

ii. What would be the equilibria if the firm ownerships were reversed (i.e. if Bob owned firm 1, and Ann owned firm 2)?

Part B

(Please answer BOTH questions from this part.)

3. *Bilateral Trade*

A single indivisible good is to be sold. There is a single buyer and a single seller. The buyer could either value the good at 100 or at 15. $\Theta_b = \{h, l\}$. The seller could value it either at 80 or at 0. $\Theta_s = \{h, l\}$. The buyer can choose any non negative number as her bid. The seller can choose any non negative number as her ask price. The players choose their actions simultaneously. The uncertainty regarding the types is captured by the following probability function, which is commonly known.

$$Pr(h,h) = 1/12, Pr(h,l) = 1/6, Pr(l,h) = 1/4, Pr(l,l) = 1/2$$

where the type profiles are of the form (θ_b, θ_s).

If the buyer's bid is no less than the seller's ask price the good is transferred to the buyer and the buyer pays the seller the price equal to *the average of the two prices*. In such a scenario trade is said to have taken place. For instance, if a buyer who values the good at a 100 bids 50 and the seller bids 30 then the buyer's payoff is $100 - (\frac{50+30}{2})$. If the seller in this case valued the good at 10, her payoff following such a bid profile would be $\frac{50+30}{2}$. If instead the high type buyer had bid 10 then no trade would have occurred. The high type buyer would have received a payoff of 0, and the seller a payoff of 10.

a) Is there a Bayes Nash equilibrium in which both types of the buyer, high and low, trade with positive probability? If so, state the equilibrium. Otherwise explain why not.

b) Is there a Bayes Nash equilibrium such that the buyer's low type bids at least as high as her value *and* the only scenario in which trade takes place is when the buyer is of the high type (value=100) and the seller is of the low type (value=0)? If so, state the equilibrium. Otherwise explain why not.

4. *VCG*

Consider the allocation of three identical and indivisible objects to three agents, $N = \{1, 2, 3\}$, each of whom can consume multiple units. Agent i values the consumption of k units of the object at v^i_k. Her valuation profile $\theta^i = (v^i_1, v^i_2, v^i_3)$ is her private information. All that is publicly known is that for each player i, $0 \le v^i_1 \le v^i_2 \le v^i_3$. Suppose the true valuations of the three players are as follows,

	θ^1	θ^2	θ^3
v_1	4	2	3
v_2	7	9	6
v_3	12	10	9

The table above shows, for instance, that player 1 has valuations $v^1_1 = 4$, $v^1_2 = 7$ and $v^1_3 = 12$.

Suppose the three players report their respective valuation profile truthfully,

a) What is the allocation of the three objects that maximizes the sum of values (utilities) across all players, for such a profile? (Who gets what?)

b) What are the transfers according to the VCG mechanism for this valuation profile?

(Note: You are only required to state the allocation and transfers for *this particular* valuation profile and not for any general valuation profile. The correct answer involves explicit numbers and not algebraic expressions.)

Now suppose instead that Player 3 chooses to lie about his valuation profile by reporting $\theta^3 = (6, 8, 12)$. All the others report truthfully.

c) What is the allocation of the three objects that maximizes the sum of *reported* values (utilities) across all players, for such a *reported* profile? (Who gets what?)

d) What are the transfers according to the VCG mechanism for this *reported* valuation profile?

e) How much does Player 3 gain or lose by lying?