Electoral Accountability and Responsive Democracy*

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Abstract

We consider a canonical two-period model of elections with adverse selection (hidden preferences) and moral hazard (hidden actions), in which neither voters nor politicians can commit to future choices. We prove existence of electoral equilibria, and we show that existence of equilibria sometimes requires mixing over policy choices. As politicians become more office motivated, voters become arbitrarily demanding, leading above average politicians to go for broke (choosing arbitrarily high policies), while below average politicians may take it easy (choosing policies near their ideal points). Under additional assumptions, the lowest politician type mixes between taking it easy and pooling with higher types by going for broke; and all politician types are re-elected with probability converging to one. Thus, electoral incentives shift to sanctioning, rather than selection, as office motivation increases.

1 Introduction

Representative democracy, by definition, entails the delegation of power by society to elected officials. A main concern for representative democracy is that elected politicians are responsive to voter preferences and produce desirable policy outcomes for citizens. Political thinkers since Madison, if not earlier, have viewed elections as an effective mechanism for achieving responsiveness.[1] The goal of this paper is to study formally the incentives provided by democratic elections

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[1] *The Federalist* 57, in particular, offers a discussion of the role of "frequent elections" in the selection of politicians and the control of politicians while in office.

and the implied linkage between voter preferences and policy outcomes. In doing so, we must move beyond the basic Downsian model of static elections, the stalwart of formal work on electoral competition, to explicitly incorporate a temporal dimension within the analysis. An active and growing literature on electoral accountability, starting with the seminal work of Barro (1973) and Ferejohn (1986), has undertaken this line of inquiry, with the goal of improving our understanding of the operation of real-world political systems and the conditions under which democracies succeed or fail. This, in turn, has the potential to facilitate the design of political institutions that produce socially desirable policy outcomes.

Nevertheless, our understanding of the fundamental interplay between disciplining incentives provided by the possibility of re-election and the temptation of opportunistic behavior in the present remains incomplete. With few exceptions, such as Fearon (1999), Ashworth (2005), and Ashworth and Bueno de Mesquita (2008), the literature on electoral accountability has paid relatively little attention to the situation in which the preferences *and* actions of politicians are unobserved by voters. Such settings combine salient aspects of real-world elections, but they present obstacles to the application of game-theoretic tools, and as a consequence, research has been conducted under special modeling assumptions about the type space, the action space, or the information held by politicians.

We present a two-period model of elections that allows us to study the dynamic incentives facing politicians, and the policy choices emerging from those incentives, in environments with realistically sparse information. We prove existence of equilibrium under general conditions, and we provide a characterization of equilibrium behavior in the model. We impose sufficient structure (satisfied in special cases of interest) that in the first period, a politician can have at most two optimal policy choices, "taking it easy" and "going for broke," and voters follow a straightforward retrospective rule: re-elect the incumbent if and only if the observed policy outcome is equal to or better than a cutoff level. The first-period office holder's choice must take account of the cutoff used by voters, and the updating of voter beliefs (and thus the voters' cutoff) depends on choices of the first period office holder via Bayes' rule; thus, electoral equilibria must solve a nontrivial fixed point problem.

As politicians become more office motivated, the re-election cutoff used by voters becomes arbitrarily demanding, and politicians become responsive to voter preferences: the incentives of re-election induce above average politicians (i.e., politician types that are better in expectation for voters than the challenger) to choose high policies, while lower politician types choose policies that are above but close to their ideal points with positive probability. Thus, there is a "marginal type" such that higher politician types go for broke, and lower politician types take it easy. Despite the increasing cutoff used by voters and the cost of choosing high policy, politicians who go for broke are re-elected with probability close to one, demonstrating a strong form of incumbency advantage for politicians above the marginal type. Under further assumptions on the curvature of politician payoff functions, we show that the marginal type is in fact the lowest politician type; this politician type necessarily mixes between taking it easy and going for broke, with the probability of the latter converging to one; and all higher types go for broke. In particular, all politician types are re-elected with probability close to one, and the incentive effects of elections shift to sanctioning (incentivizing politicians to choose high policy), rather than selection (screening higher types as office holders in the second period), as politicians become more office motivated.

When policies are interpreted as effort and voter utility is increasing in policy, our responsive democracy result has positive welfare implications for the effectiveness of elections: all above average politician types (and under stronger assumptions, *all* types) choose arbitrarily high policies, increasing the ex ante expected payoff of voters as office motivation increases; and in case voter utility is increasing and unbounded, then ex ante voter welfare increases without bound, despite the sparseness of information available to voters and the paucity of instruments that the voters can wield.

However, the welfare implications of our analysis are more nuanced when voter preferences over policies are single-peaked. The key condition for the responsiveness result is that voter preferences are strictly increasing over the ideal policies of the politician types, but we allow for the possibility that policy choices above that range are damaging to voters. For example, if politicians choose economic stimulus policies, then it may be that voter preferences are initially increasing, but that overstimulation of the economy is harmful. In this case, politicians above the marginal type still respond to electoral incentives by choosing arbitrarily high policy, but the signaling technology has negative welfare effects; in terms of the growth example, politicians have an incentive to overstimulate the economy in the first period, in order to signal to voters that they will do a better than average job of managing the economy in the second period.

A similar incentive is illustrated in Acemoglu et al.'s (2013) model of populism, in which there are two politician types (one moderate and one extreme), policy outcomes are determined by an unobserved, normally distributed shock, and the voter has quadratic utility over outcomes. They show that in order to signal that they are moderate, if office motivation is high, then both politician types choose policy to the far side of the voter.[2] We obtain the model of Acemoglu et al. (2013) as a special case of our framework, and our results show that their insight extends

[2]See also Persson and Tabellini (1990), where politicians in the first period signal their competence by choosing inflationary growth policy, for an example from the literature on political cycles.

to a much broader class of model, independent of particular functional form assumptions or restrictions on the number of politician types. See Section 8, below, for a more detailed discussion.

We conduct our research in the framework of a two-period model in which the incumbent politician in office in the first period faces a randomly chosen challenger in the second period. Variations of the two-period period model have been employed in the graduate textbooks of Persson and Tabellini (2000) and Besley (2006), providing a minimalist setting to study intertemporal incentives; in this sense the two-period model can be regarded as a "workhorse" model for the study of elections. We analyze a version of the model that is general with respect to preferences and information. We assume that politicians' preferences are private information, i.e., adverse selection is present, and we allow the structure of preferences and the number of possible politician types to be general. As well, we assume that political choices are observed by voters only with some noise, i.e., they are subject to imperfect monitoring, or moral hazard. We consider the rent-seeking environment studied in the public choice tradition of Barro (1973) and Ferejohn (1986), in which politicians have a short-run incentive to shirk from exerting effort while in office, or equivalently, to engage in rent-seeking activities that hurt other citizens. Politicians differ with regard to their preference for rent-seeking (or equivalently, they differ in their cost of effort).

We maintain the key assumption of the electoral accountability literature that neither politicians nor voters can commit to future actions, in the citizen-candidate tradition of Osborne and Slivinski (1996) and Besley and Coate (1997). An implication is that in equilibrium, both the policy choices of politicians and the re-election standard used by voters must be time consistent, in the sense that first-period choices of politicians and voters must be optimal in light of expected behavior in the future. Our responsiveness result may look superficially similar to the median voter theorem in the traditional Downsian framework, but the logic underlying it is very different: candidates cannot make binding campaign promises, and they do not compete for votes in the Downsian sense; rather, they are citizen candidates whose policy choices must maximize their payoffs in equilibrium, and the responsiveness result is driven by politicians' concern for reputation. Specifically, the desire to be re-elected can induce below average politician types to mimic above average types, and if the reward for political office is large enough, then this incentive leads above average types to choose arbitrarily high policies.

We emphasize that the equilibrium standard used by voters is optimal given politicians' choices, but it is not optimally set ex ante: voters do not set the standard before the election in order to elicit maximal effort. Because voters, like politicians, face a commitment problem, they "best respond" in equilibrium by re-electing the incumbent when the expected payoff from doing so, conditional on the observed policy outcome, exceeds the prospects of a challenger; in other words, the equilibrium standard is time consistent. This facet of the equilibrium analysis stacks the deck against our responsive democracy result, and it means that responsiveness does not rely on any assumption that voters can commit ex ante to a socially optimal standard of re-election.

In Section 2, we present the two-period electoral accountability model, and in Section 3, we define the concept of electoral equilibrium on which our analysis is focussed. In Section 4, we impose added structure on the model and take preliminary steps toward the main results, e.g., showing the existence of at most two local maximizers to the best response problem faced by each type of politician. In Section 5, we develop the two-type version of the model, along with numerical examples, to introduce themes from the general analysis. In Section 6, we prove existence and provide a partial characterization of electoral equilibria. In Section 7, we present our results on responsive democracy as politicians become office motivated. In Section 8, we discuss the relationship of our paper with the electoral accountability literature. In Section 9, we gather final remarks.

2 Electoral accountability model

We analyze a two-period model of elections involving a representative voter, an incumbent politician, and a challenger. Prior to the game, nature chooses the types of the incumbent and challenger from the finite set $T = \{1,\dots,n\}$, with $n \geqslant 2$. These types are private information—in particular, they are unobserved by the voter—and are drawn identically and independently. We let $p_j > 0$ denote the prior probability that a politician is type j. In period 1, the incumbent makes a policy choice $x_1 \in X = \mathbb{R}_+$, which is unobserved by the voter, and a policy outcome y_1 is then drawn from $Y = \mathbb{R}$ according to the distribution $F(\cdot|x_1)$. The choice of x_1 may be viewed as an effort level, but that interpretation is not needed for application of the model. In contrast to the choice x_1, the outcome y_1 is observed by the voter. Then the voter chooses between the incumbent politician and the challenger. In period 2, the winner of the election makes a policy choice $x_2 \in X$, a policy outcome y_2 is drawn from $F(\cdot|x_2)$, and the game ends. Figure 1 illustrates the timeline of events in the model.

Given policy choice x and outcome y in either period, each player obtains a payoff of $u(y)$ if not in office, while an office holder of type j receives a payoff of $w_j(x) + \beta$, where $\beta \geqslant 0$ represents the benefits of holding office. Total payoffs for the voter and politicians are the sum of per-period payoffs. We assume that for all $j = 1,\dots,n$, $w_j\colon X \to \mathbb{R}$ is twice differentiable and concave, and later we impose a minimal assumption on $u\colon Y \to \mathbb{R}$ that holds if, for example, u is strictly increasing.

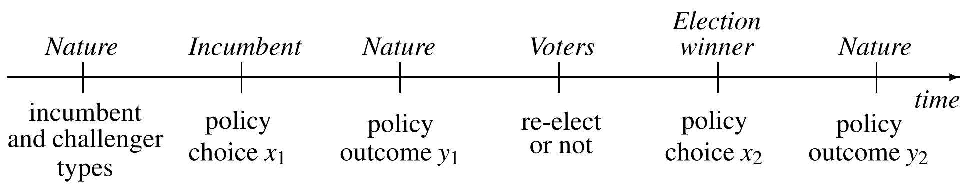

Figure 1: Timeline

We assume that marginal utilities are ordered by type: for all $j < n$, we have

(A1) $w'_j(x) < w'_{j+1}(x)$ for all x, $w'_1(0) \geqslant 0$, and $w'_n(x) < 0$ for large enough x.

Here, beyond the politician's preferences as a citizen, w_j reflects her interests as a policy maker, e.g., the cost of effort of a preference for rents captured from voters. Our assumptions imply that each politician type j has an optimal policy $\hat{x}_j$, and the ideal policies are strictly ordered according to type:

$$0 \leqslant \hat{x}_1 < \hat{x}_2 < \cdots < \hat{x}_n.$$

Moreover, we assume that the ideal politician payoffs are ordered by type: for all $j < n$, we have

$$\text{(A2)} \qquad w_j(\hat{x}_j) \quad \leqslant \quad w_{j+1}(\hat{x}_{j+1}).$$

Intuitively, the voter has increasing preferences over policy outcomes, while a politician who holds office incurs a cost for higher policy choices, and the cost is lower for higher politician types. Our assumptions admit two simple specifications that are worthy of note. One common specification is *quadratic utility,* in which case $w_j(x) = -(x-\hat{x}_j)^2$. Another specification of interest is is *exponential utility,* whereby $w_j(x) = -e^{x-\hat{x}_j} + x$.

We assume that the outcome distribution $F(\cdot|x)$ has a jointly differentiable density $f(y|x)$ and that for all $x \in X$, $F(\cdot|x)$ has full support on $Y = \mathbb{R}$ with strictly positive density. For simplicity we take the policy choice x to be a shift parameter on the density of outcomes, so, abusing notation slightly, the density can be written $f(y|x) = f(y-x)$ for some fixed density $f(\cdot)$, and the probability that the realized outcome is less than y given policy x is simply $F(y-x)$. We assume that f satisfies the standard monotone likelihood ratio property (MLRP), i.e.,

$$\text{(A3)} \qquad \frac{f(y-x)}{f(y-x')} > \frac{f(y'-x)}{f(y'-x')}$$

for all $x > x'$ and all $y > y'$. This implies that greater policy outcomes induce the voter to update favorably her beliefs about the policy adopted by the incumbent in the first period. As is well-known, the MLRP implies that the density function is unimodal, and that both the density and the distribution functions are strictly log-concave.[3] Moreover, we assume

$$\text{(A4)} \qquad \lim_{y \to -\infty} \frac{f(y-x)}{f(y-x')} \;=\; \lim_{y \to +\infty} \frac{f(y-x')}{f(y-x)} \;=\; 0$$

when $x > x'$, so that arbitrarily extreme signals become arbitrarily informative. In particular, we capture the benchmark case in which $f(\cdot)$ is a mean-zero normal density, which implies that conditional on the policy choice x, the outcome is normally distributed with mean x.

Finally, define the voter's expected utility given policy choice x by $\mathbb{E}[u(y)|x] = \int u(y)f(y-x)dy$, and we impose the minimal assumption that the voter's expected utility is strictly increasing over the range of ideal policies of the politician types: for all $x, x' \in [\hat{x}_1, \hat{x}_n]$ with $x < x'$,

$$\text{(A5)} \qquad \mathbb{E}[u(y)|x] \;<\; \mathbb{E}[u(y)|x'].$$

Given the ordering of ideal policies, (A5) is obviously satisfied if u is strictly increasing, but we allow for the possibility that the voter has a finite optimal policy above $\hat{x}_n$. Note that we can assume politicians share the voter's preferences over policy outcomes by setting

$$w_j(x) \;=\; \mathbb{E}[u(y)|x] - \zeta_j c(x)$$

for some strictly increasing, continuously differentiable, convex function $c\colon X \to \mathbb{R}_+$ and parameters ordered by type, i.e., $\zeta_j > \zeta_{j+1}$ for all $j < n$, and by letting $\mathbb{E}[u(y)|x] = x$. Of course, the latter condition holds if $u(y) = y$ and f is symmetric around zero. In this version of the model, it is natural to view policy outcomes as a level of public good and policy choices as effort or (the inverse of) corruption, and politician types then reflect different abilities to provide the public good or a distaste for corruption while in office.

3 Electoral equilibrium

As in the citizen-candidate model, we assume that neither the incumbent nor the challenger can make binding promises before an election. We also assume that

[3] See Bagnoli and Bergstrom (2005) for an in-depth analysis of log concavity and related conditions.

the voter cannot commit her vote, so that voting as well as policy making must be time consistent. Thus, our analysis focusses on perfect Bayesian equilibria of the electoral accountability model, under additional refinements to preclude implausible behavior on the part of the voter and politicians. A *strategy for the type j incumbent* is a pair (π_j^1, π_j^2), where

$$\pi_j^1 \in \triangle(X) \quad \text{and} \quad \pi_j^2 : X \times Y \to \triangle(X),$$

specifying mixtures over policy choices in period 1 and policy choices in period 2 for each possible previous policy choice and observed outcome.[4] A *strategy for the type j challenger* is a mapping

$$\gamma_j : Y \to \triangle(X),$$

specifying mixtures over policy choices in period 2 for each policy type and observed outcome. A *strategy for the voter* is a mapping

$$\rho : Y \to [0,1],$$

where $\rho(y)$ is the probability of a vote for the incumbent given outcome y. A *belief system for the voter* is a probability distribution $\mu(\cdot|y_1)$ on $T \times X$ as a function of the observed outcome.

A strategy profile $\sigma = ((\pi_j, \gamma_j)_{j \in T}, \rho)$ is *sequentially rational* given belief system μ if neither the incumbent nor the challenger can gain by deviating from the proposed strategies at any decision node, and if the voter votes for the candidate that makes her best off in expectation following all possible realizations of y_1. Beliefs μ are *consistent* with the strategy profile σ if for every y_1, the distribution $\mu(j,x|y_1)$ is derived from $(\pi_j^1)_{j \in T}$ via Bayes' rule. A *perfect Bayesian equilibrium* is a pair (σ, μ) such that the strategy profile σ is sequentially rational given the beliefs μ, and μ is consistent with σ.

Sequential rationality implies that challengers will choose their ideal policies since there are no further elections, so that γ_j assigns probability one to $\hat{x}_j$ for all y_1. This implies that the expected payoff of electing the challenger for the voter is

$$V^C = \sum_k p_k \mathbb{E}[u(y)|\hat{x}_k].$$

Similarly, sequential rationality implies that $\pi_j^2(\cdot|x_1, y_1)$ assigns probability one to $\hat{x}_j$ for all x_1 and all y_1, so henceforth we assume politicians choose their ideal

[4] The notation $\triangle(\cdot)$ indicates the set of Borel probability measures over a given subset of Euclidean space. Measurability of strategies or subsets of policies will be assumed implicitly, as needed, without further mention.

policies in the second period, and we simplify notation by dropping the superscript from π^1_j for the mixture over policies used by the type j politician in the first period. It follows that the expected payoff to the voter from re-electing the incumbent is

$$V^I(y_1) \;=\; \sum_k \mu_T(k|y_1)\mathbb{E}[u(y)|\hat{x}_k],$$

where $\mu_T(j|y_1)$ is the marginal distribution of the incumbent's type given policy outcome y_1. Thus, the incumbent is re-elected if $V^I(y_1) > V^C$ and only if $V^I(y_1) \geqslant V^C$. We say an equilibrium is *monotonic* if the voter follows a simple retrospective rule given by $\bar{y} \in \mathbb{R} \cup \{-\infty, \infty\}$ such that she re-elects the incumbent if and only if $y \geqslant \bar{y}$. The monotonicity condition follows naturally from the interpretation of outcomes as signals of politicians? actions in the first period.

Finally, an *electoral equilibrium* is any monotonic perfect Bayesian equilibrium. Electoral equilibria are then characterized by three conditions. First, updating of voter beliefs follows Bayes rule, after observing outcome y. In particular, when the policy mixtures π_j are discrete, we can write

$$\mu_T(j|y) \quad = \quad \frac{p_j \sum_x f(y-x)\pi_j(x)}{\sum_k p_k \sum_x f(y-x)\pi_k(x)}.$$

Since the outcome density is positive, every outcome is on the path of play, so Bayes' rule pins down the voter's beliefs. We henceforth summarize an electoral equilibrium by the strategy profile σ, leaving beliefs implicit. Second, the threshold $\bar{y}$ must be such that, anticipating that politicians choose their ideal policies in the second period, the expected utility of re-electing the incumbent after observing y, given the voters' belief system, is greater than or equal to $\sum_k p_k \mathbb{E}[u(y)|\hat{x}_k]$ if and only if $y \geqslant \bar{y}$. Since $\mu_T(j|y)$ is continuous in y, by the previous condition, it follows that if $\bar{y}$ is finite, then it must satisfy the indifference condition $V^I(\bar{y}) = V^C$ for the voter, or equivalently,

$$\sum_k \mu_T(k|\bar{y})\mathbb{E}[u(y)|\hat{x}_k] \quad = \quad \sum_k p_k \mathbb{E}[u(y)|\hat{x}_k]. \tag{1}$$

Third, each politician type j, knowing that she is re-elected if and only if $y \geqslant \bar{y}$, the type j incumbent's policy strategy π_j places probability one on maximizers of

$$w_j(x) + (1 - F(\bar{y} - x))[w_j(\hat{x}_j) + \beta] + F(\bar{y} - x)V^C, \tag{2}$$

so that the politician mixes over optimal actions in the first period.

4 Preliminary analysis

To facilitate the analysis, we henceforth assume that all incumbent types are in principle interested in re-election, i.e.,

(A6) $$w_1(\hat{x}_1) + \beta \ > \ V^C,$$

so that if re-election is assured by choosing their ideal policies in the first period, then the benefits of re-election outweigh the costs. Note that the incumbent can always choose her ideal policy, so it is never optimal for the politician to choose large policies x for which $w_j(x) + \beta < V^C$. By (A6) and concavity of w_j, it is never optimal to choose a policy below the politician's ideal policy, so there is at least one solution to the incumbent's problem in the first period. Denoting such a solution by x_j^*, note that $x_j^* \geqslant \hat{x}_j$, so that the necessary first order condition for a solution of the incumbent's maximization problem (2) is

$$w_j'(x_j^*) \quad = \quad -f(\overline{y} - x_j^*)[w_j(\hat{x}_j) + \beta - V^C]. \tag{3}$$

That is, the marginal disutility in the current period from increasing the policy choice is just offset by the marginal utility in the second period, owing to the politician's increased chance of re-election. By the assumption that $f(\cdot)$ is everywhere positive, with (A6), the right-hand side of (3) is negative, and we see that for an arbitrary cutoff $\overline{y}$, the politician optimally exerts a positive amount of effort, i.e., chooses $x_j^* > \hat{x}_j$, in the first term of office.

We can gain some insight into the incumbent's problem and the role of mixing in equilibrium by reformulating it in terms of optimization subject to an inequality constraint. Define a new objective function

$$W_j(x,r) \quad = \quad w_j(x) + r[w_j(\hat{x}_j) + \beta - V^C],$$

which is the expected utility if the politician chooses policy x and is re-elected with probability r, minus a constant term corresponding to the current enjoyment of office. Note that W_j is concave in (x,r) and quasi-linear in r. Of course, given x, the re-election probability is in fact pinned down as $1 - F(\overline{y} - x)$. Defining the constraint function

$$g(x,r) \quad = \quad 1 - F(\overline{y} - x) - r,$$

we can then formulate the politician's optimization problem as

$$\max_{(x,r)} W_j(x,r)$$
$$\text{s.t. } g(x,r) \geqslant 0,$$

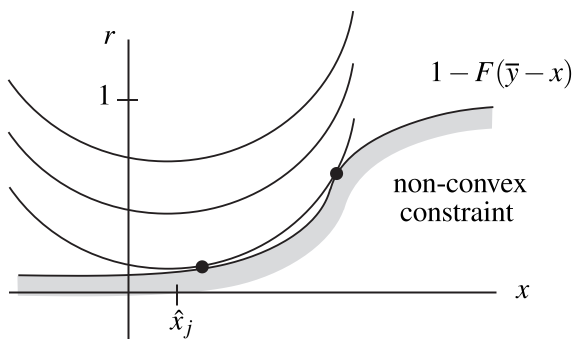

Figure 2: Politician's optimization problem

which has the general form depicted in Figure 2. Here, the objective function is well-behaved, but the constraint set inherits the natural non-convexity of the distribution function F, leading to the possibility of multiple solutions. This, in turn, can lead to multiple optimal policies and the necessity of mixing in equilibrium; see Figure 2 for an illustration of this multiplicity.

One of our contributions is a condition that is satisfied in environments of interest and limits the need for mixing to at most two policy choices for each type.[5] Assume that for all j, all finite $\overline{y}$, and all $x, \tilde{x}, z$ with $\hat{x}_j < x < \tilde{x} < z$, we have the following condition:

$$\text{(A7)} \qquad \text{if } \frac{w_j''(x)}{w_j'(x)} \leqslant -\frac{f'(\overline{y}-x)}{f(\overline{y}-x)} \text{ and } \frac{w_j''(z)}{w_j'(z)} \leqslant -\frac{f'(\overline{y}-z)}{f(\overline{y}-z)}, \quad \text{then } \frac{w_j''(\tilde{x})}{w_j'(\tilde{x})} < -\frac{f'(\overline{y}-\tilde{x})}{f(\overline{y}-\tilde{x})}.$$

That is, the set of $x > \hat{x}_j$ such that $w_j''(x)/w_j'(x) \leqslant -f'(\overline{y}-x)/f(\overline{y}-x)$ is convex, and if x and z satisfy the inequality, then every policy between them satisfies it strictly. To understand this condition, note that by log concavity of $f(\cdot)$ from (A3), the term $-f'(\overline{y}-x)/f(\overline{y}-x)$ is strictly decreasing in x, and thus (A7) is satisfied if the coefficient of absolute risk aversion, $w_j''(x)/w_j'(x)$, does not decrease too fast to the right of the type j politicians' ideal policy. To illustrate, when the utility function w_j is quadratic, the coefficient of absolute risk aversion is $1/(x-\hat{x}_j)$, and when the density f is standard normal, the likelihood ratio $-f'(\overline{y}-x)/f(\overline{y}-x)$ simplifies to $\overline{y}-x$. Thus, (A7) is satisfied in the quadratic-normal special case,

[5]The possibility of multiple optimizers has a counterpart in static models of elections with probabilistic voting, where log concavity is sufficient to ensure existence of equilibria in pure strategies (cf. Roemer (1997) and Bernhardt et al. (2009)).

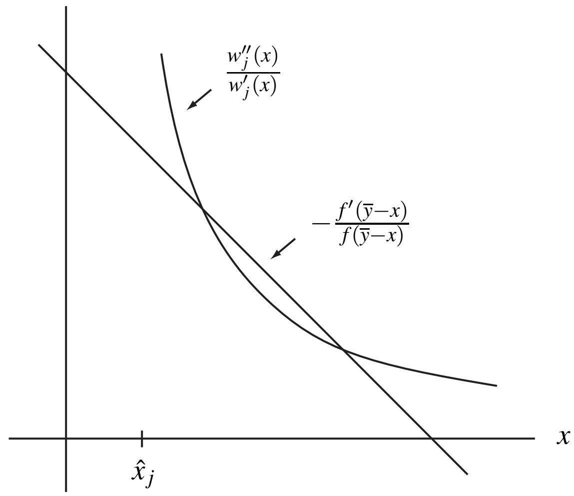

Figure 3: Quadratic-normal special case

depicted in Figure 3. Likewise, in the case of exponential utility, the coefficient of risk aversion is $1/(1-\exp(\hat{x}_j - x))$, which is decreasing and convex, and again (A7) is satisfied.

The usefulness of (A7) is delineated in the next result, which implies that for arbitrary cutoffs, each type of incumbent has at most two optimal policies as a function of the cutoff. The greater solution to the incumbent's optimization problem, which is denoted $x_j^*(\overline{y})$, corresponds to "going for broke," while the least solution, denoted $x_{*,j}(\overline{y})$, corresponds to "taking it easy." When these two policy choices coincide, the politician has a unique optimal policy; a gap between the two choices reflects the possibility that the increase in effort involved in going for broke is just offset by the increase in probability of being re-elected.[6]

Proposition 1. *Assume (A1), (A2), (A6) and (A7). For every cutoff $\overline{y} \in Y$ and every type j, there are at most two optimal policies, i.e., two maximizers of the objective function (2). For every type j, the greatest and least optimal policies for type j, $x_j^*(\cdot)$ and $x_{*,j}(\cdot)$ are, respectively, upper and lower semi-continuous functions of $\overline{y}$.*

Proof. Suppose toward a contradiction that there are three distinct local maximizers of the type j politician's objective function, say x', x'', and x''' with $x' < x'' < x'''$. Thus, there are local minimizers z' and z'' such that $x' < z' < x'' < z'' < x'''$. With

[6]Assumption (A7) implies an upper bound of two local maximizers of a politician's objective function, but the condition can be relaxed to allow for any fixed upper bound.

(A2) and (A6), inspection of the first order condition (3) at $x = z', z''$ reveals that $w_j'(z') < 0$ and $w_j'(z'') < 0$, so we can write the first order condition at z' and z'' as

$$w_j(\hat{x}_j) + \beta - V^C \;=\; -\frac{w_j'(z')}{f(\overline{y}-z')} \;=\; -\frac{w_j'(z'')}{f(\overline{y}-z'')}.$$

By the necessary second order condition for a local minimizer, the second derivative at z' satisfies

$$0 \;\leqslant\; w_j''(z') - f'(\overline{y}-z')[w_j(\hat{x}_j) + \beta - V^C] \;=\; w_j''(z') - f'(\overline{y}-z')\left[-\frac{w_j'(z')}{f(\overline{y}-z')}\right],$$

or equivalently,

$$\frac{w_j''(z')}{w_j'(z')} \;\leqslant\; -\frac{f'(\overline{y}-z')}{f(\overline{y}-z')}.$$

Similarly, we have

$$\frac{w_j''(z'')}{w_j'(z'')} \;\leqslant\; -\frac{f'(\overline{y}-z'')}{f(\overline{y}-z'')}.$$

Since x'' is a local maximizer, the first order condition holds at x'', and the second derivative at x'' is non-positive, but then we have

$$\frac{w_j''(x'')}{w_j'(x'')} \;\geqslant\; -\frac{f'(y-x'')}{f(y-x'')},$$

contradicting (A7). We conclude that the objective function has at most two local maximizers, and therefore there are at most two optimal policies for type j.

From previous arguments and (A1), optimal policies for type j are bounded below by $\hat{x}_j \geqslant 0$ and above by $\overline{x}_j > \hat{x}_j$ such that $w_j(\overline{x}_j) + \beta = V^C$. A standard application of Berge's theorem of the maximum (see e.g. Border (1985), Theorem 12.1) implies that the correspondence of optimal best responses is nonempty-valued and is upper hemi-continuous in $\overline{y}$. Since the correspondence of optimal best-responses includes at most two policies for each cutoff, upper hemi-continuity of the best response correspondence is equivalent to the greatest and least optimal policies for type j, $x_j^*(\cdot)$ and $x_{*,j}(\cdot)$ being, respectively, upper and lower semi-continuous functions of $\overline{y}$. □

The idea of the proof is that, at any local minimizer of the politician's objective function in (2), the likelihood ratio must exceed the coefficient of absolute risk aversion. Then (A7) implies that there is at most one local minimizer of the politician's objective function, so that the objective function has either one or two maximizers. In terms of Figure 3, there is at most one maximizer below the region of policy choices such that the likelihood ratio exceeds the coefficient of absolute risk aversion, and at most one maximizer above that region.

We can visualize the effect of a change in the cutoff on the politician's objective function, for the quadratic-normal model, using Figure 3. Recall that in the normal case, the ratio $-\frac{f'(\overline{y}-x)}{f(\overline{y}-x)}$ in the figure is equal to $\overline{y}-x$. If the cutoff $\overline{y}$ is small enough, then the ratio is always below the coefficient of absolute risk aversion so the objective function is quasiconcave and has a unique maximizer. If the cutoff is larger, so that the two curves cross as drawn in the figure, then two local maximizers are possible. Moreover, as the cutoff increases, one local maximizer moves toward the ideal policy, while the other—if it exists—must grow large.

In Figure 4, we illustrate the politician's objective function in the example with quadratic utility and standard normal density, for a politician with $\hat{x}_j = 1$ and $\beta - V^C = 20$, and for $\overline{y}$ taking values in 3.4, 4.21 and 5. In the first case, there is a unique local maximum, which is the global optimum, near 4.11; in the second case there are two local maxima, one near 1 and the other at 4.51; while in the third case, there is again a unique local maximum—hence the global optimum—near 1. In Figure 5, we have illustrated the maximizers of the politician's objective function for the same example. Note that the greatest and least optimal policies, $x_j^*(\overline{y})$ and $x_{*,j}(\overline{y})$, coincide for all values of the cutoff except at single cutoff near 4.21. Although a value of the cutoff such that the objective function has two global optima may appear to be a knife-edge possibility, the equilibrium cutoff is endogenous, and we will see that equilibrium existence sometimes requires that some politician type mixes between "taking it easy" and "going for broke."

The next proposition establishes that the incumbent's objective function satisfies the important property that differences in payoffs are monotone in type. We say that $W_j(x, 1-F(\overline{y}-x))$ is *supermodular* in (j,x) if for all (j,x) and all (k,z) with $j > k$ and $x > z$, we have

$$\begin{aligned} & W_j(x,1-F(\overline{y}-x)) - W_j(z,1-F(\overline{y}-z)) \\ & \quad > \quad W_k(x,1-F(\overline{y}-x)) - W_k(z,1-F(\overline{y}-z)). \end{aligned}$$

An important implication of supermodularity is that given an arbitrary value $\overline{y}$ of the cutoff, the maximizers of (2) are ordered by type—a property that will be key for establishing existence of equilibrium.

Proposition 2. *Assume (A1), (A2), and (A6). For every cutoff $\overline{y}$, the incumbent's objective function, $W_j(x, 1-F(\overline{y}-x))$, is supermodular in (j,x). Moreover, maxi-*

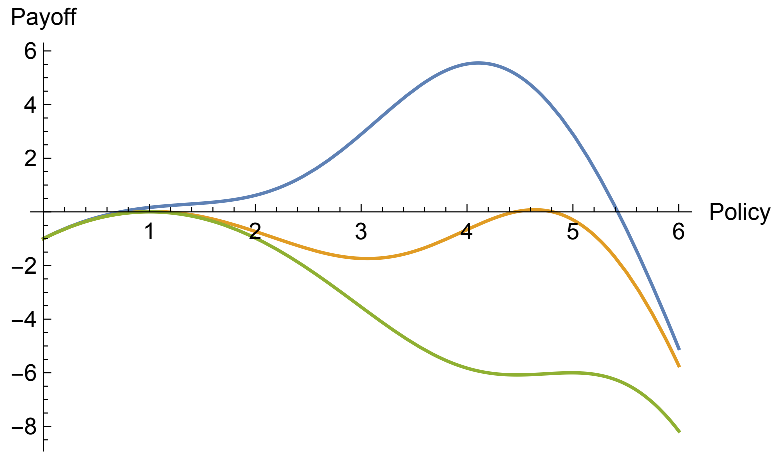

Figure 4: Politician's payoff function for different voter cutoffs

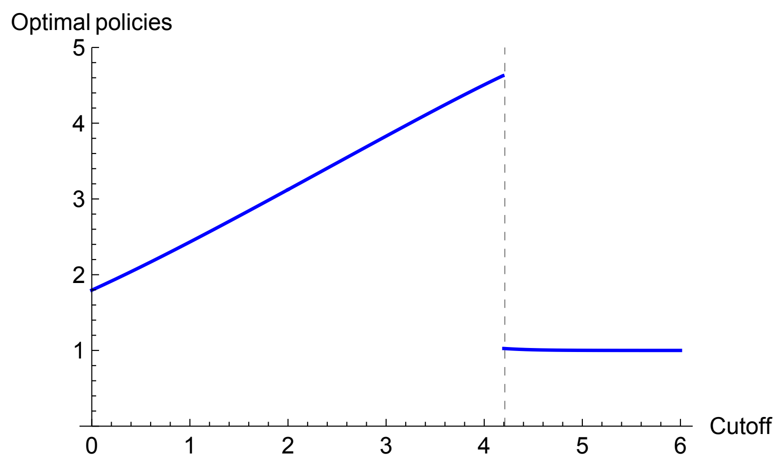

Figure 5: Politician's optimal policies as function of voter cutoff

mizers of (2) are strictly ordered by type, i.e.,

$$x_j^*(\bar{y}) \; < \; x_{*,j+1}(\bar{y}), \quad \textit{for all } j < n.$$

Proof. Consider $j > k$ and $x > z$. To establish supermodularity, we must show that

$$\begin{aligned} & w_j(x) - w_j(z) + (F(\bar{y}-z) - F(\bar{y}-x))[w_j(\hat{x}_j) + \beta - V^C] \\ & > \; w_k(x) - w_k(z) + (F(\bar{y}-z) - F(\bar{y}-x))[w_k(\hat{x}_k) + \beta - V^C]. \end{aligned}$$

Since $x > z$, we have $F(\bar{y}-z) - F(\bar{y}-x) > 0$. Using (A2) and (A6),

$$\begin{aligned} & (F(\bar{y}-z) - F(\bar{y}-x))[w_j(\hat{x}_j) + \beta - V^C] \\ & > \; (F(\bar{y}-z) - F(\bar{y}-x))[w_k(\hat{x}_k) + \beta - V^C]. \end{aligned}$$

In addition, continuous differentiability of w_j and (A1) imply $w_j(x) - w_j(z) > w_k(x) - w_k(z)$, as required. The ordering of constrained maximizers of $W_j(x, 1 - F(\bar{y}-x))$ then follows from standard arguments. □

The ordering of optimal policies implied by supermodularity is very useful in combination with the fact that, as shown below, given arbitrary policy choices $x_1 < x_2 < \cdots < x_n$ of the politician types in the first period, there is a unique outcome, which we denote $y^*(x_1, \ldots, x_n)$, such that conditional on realizing this value, the voter is indifferent between re-electing the incumbent and electing a challenger. Moreover, this extends to the case of mixed policy strategies $\pi_1, \ldots, \pi_n$ with discrete supports that are strictly ordered by type, i.e., for all $j < n$,

$$\max\{x : \pi_j(x) > 0\} \quad < \quad \min\{x : \pi_{j+1}(x) > 0\}.$$

That is, given such mixed policy strategies, there is a unique solution to the voter's indifference condition in (1), and we let $y^*(\pi_1, \ldots, \pi_n)$ denote the solution to the voter's indifference condition as a function of policy choices. In addition to uniqueness, the next proposition establishes that the cutoff is continuous in policy strategies and lies between the choices of the type 1 and type n politicians, shifted by the mode of the density of $f(\cdot)$, which we denote by $\hat{z}$.

Proposition 3. *Assume (A3) and (A4). For all mixed policy strategies $\pi_1, \ldots, \pi_n$ with discrete supports that are strictly ordered by type and for all belief systems μ derived via Bayes rule, there is a unique solution to the voter's indifference condition (1), and the solution $y^*(\pi_1, \ldots, \pi_n)$ is continuous as a function of mixed policies. Moreover, this solution lies between the extreme policy choices shifted by the mode of the outcome density, i.e.,*

$$\min\{x : \pi_1(x) > 0\} + \hat{z} \; < \; y^*(\pi_1, \ldots, \pi_n) \; < \; \max\{x : \pi_n(x) > 0\} + \hat{z}.$$

Proof. For existence of a solution to the indifference condition, fix $\pi_1, \ldots, \pi_n$ with supports that are strictly ordered by type, and note that the left-hand side of (1) is continuous in $\overline{y}$. Let $x_n = \min\{x : \pi_n(x) > 0\}$ be the lowest policy chosen with positive probability by the type n politicians. For all $j < n$ and all x with $\pi_j(x) > 0$, (A4) implies that $f(\overline{y}-x)/f(\overline{y}-x_n) \to 0$ as $\overline{y} \to \infty$. Thus, using (A4),

$$\mu_T(j|\overline{y}) = \frac{p_j \sum_x f(\overline{y}-x)\pi_j(x)}{\sum_k p_k \sum_x f(\overline{y}-x)\pi_k(x)} \leqslant \frac{p_j}{p_n} \sum_x \frac{f(\overline{y}-x)}{f(\overline{y}-x_n)} \frac{\pi_k(x)}{\pi_n(x_n)} \to 0$$

as $\overline{y} \to \infty$, which implies that $\mu_T(n|\overline{y})$ goes to one as the cutoff increases. In words, when the policies of the politicians are ordered by type, high realizations of the outcome become arbitrarily strong evidence that the incumbent is the best possible type. Similarly, $\mu_T(1|\overline{y})$ goes to one as $\overline{y}$ decreases without bound. Thus, the left-hand side of (1) approaches $\mathbb{E}[u(y)|\hat{x}_n]$ when the cutoff is large, and it approaches $\mathbb{E}[u(y)|\hat{x}_1]$ when the cutoff is small, and existence of a solution follows from the intermediate value theorem.

To show uniqueness, we claim that the left-hand side of (1) is strictly increasing in $\overline{y}$. Since higher types choose better policies for the voter, to prove the claim it is enough to show that $\mu_T(\cdot|\overline{y})$ exhibits first order stochastic dominance over $\mu_T(\cdot|\overline{y}')$ for $\overline{y} > \overline{y}'$; we claim the slightly stronger condition that for each $1 \leqslant j \leqslant n$, $\overline{y} > \overline{y}'$ implies

$$\sum_{k \geqslant j} \mu_T(j|\overline{y}) > \sum_{k \geqslant j} \mu_T(j|\overline{y}').$$

This is the case if

$$\frac{\sum_{k=j}^n p_k \sum_x f(\overline{y}-x)\pi_k(x)}{\sum_{m=1}^n p_m \sum_{x'} f(\overline{y}-x')\pi_m(x')} > \frac{\sum_{k=j}^n p_k \sum_x f(\overline{y}'-x)\pi_k(x)}{\sum_{m=1}^n p_m \sum_x f(\overline{y}'-x')\pi_m(x')},$$

or equivalently, cancelling repeated terms,

$$\sum_{m=1}^{j-1} \sum_{k=j}^{n} p_m p_k \sum_{x'} \sum_x \pi_m(x')\pi_k(x) f(\overline{y}'-x') f(\overline{y}-x)$$
$$> \sum_{m=1}^{j-1} \sum_{k=j}^{n} p_m p_k \sum_{x'} \sum_x \pi_m(x')\pi_k(x) f(\overline{y}-x') f(\overline{y}'-x).$$

Since supports are strictly ordered by type, $m < k$ and $\pi_m(x')\pi_k(x) > 0$ imply $x > x'$. From (A3), then,

$$f(\overline{y}'-x') f(\overline{y}-x) > f(\overline{y}-x') f(\overline{y}'-x)$$

for $\pi_m(x')\pi_k(x) > 0$ and $\overline{y} > \overline{y}'$, and the desired inequality follows.[7] Standard continuity arguments imply that $y^*(\pi_1, \ldots, \pi_n)$ is continuous as a function of mixed policy strategies with discrete supports.

To obtain the upper bound on the cutoff, consider any $\overline{y} \geqslant \max\{x : \pi_n(x) > 0\} + \hat{z}$. Recall that the posterior probability that the politician is type j, conditional on observing $\overline{y}$, is

$$\mu_T(j|\overline{y}) \quad = \quad \frac{p_j \sum_x f(\overline{y}-x)\pi_j(x)}{\sum_k p_k \sum_x f(\overline{y}-x)\pi_k(x)}.$$

Note that for all $k > j$ and all policies x_j with $\pi_j(x_j) > 0$ and x_k with $\pi_k(x_k) > 0$, we have $\hat{z} \leqslant \overline{y} - x_k < \overline{y} - x_j$. Since $f(\cdot)$ is single-peaked by (A3), we see that for all $x_1, \ldots, x_n$ such that each x_k is in the support of π_k, we have

$$f(\overline{y}-x_1) \ < \ f(\overline{y}-x_2) \ < \ \cdots \ < \ f(\overline{y}-x_n).$$

Therefore, the coefficients on prior beliefs are ordered by type, i.e.,

$$\frac{\sum_x f(\overline{y}-x)\pi_1(x)}{\sum_k p_k \sum_x f(\overline{y}-x)\pi_k(x)} \ < \ \cdots < \ \frac{\sum_x f(\overline{y}-x)\pi_n(x)}{\sum_k p_k \sum_x f(\overline{y}-x_k)\pi_k(x)},$$

and we conclude that the posterior distribution $\mu_T(\cdot|\overline{y})$ first order stochastically dominates the prior, contradicting the indifference condition. An analogous argument derives a contradiction for the case $\overline{y} \leqslant \min\{x : \pi_1(x) > 0\} + \hat{z}$. □

5 Two-type model

For the special case of two types, we can calculate electoral equilibria explicitly, and we can demonstrate the necessity of mixing when politicians are sufficiently office motivated. In the two-type model, the voter's cutoff is simply the solution to the equation $\mu_T(2|y) = p_2$, so that conditional on the cutoff $y^*(\pi_1, \ldots, \pi_n)$, the probability that the incumbent is the high type is just equal to the prior probability. If policy strategies are degenerate, then we let x_1 and x_2 be the policies chosen by the two types, so that $y^*(x_1, x_2)$ solves the equation

$$p_2 \quad = \quad \frac{p_2 f(y-x_2)}{p_1 f(y-x_1) + p_2 f(y-x_2)},$$

or after manipulating, the likelihood of y is the same given the policy choices of the politician types, i.e., $f(y-x_1) = f(y-x_2)$. Adding the assumption that the density

[7] Banks and Sundaram (1998) develop a similar argument in a related problem (Lemma A.6).

$f(\cdot)$ is symmetric around zero, the cutoff is simply the midpoint of the politicians' choices, i.e.,

$$y^*(x_1,x_2) \;=\; \frac{x_1+x_2}{2}.$$

The preceding observations allow us to graphically depict an electoral equilibrium in pure policy strategies for the two-type model. In Figure 6, we draw the indifference curves of the type 1 and type 2 politicians through their optimal policies, x_1^* and x_2^*, given the constraint set determined by the cutoff y^*. This is reflected in the tangency condition at each optimal policy. Moreover, the voter's indifference condition implies that the likelihood of outcome y^* is equal given either optimal policy, and this implies that the two tangent lines have equal slopes. Indeed, using the first order condition for office holders of types 1 and 2, we have

$$\frac{w_1'(x_1^*)}{w_1(\hat{x}_1)+\beta-V^C} \;=\; -f(y^*-x_1^*) \;=\; -f(y^*-x_2^*) \;=\; \frac{w_2'(x_2^*)}{w_2(\hat{x}_2)+\beta-V^C},$$

as claimed. Note that when the office benefit β increases and the cutoff $\overline{y}$ is held fixed, the indifference curves of the politician types become flatter, and optimal policies will move to the right, suggesting that higher office benefit leads to greater policy responsiveness. Of course, the voter's cutoff is itself endogenous and will respond to variation of parameters. If voters become more demanding when office benefit increases, so that $\overline{y}$ increases, it may then be that the overall effect is that some politician types reduce effort when office motivation is higher.

As an example, consider the quadratic-normal case with two types, e.g., $w_j(x) = -(x-\hat{x}_j)^2$ for $j=1,2$, and f equal to the standard normal density. Assume that $\beta > V^C$, and that the voter is risk neutral, i.e., $u(y)=y$ for all y. From the necessary first order condition, we deduce that an equilibrium in pure policies must satisfy

$$x_j^* \;=\; x_j(\beta) \;=\; \hat{x}_j + \left(\frac{\beta-V^C}{2}\right) f\left(\frac{\hat{x}_2-\hat{x}_1}{2}\right) \tag{4}$$

for $j=1,2$, and

$$y^* \;=\; y(\beta) \;=\; \frac{\hat{x}_1+\hat{x}_2}{2} + \left(\frac{\beta-V^C}{2}\right) f\left(\frac{\hat{x}_2-\hat{x}_1}{2}\right).$$

In terms of Figure 6, in an equilibrium in pure strategies, as β increases, the curve $1-F(y^*-x)$ moves to the right in parallel with the points x_1^*, x_2^*, and y^*.

We can check that indeed there is an equilibrium in pure strategies for low values of office benefit, if the ideal policies of the two types are not too far apart. Consider again Figure 3. For $\overline{y} < \hat{x}_j + 1$, the coefficient of absolute risk aversion

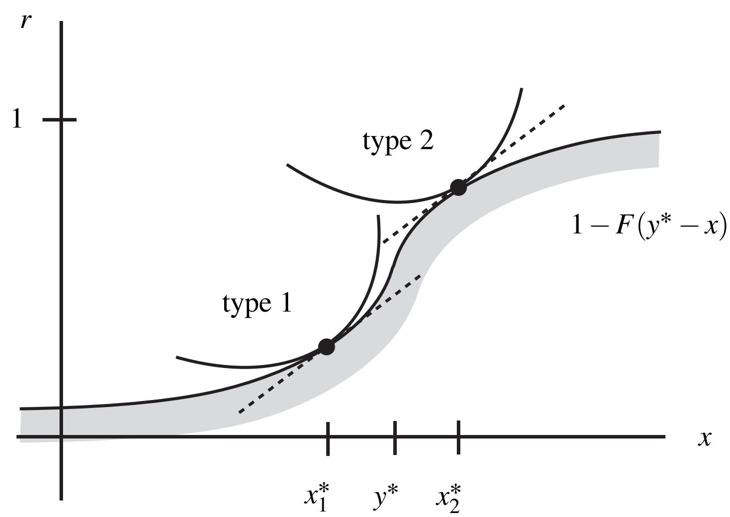

Figure 6: Electoral equilibrium in pure strategies with two types

$\frac{w_j''(x)}{w_j'(x)}$ for the type j politician, is above the ratio $-\frac{f'(\bar{y}-x)}{f(\bar{y}-x)}$ for all $x > \hat{x}_j$, so that the objective function of type j is quasi-concave. Thus, if $y(\beta) < \hat{x}_1 + 1$, or equivalently, if

$$\beta \;<\; V^C + \frac{2-(\hat{x}_2-\hat{x}_1)}{f\left(\frac{\hat{x}_2-\hat{x}_1}{2}\right)},$$

then $x_1^* = x_1(\beta)$, $x_2^* = x_2(\beta)$, and $y^* = y(\beta)$ determine a pure strategy equilibrium. However, if

$$\beta \;>\; V^C + \frac{2}{-f'\left(\frac{\hat{x}^2-\hat{x}_1}{2}\right)}, \tag{5}$$

then the second order condition of the problem of type 1 fails at $x_1(\beta)$. In fact, for high enough office motivation, the solution $x_1(\beta)$ of the type 1 politician's first order condition, from (4), becomes a local *minimum*, and thus a pure strategy equilibrium fails to exist.

For a numerical illustration, let the two politician types have ideal policies $\hat{x}_1 = 1$, $\hat{x}_2 = 2$, and set $\beta - V^C = 20$. The problem of the type 1 politician is depicted in Figures 4 and 5. We can construct an equilibrium recursively, beginning with the observation that inequality (5) holds, so equilibria will necessarily involve mixing. First, we obtain the unique cutoff such that the type 1 politician has two distinct optimal policies; in this example, for $y^* \approx 4.21$, the type 1 politician has optimal policies $x_{*,1} \approx 1$ and $x_1^* \approx 4.51$. Solving the first-order condition of the type 2 politician, we obtain $x_2^* \approx 4.98$. Finally, we back out the mixing probabilities for the type 1 politician as determined by the voter's indifference condition to obtain $\pi_1(x_{*,1}) \approx 0.21$ and $\pi_1(x_1^*) \approx 0.79$. The probabilities of reelection for type 1 are approximately 0 and 0.62 depending on whether the politician adopts $x_{*,1}$ or x_1^*, and the probability of reelection for type 2 is 0.78.

Several features of the above construction are noteworthy and, by results presented in the following sections, generalize beyond the two-type model and the functional forms assumed here. First, beyond the obvious fact that an electoral equilibrium exists, the voter's cutoff is demanding, in the sense that it substantially exceed the ideal policies of the politicians. Second, the type 2 politician has a unique optimal policy choice, while the type 1 politician has two optimal policies and mixes with positive probability on each. Third, both politician types are responsive, as their greater optimal policy choices are also significantly above the ideal policy choices, while the type 1 politician's lower optimal policy is close to the ideal policy. Fourth, conditional on the greater policy choices x_1^* and x_2^*, the probability of re-election is relatively high for both types and becoming close to one for the type 2 politician. Last, the higher policies x_1^* and x_2^* are relatively close to each other. Theorems 1–3 show that these are general features of electoral equilibria. In particular, as politicians become more office motivated, the equilibrium cutoff y^* increases without bound; the greater optimal policy choice of the type 1 politician and the unique optimal policy choice of the type 2 politician become arbitrarily high; the type 1 politician mixes between its two optimal policies, with the probability of going for broke converging to one; the re-election probabilities $1 - F(y^* - x_j^*)$ converge to one for both types; and the greater optimal policies of the politicians become close to each other. These observations are summarized in the following proposition.

Proposition 4. *In the two-type model with quadratic politician payoffs, standard normal density, and risk neutral voter, there is an electoral equilibrium. Let the office benefit become arbitrarily large. Then for every selection of electoral equilibria* σ*:*

(i) the voter's cutoff y^* *grows without bound, i.e.,* $y^* \to \infty$*;*

(ii) the type 1 politician mixes with positive probability on exactly two policies, $x_{*,1}$ *and* x_1^**, and the type 2 politician puts probability one on a policy* x_2^* *such that* $x_{*,1} < x_1^* < x_2^*$*;*

(iii) The lowest policy of the type 1 politician converges to type 1's ideal policy,

the highest policies of both types increase without bound, i.e., $x_{*,1} \to \hat{x}_1$ *and* $x_1^*, x_2^* \to \infty$*;*

(iv) the probability of reelection of both types converge to one, and in particular, the probability that the type 1 politician chooses the highest optimal policy converges to one, i.e., $1 - F(y^* - x_j^*) \to 1$ *for* $j = 1,2$ *and* $\pi_1(x_1^*) \to 1$*;*

(v) the highest optimal policies of the two types become arbitrarily close, i.e., $x_2^* - x_1^* \to 0$.

6 Existence and characterization of electoral equilibria

Our first general result establishes existence of electoral equilibrium, along with a partial characterization of equilibria. Importantly, electoral equilibria must solve a complicated fixed point problem: optimal policy choices of politician types depend on the cutoff used by the voter, and the cutoff used by the voter depends, via Bayes rule, on the policy choices of politician types. Nevertheless, we rely on Propositions 1–3 to provide a fixed point argument that overcomes the existence problem.[8]

Theorem 1. *Assume (A1)–(A7). Then there is an electoral equilibrium, and every electoral equilibrium is given by mixed policy strategies* $\pi_1^*, \ldots, \pi_n^*$ *and a finite cutoff* y^* *such that:*

(i) each type j politician mixes over policies using π_j^**, which places positive probability on at most two policies, say* x_j^* *and* $x_{*,j}$*, where* $\hat{x}_j < x_{*,j} \leqslant x_j^*$*,*

(ii) the supports of policy strategies are strictly ordered by type, i.e., for all $j < n$*, we have* $x_j^* < x_{*,j+1}$*,*

(iii) the voter re-elects the incumbent if and only if $y \geqslant y^*$*, where the cutoff lies between the extreme policies shifted by the mode of the outcome density, i.e.,* $x_{*,1} + \hat{z} < y^* < x_n^* + \hat{z}$.

Proof. In proving the proposition, we must address three technical subtleties. The first is that when supports of mixed policy choices are only weakly ordered, the left-hand side of (1) is only weakly increasing, so that the equality has a closed, convex (not necessarily singleton) set of solutions. In fact, if all politician types choose the same policy with probability one, then updating does not occur and

[8] We use Proposition 1 in our proof of equilibrium existence, but the key property required is that the number of optimal policies have a finite upper bound that is uniform across voter cutoffs. Assumption (A7) can be relaxed in this direction to provide more general conditions for existence.

incumbents are always re-elected, so that the voter's cutoff is negatively infinite. As policy choices of politician types converge to the same policy, this means that the cutoff either jumps discontinuously (from a bounded, finite level) or diverges to negative infinity. We circumvent this problem by deriving a positive lower bound on the distance between optimal policy choices of the different types. Indeed, we first observe that equilibrium policy choices are bounded above by $\overline{x} > \hat{x}_n$ such that $V^C = w_n(\overline{x}) + \beta$. Indeed, from (A1) and (A2), $\overline{x} > \overline{x}_j$ for $j < n$, where $\overline{x}_j > \hat{x}_j$ such that $V^C = w_j(\overline{x}_j) + \beta$. That is, if the type n politician is indifferent between choosing her ideal policy with no chance of re-election and choosing $\overline{x}$ and win with certainty, then no policy above $\overline{x}$ can be optimal for any type given any cutoff.

Next, given any cutoff $\overline{y}$ and any type j politician, there are at most two optimal policies, by Proposition 1, and each satisfies the first order condition (3). Note that $f(\overline{y}-x) \to 0$ uniformly on $[0,\overline{x}]$ as $|\overline{y}| \to \infty$, and from the first order condition, this implies that the optimal policies of the type j politician converge to the ideal policy, i.e., $x^*_j(\overline{y}) \to \hat{x}_j$ and $x_{*,j}(\overline{y}) \to \hat{x}_j$. Thus, we can choose a sufficiently large interval $[y_L, y_H]$ and $\varepsilon' > 0$ such that for all $\overline{y}$ outside the interval, optimal policies differ across types by at least ε', i.e., for all $j < n$, we have $|x_{*,j+1}(\overline{y}) - x^*_j(\overline{y})| > \varepsilon'$. By upper semi-continuity of $x^*_j(\cdot)$ and lower semi-continuity of $x_{*,j+1}(\cdot)$, the function $|x_{*,j+1}(\overline{y}) - x^*_j(\overline{y})|$ is lower semi-continuous and therefore attains its minimum on the (nonempty, compact) interval $[y_L, y_H]$. Since, from Propositions 1 and 2, $x_{*,j+1}(\overline{y}) > x^*_j(\overline{y})$ for all $\overline{y}$, this minimum is positive. Thus, there exists $\varepsilon'' > 0$ such that for all $\overline{y} \in [y_L, y_H]$, optimal policies differ by at least ε''. Finally, we set $\varepsilon = \min\{\varepsilon', \varepsilon''\}$ to establish the desired lower bound.

We are interested in the profiles $(\pi_1, \ldots, \pi_n)$ such that for all politician types j, π_j places positive probability on at most two alternatives, and the supports of mixed policy strategies are strictly ordered by type and separated by a distance of at least ε, i.e., for all $j < n$ and all policies x_j with $\pi_j(x_j) > 0$ and x_{j+1} with $\pi_{j+1}(x_{j+1}) > 0$, we have $x_j + \varepsilon \leqslant x_{j+1}$. It is convenient to represent such a profile by a $3n$-tuple (x,z,r), where $x = (x_1, \ldots, x_n) \in [0,\overline{x}]^n$, $z = (z_1, \ldots, z_n) \in [0,\overline{x}]^n$, and $r = (r_1, \ldots, r_n) \in [0,1]^n$. In addition, we require that for all j, we have $x_j \leqslant z_j$, and that for all $j < n$, we have $z_j + \varepsilon \leqslant x_{j+1}$. We then associate (x,z,r) with the profile of mixed policy strategies such that the type j politician places probability r_j on x_j and the remaining probability $1 - r_j$ on z_j. Letting D^ε consist of all such $3n$-tuples (x,z,r), we see that D^ε is nonempty, convex, and compact. Using this representation, we can define (abusing notation slightly) the induced cutoff $y^*(x,z,r)$, which is continuous as a function of its arguments.

The second difficulty is that the set Y of policy outcomes is not compact, so that the voter's cutoff is, in principle, unbounded. To circumvent this problem, we note that by continuity of the function $y^*(\cdot)$ the image $y^*(D^\varepsilon)$ is compact, and we can let $\overline{Y}$ be a closed interval containing this image. The existence proof then proceeds with an application of a fixed point theorem that relaxes Kakutani's conditions. We define the correspondence $\Phi: D^\varepsilon \times \overline{Y} \rightrightarrows D^\varepsilon \times \overline{Y}$ so that for each $(x,z,r,\overline{y})$, the value of Φ consists of $(3n+1)$-tuples $(\tilde{x},\tilde{z},\tilde{r},\tilde{y})$ such that for every politician type j, the mixed policy strategy represented by $(\tilde{x}_j,\tilde{z}_j,\tilde{r}_j)$ is optimal given $\overline{y}$, and $\tilde{y}$ is the unique cutoff induced by the indifference condition:

$$\Phi(x,z,r,\overline{y}) = \left\{ (\tilde{x},\tilde{z},\tilde{r},\tilde{y}) \in D^\varepsilon \times \overline{Y} \;\middle|\; \begin{array}{c} \text{for all } j, \tilde{x}_j \leqslant \tilde{z}_j, \\ \tilde{r}_j > 0 \Rightarrow \tilde{x}_j \in \{x_{*,j}(\overline{y}), x_j^*(\overline{y})\}, \\ \tilde{r}_j < 1 \Rightarrow \tilde{z}_j \in \{x_{*,j}(\overline{y}), x_j^*(\overline{y})\}, \\ \text{and } \tilde{y} = y^*(x,z,r) \end{array} \right\}.$$

Of note, we require that the first policy coordinate $\tilde{x}_j$ is less than or equal to the second, $\tilde{z}_j$, and we require that these are optimal when chosen with positive probability.

To deduce the existence of a fixed point of Φ, we first verify that the correspondence is upper hemi-continuous with closed values, i.e., it has closed graph. This property is not immediately obvious, because optimal policies are not unique, and the functions $x_j^*(\cdot)$ and $x_{*,j}(\cdot)$ are not continuous. It is important that we allow for the possibility that $\tilde{x}_j = \tilde{z}_j$, in which case both policies are equal to either the least optimal policy $x_{*,j}(\overline{y})$ or to the greatest optimal policy $x_j^*(\overline{y})$. Of course, these policies can coincide as well. Let $\{(x^m,z^m,r^m,\overline{y}^m)\}$ be any sequence converging to $(x,z,r,\overline{y})$ in $D^\varepsilon \times \overline{Y}$, and consider a corresponding sequence $\{(\tilde{x}^m,\tilde{z}^m,\tilde{r}^m,\tilde{y}^m)\}$ such that $(\tilde{x}^m,\tilde{z}^m,\tilde{r}^m,\tilde{y}^m)$ belongs to $\Phi(x^m,z^m,r^m,\overline{y}^m)$ for all m and $(\tilde{x}^m,\tilde{z}^m,\tilde{r}^m,\tilde{y}^m) \to (\tilde{x},\tilde{z},\tilde{r},\tilde{y})$. We must show that $(\tilde{x},\tilde{z},\tilde{r},\tilde{y}) \in \Phi(x,z,r,\overline{y})$. Since limits preserve weak inequalities, it is immediate that for all j, we have $\tilde{x}_j \leqslant \tilde{z}_j$, and continuity of $y^*(\cdot)$ implies $\tilde{y} = y^*(x,z,r)$. It remains to establish optimality of policies. It remains to establish optimality of policies adopted with positive probability, and we consider x_j, as the argument for z_j is analogous. To this end, suppose $\tilde{r}_j > 0$, so that for sufficiently high m, we also have $\tilde{r}_j^m > 0$, implying $\tilde{x}_j^m \in \{x_{*,j}(\overline{y}^m), x_j^*(\overline{y}^m)\}$. Since the best response correspondence is upper hemi-continuous (Proposition 1), $\tilde{x}_j$ is an optimal policy for the type j politician given cutoff $\overline{y}$. If $\tilde{x}_j \notin \{x_{*,j}(\overline{y}), x_j^*(\overline{y})\}$, then this implies the politician has at least three optimal policies, contradicting Proposition 1. Thus, $\tilde{x}_j$ is either the least or greatest optimal policy given $\overline{y}$, as desired.

This formulation yields a correspondence that is defined on a convex and compact domain and that is upper hemi-continuous and has nonempty, closed values. The typical application of Kakutani's fixed point theorem also proceeds by verifying convex values of the correspondence, and this leads to the third difficulty: Φ does not have this property. In particular, this property fails if $(x,z,r,\overline{y})$ is such that $x_j^*(\overline{y}) \neq x_{*,j}(\overline{y})$ for some j. Nevertheless, the values of the correspondence are contractible, and this is sufficient for existence of a fixed point. A subset $C \subseteq \Re^d$ of Euclidean space is *contractible* if there is an element $\overline{c} \in C$ and a continuous mapping $h: C \times [0,1] \to C$ such that for all $c \in C$, $h(c,0) = c$ and $h(c,1) = \overline{c}$. That is, the set can be continuously deformed to a single element. Convex sets are contractible, but convexity is not necessary for contractibility. It is straightforward to see that $\Phi(x,z,r,\overline{y})$ is contractible to the element $(\hat{x},\hat{z},\hat{r},\hat{y})$ such that: for all j,

- $\hat{x}_j = x_{*,j}(\overline{y})$,
- $\hat{z}_j = x^*_j(\overline{y})$,
- $\hat{r}_j = 1$,

where of course $\hat{y} = y^*(x,z,r)$ is fixed by construction. To reduce notation, we provide an informal description of the mapping h, breaking the unit interval into four components. Consider any $(\tilde{x},\tilde{z},\tilde{r},\tilde{y}) \in \Phi(x,z,r,\overline{y})$. For $t \in [0,.25]$, we continuously adjust each $\tilde{r}_j$ by dropping these values to zero. For $t \in (.25,.5]$, we continuously adjust each $\tilde{x}_j$ to $x_{*,j}(\overline{y})$. This adjustment requires that $\tilde{x}_j$ take sub-optimal values, but because the probability on $\tilde{x}_j$ is zero, we remain in the value of the correspondence. For $t \in (.5,.75]$, we continuously adjust each $\tilde{r}_j$ to one. And for $t \in (.75,1]$, we continuously adjust each $\tilde{z}_j$ to $x^*_j(\overline{y})$. This completes the construction, and we conclude that the values of Φ are contractible.

The correspondence Φ is upper hemi-continuous with nonempty, closed, contractible values, and the domain $D^\varepsilon \times \overline{Y}$ is nonempty, compact, and convex. Therefore, the Eilenberg-Montgomery fixed point theorem (see McLennan (2014), Theorem 14.1.5) implies that Φ has a fixed point,[9] (x^*,z^*,r^*,y^*), which yields an electoral equilibrium. Finally, the characterization results in (i)–(iii) follow directly from Propositions 1–3. □

7 Responsive democracy

We now consider the possibility of responsive democracy, meaning that incumbents choose high levels of policy, despite the short run temptation to choose their ideal policies. Our results in this section provide a characterization of electoral equilibria when office benefit is high. Under general conditions, we find that the voter becomes arbitrarily demanding, in the sense that the equilibrium cutoff diverges to infinity, that the policy choices of all politician types become close to their ideal

[9] The Eilenberg-Montgomery fixed point theorem holds for a domain that is a nonempty compact absolute retract. Every compact, convex set is an absolute retract (McLennan, 2014), so this assumption is satisfied automatically.

policy or arbitrarily high, and that, in fact, all above average types choose arbitrarily high policies. Under additional restrictions on the curvature of politician payoffs, we find that the type 1 politician mixes with positive probability between taking it easy and pooling with all other politician types by going for broke. Moreover, the probability of shirking by the type 1 politician goes to zero, and the probability that an incumbent of any type is re-elected goes to one, demonstrating a form of incumbency advantage.

We now make use of a standard Inada-type condition: for all j,

(A8) $$\lim_{x\to\infty} w'_j(x) \;=\; -\infty.$$

Intuitively, we require the marginal cost of effort to increase without bound as effort increases. This assumption is satisfied in the quadratic and exponential cases and many other cases of interest. Let $G = \{j : \mathbb{E}[u(y)|\hat{x}_j] > V^C\}$ denote the set of *above average* types, which are such that the expected utility from their ideal policy exceeds the expected utility from a challenger. Let $\ell = \min G$ be the smallest above average type.

Before proceeding to the formal analysis, we comment on the welfare implications of our responsiveness results. Under the natural interpretation of policy as effort, we would assume that voter preferences are monotonic in policy outcomes, so that u is strictly increasing. Given the short time horizon (and limited ability of the voter to sanction politicians), and given the divergence in preferences between the voter and politicians, the prospects for well-functioning democratic elections may seem dim. Nevertheless, when β is large, so that politicians are substantially office-motivated, we obtain a positive welfare result. Letting $\lim_{y\to\infty} u(y) = \overline{u}$, our analysis implies that the voter's ex ante expected payoff in the first period is bounded below by

$$\sum_{j\notin G} p_j \mathbb{E}[u(y)|\hat{x}_j] \quad + \quad \sum_{j\in G} p_j \overline{u}.$$

Of course, if the voter's utility function is bounded above, then an immediate implication, since type n is above average and $p_n > 0$, is that the voter's expected utility from politicians' choices in the first period increases without bound as office benefit becomes large.

However, our analysis admits more general voter utilities, including the possibility that u is single-peaked, so that high policy choices are actually damaging to the voter. In this case, the previous lower bound still holds, but it may be unrestrictive. If u is single-peaked and not bounded below, then $\overline{u} = -\infty$, and the ex ante expected payoff of the voter decreases without bound, as politicians choose high policies at great cost in order to signal to voters that they are an above average type. At work is the fact that current policy outcomes are used only to update beliefs about the incumbent's type; utility from current policy (whether high or low) is "sunk," and is not used to reward or punish politicians.

The next theorem states our initial result on responsive democracy. Note that condition (A8) is used only for part (iii) of the result.

Theorem 2. *Assume (A1)–(A8). Let the office benefit β become arbitrarily large. Then for every selection of electoral equilibria σ, the voter's cutoff diverges to infinity; the policy choices of each politician type either accumulate at their ideal policy or increase without bound; and the policy choices of all above average types increase without bound:*

(i) $y^* \to \infty$;

(ii) *for all j, all $\varepsilon > 0$, and sufficiently large β, we have either*

$$x_{*,j} = x_j^* \in (\hat{x}_j, \hat{x}_j + \varepsilon) \cup (\tfrac{1}{\varepsilon}, \infty) \quad \text{or} \quad x_{*,j} \in (\hat{x}_j, \hat{x}_j + \varepsilon) \text{ and } x_j^* \in (\tfrac{1}{\varepsilon}, \infty);$$

(iii) $x_{\ell-1}^* \to \infty$ *and thus for all $j \geqslant \ell$, we have $x_{*,j} = x_j^* \to \infty$.*

Proof. Consider an electoral equilibrium as β becomes large. By Theorem 1, each politician type j mixes between two policies, x_j^* and $x_{*,j}$, and the voter uses a finite cutoff y^*. Suppose there is a subsequence such that y^* is bounded above, say $y^* \leqslant \overline{y}$. By Theorem 1, the equilibrium cutoff lies in the compact set $[\hat{x}_1 + \hat{z}, \overline{y}]$. Then the first order condition for the type 1 politician in (3) implies that $x_{*,1} \to \infty$, and in particular, we have $x_{*,1} > \overline{y} - \hat{z}$ for large enough β, but this contradicts $x_{*,1} + \hat{z} \leqslant y^* \leqslant \overline{y}$. We conclude that y^* diverges to infinity, which proves (i).

To prove (ii), suppose there is a type j, an $\varepsilon > 0$, and a subsequence of office benefit levels such that $\hat{x}_j + \varepsilon \leqslant x_j^* \leqslant \frac{1}{\varepsilon}$. Going to a subsequence, we can assume $x_j^* \to \tilde{x}_j$ such that $\hat{x}_j < \tilde{x}_j < \infty$. Then for sufficiently large β, we have $\hat{x}_j < x_j^*$. For these parameters, the payoff to the type j politician from choosing $\hat{x}_j$ instead of x_j^* is non-positive, and thus we note that

$$(F(y^* - \hat{x}_j) - F(y^* - x_j^*))[w_j(\hat{x}_j) + \beta - V^C] \quad \geqslant \quad w_j(\hat{x}_j) - w_j(x_j^*).$$

That is, the current gains from choosing the ideal policy are offset by future losses. Since $y^* \to \infty$, the limit of

$$\frac{F(y^* - x_j^*) - F(y^* - \tilde{x}_j - 1)}{F(y^* - \hat{x}_j) - F(y^* - x_j^*)}$$

as β becomes large is indeterminate, and by L'Hôpital's rule, the limit is equal to

$$\lim \frac{f(y^* - x_j^*) - f(y^* - \tilde{x}_j - 1)}{f(y^* - \hat{x}_j) - f(y^* - x_j^*)} = \lim \frac{f(y^* - \tilde{x}_j - 1)\left(\frac{f(y^* - x_j^*)}{f(y^* - \tilde{x}_j - 1)} - 1\right)}{f(y^* - x_j^*)\left(\frac{f(y^* - \hat{x}_j)}{f(y^* - x_j^*)} - 1\right)} = \infty,$$

where we use (A3) and (A4). Then, however, the future gain from choosing $\tilde{x}_j + 1$ instead of x_j^* strictly exceeds current losses, i.e.,

$$(F(y^* - x_j^*) - F(y^* - \tilde{x}_j - 1))[w_j(\hat{x}_j) + \beta - V^C] > w_j(x_j^*) - w_j(\tilde{x}_j + 1), \quad (6)$$

for high enough β. To be specific, let

$$\begin{aligned} A &= w_j(\hat{x}_j) + \beta - V^C, \\ B &= w_j(\hat{x}_j) - w_j(x_j^*), \text{ and} \\ C &= w_j(x_j^*) - w_j(\tilde{x}_j + 1), \end{aligned}$$

where A is evaluated at sufficiently large β. Note that since $\hat{x}_j < \tilde{x}_j < \infty$, we have $\lim B > 0$ and $\lim C < \infty$. We have noted that $(F(y^* - \hat{x}_j) - F(y^* - x_j^*))A \geqslant B$ for sufficiently large β, and we have shown that as β becomes large, we have

$$\frac{F(y^* - x_j^*) - F(y^* - \tilde{x} - 1)}{F(y^* - \hat{x}_j) - F(y^* - x_j^*)} > \frac{C}{B}.$$

Combining these facts, we have

$$(F(y^* - \hat{x}_j) - F(y^* - x_j^*))A\left(\frac{F(y^* - x_j^*) - F(y^* - \tilde{x} - 1)}{F(y^* - \hat{x}_j) - F(y^* - x_j^*)}\right) > B\left(\frac{C}{B}\right),$$

which yields (6) for large β. This gives the type j politician a profitable deviation from x_j^*, a contradiction. A similar argument holds for $x_{*,j}$. It follows that for all j, all $\varepsilon > 0$, and sufficiently large β, we have $\{x_{*,j}, x_j^*\} \subset (\hat{x}_j, \hat{x}_j + \varepsilon) \cup (\frac{1}{\varepsilon}, \infty)$,

To establish that $\{x_{*,j}, x_j^*\} \subset (\hat{x}_j, \hat{x}_j + \varepsilon)$ for all $\varepsilon > 0$, and sufficiently large β, implies $x_{*,j} = x_j^*$ for sufficiently large β, suppose otherwise. Then there must be a a sequence of equilibria such that along that sequence $x_{*,j} \neq x_j^*$ and $\{x_{*,j}, x_j^*\} \subset (\hat{x}_j, \hat{x}_j + \varepsilon)$ for some type j for all ε and for sufficiently large β. Using part (i), we can find a subsequence of equilibria for increasing values of the office benefit such that along that subsequence the voter cutoff is strictly increasing in β. In each equilibrium in the sequence there must be a local minimizer located in between

$x_{*,j}$ and x_j^*. Note that a local minimizer must satisfy the necessary second order condition:

$$\frac{w_j''(x)}{w_j'(x)} \leqslant -\frac{f'(y-x)}{f(y-x)}$$

for $y = y^*$. Let $\tilde{X}(y) \subseteq X$ denote the set of policies satisfying the inequality above for a given voter cutoff. By (A7), $\tilde{X}(y)$ is convex for any y. From the necessary second order condition for a maximizer, we get

$$\frac{w_j''(x_{*,j})}{w_j'(x_{*,j})} \geqslant -\frac{f'(y^*-x_{*,j})}{f(y^*-x_{*,j})} \quad \text{and} \quad \frac{w_j''(x_j^*)}{w_j'(x_j^*)} \geqslant -\frac{f'(y^*-x_j^*)}{f(y^*-x_j^*)}.$$

Since there must be a minimizer in the interval $[x_{*,j}, x_j^*]$, these two inequalities imply $\varnothing \neq \tilde{X}(y^*) \subseteq [x_{*,j}, x_j^*]$. Now fix one value of β, say β', and let x' denote the minimizer in between $x'_{*,j}$ and $x_j^{'*}$, so that $x' > \hat{x}_j$ and $x' \in \tilde{X}(y^{'*})$. Since f is log-concave by (A3) and y^* is increasing in the value of the office, for any $\beta > \beta'$ we get $x' \in \tilde{X}(y^*)$. But then $x' \in [x_{*,j}, x_j^*]$ for any $\beta > \beta'$, which implies $x_{*,j} \leqslant x' \leqslant x_j^*$ for all $\beta > \beta'$, so that $x_j^* > \hat{x}_j + \varepsilon$ for small enough ε, a contradiction. A similar argument establishes that $\{x_{*,j}, x_j^*\} \subset (\frac{1}{\varepsilon}, \infty)$ implies $x_{*,j} = x_j^*$ for sufficiently large β.

To prove (iii), suppose that $x_{\ell-1}^*$ does not diverge to infinity. By (ii), there is a subsequence such that $x_{\ell-1}^* \to \hat{x}_{\ell-1}$. Now fix politician type $j \leqslant \ell - 1$, and note that since equilibrium policy choices are ordered by type, we have $x_j^* \to \hat{x}_j$. Using the expression for Bayes rule, the posterior probability of type $j \leqslant \ell - 1$ conditional on observing y^* satisfies

$$\mu_T(j|y^*) \quad = \quad \frac{p_j \sum_x f(y^*-x)\pi_j(x)}{\sum_k p_k \sum_x f(y^*-x)\pi_k(x)} \leqslant \frac{p_j f(y^*-x_j^*)}{\sum_{k\geqslant \ell} p_k \sum_x f(y^*-x)\pi_k(x)},$$

where the inequality uses $y^* \to \infty$ with (A3) and (A4). Note that

$$\sum_{k\geqslant \ell} p_k \sum_x f(y^*-x)\pi_k(x) \quad = \quad \sum_{k\geqslant \ell} p_k[f(y^*-x_k^*)\pi_k(x_k^*) + f(y^*-x_{*,k})\pi_k(x_{*,k})].$$

Dividing by $f(y^*-x_j^*)$, we obtain the expression

$$\sum_{k\geqslant \ell} p_k \left[\frac{f(y^*-x_k^*)}{f(y^*-x_j^*)}\pi_k(x_k^*) + \frac{f(y^*-x_{*,k})}{f(y^*-x_j^*)}\pi_k(x_{*,k})\right].$$

By (A3) and (A4), if $x_k^* \to \hat{x}_k$, then we have $\frac{f(y^*-x_k^*)}{f(y^*-x_j^*)} \to \infty$. By (ii), the remaining case is $x_k^* \to \infty$. Note that in this case, (A8) implies $w_k'(x_k^*) \to -\infty$, and thus the first order condition in (3) implies that $f(y^*-x_k^*)\beta \to \infty$. The first order condition for type j implies $f(y^* - x_j^*)\beta \to 0$, and we infer that $\frac{f(y^*-x_k^*)}{f(y^*-x_j^*)} \to \infty$. Similarly, $\frac{f(y^*-x_{*,k})}{f(y^*-x_j^*)} \to \infty$ for all $k \geqslant \ell$. Thus, we have

$$\mu_T(j|y^*) \;\leqslant\; \frac{p_j}{\sum_{k\geqslant\ell} p_k \sum_x \frac{f(y^*-x)}{f(y^*-x_j^*)}\pi_k(x)} \;\to\; 0.$$

We conclude that the voter's posterior beliefs conditional on y^* place probability arbitrarily close to one on above average types $j \geqslant \ell$, contradicting the indifference condition in (1). Therefore, we have $x^*_{\ell-1} \to \infty$, and since policy choices are ordered by type, this implies that $x_{*,j} \to \infty$ for all $j \geqslant \ell$. □

Theorem 2 implies that, as the office benefit goes to infinity, for every equilibrium sequence there is a "marginal type" $m \leqslant \ell - 1$ such that the policy choices of all types above the marginal type go to infinity, the policy choices of all types below the marginal type converge to their ideal policies, the highest optimal policy of the marginal type goes increases without bound, and the lowest optimal policy (if different from the highest) converges to the marginal type's ideal point. Note that for a type $j < m$ below the marginal type, we have $x_j^* \to \hat{x}_j$, and since $y^* \to \infty$, by Theorem 2, we have $F(y^* - x_j^*) = 1$. Thus, the probability of reelection of every type below the marginal type converges to zero.

Corollary 1. *Under the assumptions of Theorem 2, for office benefit* β *is sufficiently large and for every electoral equilibrium* σ*, there is a marginal politician type (which may depend on* σ*)* $m \leqslant \ell - 1$ *such that:*

(i) for all $j < m$*,* $x_j^* = x_{*,j} \to \hat{x}_j$ *and* $1 - F(y^* - x_j^*) \to 0$*;*

(ii) for all $j > m$*,* $x_j^* = x_{*,j} \to \infty$*;*

(iii) $x_m^* \to \infty$ *and if* $x_m^* \neq x_{*,m}$*, then* $x_{*,m} \to \hat{x}_m$*.*

A possibility left open in Theorem 2 is that all types go for broke, and it does not touch on the probability that the incumbent politician is re-elected. To further characterize the equilibrium behavior of politicians for large office benefit, we slightly strengthen assumption (A8): for all j,

(A9) $$\lim_{x\to\infty} \frac{w_j''(x)}{w_j'(x)} \;=\; 0.$$

Note that (A9) is satisfied if (A8) holds and $\lim_{x\to\infty} w_j''(x) > -\infty$, as in the quadratic case and other cases of interest.[10] The following theorem, in combination with previous results, shows that under the latter assumption, the probability of reelection of every type above the marginal type converges to one, and there is at least one type whose lowest equilibrium policy converges to that type's ideal point.

Theorem 3. *Assume (A1)–(A9). Let the office benefit β become arbitrarily large. Then for every selection of electoral equilibria σ, the least equilibrium policy $x_{*,1}$ of the type 1 politician converges to that type's ideal policy; the difference between the highest equilibrium policy and the voter cutoff goes to infinity for every type whose highest equilibrium policy goes to infinity; and in consequence the probability of reelection conditional on adopting that policy converges to one:*

(i) $x_{*,1} \to \hat{x}_1$;

(ii) if $x_j^* \to \infty$, *then both* $x_j^* - y^* \to \infty$ *and* $1 - F(y^* - x_j^*) \to 1$.

Proof. For part (i), note that, from part (ii) of Theorem 2, either $x_{*,1} \to \hat{x}_1$ or $x_{*,1} = x_1^* \to \infty$. Suppose toward a contradiction that $x_{*,1}$ increases without bound. From the necessary first and second order conditions of the type 1 politician's problem, we have

$$\frac{w_1''(x_{*,1})}{w_1'(x_{*,1})} \geqslant -\frac{f'(y^* - x_{*,1})}{f(y^* - x_{*,1})}.$$

From (A9), the left-hand side of the above inequality converges uniformly to zero from above as $x_{*,1} \to \infty$. Since (A3) implies f is log-concave, it follows that $-f'(z)/f(z)$ is strictly increasing in z, and moreover $-f'(z)/f(z) > 0$ if and only if $z > \hat{z}$. From Proposition 3, we have $y^* - x_{*,1} > \hat{z}$, where $\hat{z}$ is the mode of the outcome density, so that the right-hand side of the inequality above is strictly positive. We conclude that $y^* - x_{*,1}$ must converge to $\hat{z}$ from above as $x_{*,1} \to \infty$, so the probability of reelection of the lowest type must converge to $1 - F(\hat{z})$. Now consider the indifference curve through $\hat{x}_1$, given by (x, r) pairs satisfying

$$w_1(x) + r[w_1(\hat{x}_1) + \beta - V^C] = w_1(\hat{x}_1),$$

or equivalently, by $r = r_1(x)$, where

$$r_1(x) \equiv \frac{w_1(\hat{x}_1) - w_1(x)}{w_1(\hat{x}_1) + \beta - V^C}.$$

[10] If instead $w_j''(x)$ goes to $-\infty$, then by L'Hôpital's rule the assumption is satisfied if (A8) holds and $w_j'''(x)$ is bounded, and so on for higher order derivatives. Assumption (A9) fails in the exponential case, in which all higher order derivatives of w_j go to $-\infty$.

Let $\tilde{x}$ be defined by $r_1(\tilde{x}) = 1 - F(y^* - x_{*,1})$, so that

$$\tilde{x} \quad = \quad w_1^{-1}(F(y^* - x_{*,1})w_1(\hat{x}_1) - (1 - F(y^* - x_{*,1}))(\beta - V^C)).$$

We claim that $\tilde{x} < x_{*,1}$ for large β, so that the type 1 politician would be better off by adopting the ideal policy and being reelected with probability zero than by adopting the policy $x_{*,1}$ and being reelected with probability $1 - F(y^* - x_{*,1})$. Since the politician's indifference curves are vertically parallel and convex, it is enough to check that $r_1'(\tilde{x}) < f(y^* - x_{*,1})$ for large β or equivalently

$$-\frac{w_1'(w_1^{-1}(F(y^* - x_{*,1})w_1(\hat{x}_1) - (1 - F(y^* - x_{*,1}))(\beta - V^C)))}{w_1(\hat{x}_1) + \beta - V^C} \quad < \quad f(y^* - x_{*,1}).$$

This holds for large enough β if and only if

$$\lim_{\beta \to \infty} -\frac{w_1'\left(w_1^{-1}\left(F(\hat{z})w_1(\hat{x}_1) - (1 - F(\hat{z}))(\beta - V^C)\right)\right)}{w_1(\hat{x}_1) + \beta - V^C} \quad < \quad f(\hat{z}),$$

which holds if and only if

$$\lim_{z \to \infty} \frac{w_1''(z)}{w_1'(z)} \quad < \quad f(\hat{z}),$$

which holds by (A9). This contradicts optimality of $x_{*,1}$ and establishes part (i).

For part (ii), assume x_j^* increases without bound. We claim that either $y^* - x_j^* \to -\infty$ or $y^* - x_j^* \to \infty$. To prove this, suppose toward a contradiction that there is a subsequence of electoral equilibria such that $y^* - x_j^* \to K$ with K finite. Along this subsequence, the probability of reelection conditional on adopting x_j^* converges to $1 - F(K)$. As in the previous argument, consider the indifference curve through $\hat{x}_j$, given by

$$w_j(x) + r[w_j(\hat{x}_j) + \beta - V^C] \quad = \quad w_j(\hat{x}_j),$$

or equivalently, by $r = r_j(x)$, where

$$r_j(x) \quad \equiv \quad \frac{w_j(\hat{x}_j) - w_j(x)}{w_j(\hat{x}_j) + \beta - V^C}.$$

Let $\tilde{x}$ be defined by $r_j(\tilde{x}) = 1 - F(y^* - x_j^*)$, so that

$$\tilde{x} \quad = \quad w_j^{-1}(F(y^* - x_j^*)w_n(\hat{x}_j) - (1 - F(y^* - x_j^*))(\beta - V^C)).$$

We claim that $\tilde{x} < x_j^*$ for large β, so that the politician of type j would be better off by adopting the ideal policy and being reelected with probability zero than by adopting the policy x_j^* and being reelected with probability $1 - F(y^* - x_j^*)$. Since the politician's indifference curves are vertically parallel and convex, it is enough to check that $r_j'(\tilde{x}) < f(y^* - x_j^*)$ for large β or equivalently

$$-\frac{w_j'(w_j^{-1}(F(y^* - x_j^*)w_j(\hat{x}_j) - (1 - F(y^* - x_j^*))(\beta - V^C)))}{w_j(\hat{x}_j) + \beta - V^C} \quad < \quad f(y^* - x_j^*).$$

This holds for large enough β if and only if

$$\lim_{\beta\to\infty} -\frac{w_j'\left(w_1^{-1}\left(F(K)w_j(\hat{x}_j) - (1 - F(K))(\beta - V^C)\right)\right)}{w_j(\hat{x}_j) + \beta - V^C} \; < \; f(K),$$

which holds if and only if

$$\lim_{z\to\infty} \frac{w_j''(z)}{w_j'(z)} \quad < \quad f(K),$$

which holds by (A9). To complete the proof of part (ii), it remains to show that $y^* - x_j^*$ cannot diverge to infinity if x_j^* goes to infinity as β grows arbitrarily large. To see this, note that the first and second order condition for x_j^* to be maximum imply

$$\frac{w_j''(x_j^*)}{w_j'(x_j^*)} \; \geqslant \; -\frac{f'(y^* - x_j^*)}{f(y^* - x_j^*)},$$

which cannot hold for large enough β since the left-hand side converges to zero by (A9), but the right hand side is increasing since f is log-concave by (A3), and is positive for $y^* - x_j^* > \hat{z}$. □

By assumption (A1), we have $\frac{w_j'(x)}{w_{j+1}'(x)} < 1$ for large enough x, so that marginal disutility of effort is ranked according to type, and for some functional forms, such as quadratic, this ratio goes to one as x increases. We end by exploring the implications of this restriction: for all $j < n$,

(A10) $$\lim_{x\to\infty} \frac{w_{j+1}'(x)}{w_j'(x)} \; = \; 1.$$

An advantage of (A10) is that, with our earlier assumptions, it allows us to identify the marginal type as type 1. As office benefit becomes large, the type 1 politician has two optimal policy choices: take it easy by choosing policies arbitrarily close to her ideal point, or go for broke by choosing arbitrarily high policies. The type 1 politician must shirk with positive probability, for otherwise, the prior would stochastically dominate the voter's beliefs conditional on the cutoff y^*. Nevertheless, the probability of shirking goes to zero, and the probability that the type 1 politician is reelected goes to one as office benefit increases—despite the fact that the voter becomes arbitrarily demanding. Moreover, the high policy choices of the types become arbitrarily close to each other when politicians are sufficiently office motivated. An implication is that under (A1)–(A10), the effect of electoral incentives shifts away from selection to sanctioning: in the limit, all types choose approximately the same high policy in the first period, and all types are reelected with probability one.

Theorem 4. *Assume (A1)–(A10). Let the office benefit β become arbitrarily large. Then for every selection of electoral equilibria σ, the type 1 politician mixes between taking it easy and going for broke, and the probability of shirking goes to zero; for all politician types, the probability of reelection goes to one; and the greatest optimal policies of the types become arbitrarily close:*

(i) $x_{*,1} \to \hat{x}_1$, $x_1^* \to \infty$, *and* $\pi_1(x_1^*) \to 1$*;*

(ii) for all $j = 1, \ldots, n$, $1 - F(y^* - x_j^*) \to 1$*;*

(iii) $x_n^* - x_1^* \to 0$*;*

(iv) $0 < \pi_1(x_1^*) < 1$.

Proof. Part (ii) follows from part (i) and Theorem 3. Thus, we focus on parts (i), (iii), and (iv). Note that $x_{*,1} \to \hat{x}_1$ follows from Theorem 3. Now, by Corollary 1, there is a marginal type m such that $x_{*,j} = x_j^* \to \hat{x}_j$ for all $j < m$, $x_{*,j} = x_j^* \to \infty$ for all $j > m$, and $x_m^* \to \infty$. Let

$$\begin{aligned} A_j &= p_j \frac{f(y^* - x_{*,j})}{f(y^* - x_m^*)}, \text{ for all } j < m, \\ B &= p_m \pi_m(x_{*,m}) \frac{f(y^* - x_{*,m})}{f(y^* - x_m^*)} \\ C &= p_m \pi_m(x_m^*) \\ D_j &= p_j \frac{f(y^* - x_j^*)}{f(y^* - x_m^*)}, \text{ for all } j > m, \end{aligned}$$

and define

$$A = \sum_{j:j<m} A_j \quad \text{and} \quad D = \sum_{j:j>m} D_j.$$

Then the indifference condition for the voter conditional on y^* can be written as

$$\frac{\sum_{j:j<m} A_j \mathbb{E}[u(y)|\hat{x}_j] + (B+C)\mathbb{E}[u(y)|\hat{x}_m] + \sum_{j:j>m} D_j \mathbb{E}[u(y)|\hat{x}_j]}{A+B+C+D} = V^C. \quad (7)$$

By the first order conditions for the type $j \leqslant m$ and type m politicians, we have

$$\frac{f(y^* - x_{*,j})}{f(y^* - x_m^*)} = \frac{w_j'(x_{*,j})}{w_m'(x_m^*)} \to 0, \quad (8)$$

and thus $A, B \to 0$.

We claim that for all for all $j = m, \ldots, n-1$, we have

$$\lim \frac{f(y^* - x_j^*)}{f(y^* - x_m^*)} = 1.$$

Indeed, for β sufficiently large, we have $y^* < x_m^* \leqslant x_j^*$, and then single-peakedness of f implies that the above limit is less than or equal to one. For the opposite inequality, the first order condition for the type m and type j politician imply

$$\frac{f(y^* - x_j^*)}{f(y^* - x_m^*)} = \frac{w_j'(x_j^*)}{w_m'(x_m^*)} \geqslant \frac{w_j'(x_m^*)}{w_m'(x_m^*)},$$

where we use the facts that w_j is concave and that $x_j^* \geqslant x_m^*$. Since $x_m^* \to \infty$, (A10) implies that the limit of the right-hand side of the preceding inequality equals one, and the claim follows.

Note that the left-hand side of (7) can be written as an expectation with respect to a probability distribution $q = (q_1, \ldots, q_n)$, namely, $\sum_{j=1}^n q_j \mathbb{E}[u(y)|\hat{x}_j]$, where

$$\begin{aligned} q_j &= \frac{A_j}{A+B+C+D}, \text{ for all } j < m, \\ q_m &= \frac{B+C}{A+B+C+D}, \\ q_j &= \frac{D_j}{A+B+C+D}, \text{ for all } j > m. \end{aligned}$$

Going to a subsequence along which these terms converge, we can assume that $q \to \tilde{q}$ as β becomes large. By (8), we have $\tilde{q}_j = 0$ for all $j < m$. Moreover, (8) implies that $B \to 0$, and thus we have

$$\tilde{q}_m = \lim \frac{p_m \pi_m(x_m^*)}{A+B+C+D}.$$

From our claim, it follows that for all $j = m+1, \ldots, n-1$, we have

$$\frac{\tilde{q}_{j+1}/p_{j+1}}{\tilde{q}_j/p_j} \;=\; \lim \frac{D_{j+1}/p_{j+1}}{D_j/p_j} \;=\; 1.$$

Moreover, using $B \to 0$ and setting $j = m$ in our claim, we have

$$\frac{\tilde{q}_{m+1}/p_{m+1}}{\tilde{q}_m/p_m} \;=\; \lim \frac{D_{m+1}/p_{m+1}}{(B+C)/p_m} \;=\; \frac{1}{\lim \pi_m(x_m^*)}$$

if $\lim \pi_m(x_m^*) > 0$, and $\tilde{q}_m = 0$ otherwise. Therefore, we have shown that for all $j = m+1, \ldots, n$, we have $\frac{\tilde{q}_{j+1}}{p_{j+1}} \geqslant \frac{\tilde{q}_j}{p_j} \geqslant \frac{\tilde{q}_m}{p_m}$. If either $m > 1$ or both $m = 1$ and $\lim \pi_m(x_m^*) < 1$, then the limiting distribution $\tilde{q}$ stochastically dominates the prior, and we have

$$\lim \sum_{j=1}^{n} q_j \mathbb{E}[u(y)|\hat{x}_j] \;=\; \sum_{j=1}^{n} \tilde{q}_j \mathbb{E}[u(y)|\hat{x}_j] \;>\; V^C,$$

contradicting the voter's indifference condition for large enough β. This establishes part (i).

To prove part (iii), suppose toward a contradiction that $x_n^* - x_1^* \not\to 0$. Going to a subsequence, we can assume $x_n^* = x_1^* \to \Delta > 0$. From the first order condition for the type 1 and type n politician, we have

$$\frac{w_n'(x_n^*)}{w_1'(x_1^*)} \;=\; \frac{f(y - x_n^*)}{f(y - x_1^*)} \;\to\; 0,$$

where the limit follows from $\Delta > 0$ and (A4). By $x_1^* \to \infty$ and (A10), we have

$$\liminf \frac{w_n'(x_n^*)}{w_1'(x_1^*)} \;\geqslant\; \lim \frac{w_n'(x_1^*)}{w_1'(x_1^*)} \;\geqslant\; 1,$$

where the first inequality uses concavity of w_n and $x_n^* \geqslant x_1^*$. This contradiction establishes (iii).

To prove part (iv), we must argue that $0 < \pi_1(x_1^*) < 1$ for β sufficiently large. First, note that $\pi_m(x_m^*) \to 1$, so that $\pi_m(x_m^*) > 0$ holds for sufficiently large β. Suppose toward a contradiction that $\pi_1(x_1^*) = 1$ along some subsequence. By Proposition 3, we must have $x_1^* + \hat{z} < y^*$, or equivalently, $x_1^* - y^* < -\hat{z}$, but Theorem 3 implies that $x_1^* - y^* \to \infty$ a contradiction. □

8 Literature review

Closely related to our work is Fearon's (1999) two-period model of "selection and sanctioning," with several important differences. First, Fearon assumes that there are just two types, that utility is quadratic, and that incentive constraints bind for only one type. Second, he assumes that a random shock is added directly to the voter's utility, and not to the underlying policy outcome; thus, Fearon's model cannot generally be interpreted as capturing an uncertain linkage between policy and observable variables, such as employment status, inflation, etc., on which voters might base their decisions.[11] Third, Fearon focuses his analysis on pure strategy equilibria, but he does not prove existence of such an equilibrium. At issue is the possibility of non-convexities in the first-term office holder's objective function; this is exemplified in our Figure 2, and it arises in Fearon's model through non-concavity of the objective function in his equation (1). In our model, as in his, a policy satisfying first order conditions need not be a global maximizer of the politician's problem, even if it satisfies the second order condition. It is well-known that in non-convex games, existence of pure strategy equilibria is problematic, and this is true, in particular, of the electoral accountability model: it may be that none of the candidates for equilibrium identified by Fearon are, in fact, equilibria. In contrast, we impose sufficient structure on the model, in the form of (A1)–(A7), to establish existence of electoral equilibria and that mixing is limited to at most two policy choices for each politician type.[12]

Recently, Acemoglu et al. (2013) analyze a model in which policy outcomes lie on an ideological spectrum, and there are two citizen types: an honest type (with ideal policy equal to the median voter) and a dishonest type, which can be lobbied by a right-wing interest group. The effect of lobbying is to shift the ideal policy of the dishonest politician type, leading to a special case of our framework such that two politician types differ with respect to their effective ideal policies—one type being moderate, and the other right-wing, with ideal policy to the right of the median. The first-period politician's type is unobserved by the median; the politician makes a policy choice that is observed with noise; and the median voter then decides between re-electing the incumbent or electing an unknown challenger in the second period. Assuming quadratic utility and that the noise term is normally distributed with mean zero and variance that is sufficiently high relative to office

[11] The distinction between utility and policy outcomes disappears when the voter is risk neutral, i.e., u is affine linear, but the approaches are not equivalent in the general case, when the voter may be risk averse or risk loving.

[12] A recent book by Achen and Bartels (2016) offers a version of Fearon's model in an Appendix; the same remark applies, namely, Achen and Bartels assume that a solution to first order conditions yields a global maximizer.

benefit,[13] the authors show that there is a unique equilibrium in pure strategies, and in equilibrium the politicians choose policies to the left of the median.

The extent of this "populist" bias increases in office benefit, but a drawback is that the case of highly office motivated politicians is precluded by the authors' dispersion assumption on the noise density; without this assumption, equilibria in pure strategies will fail to exist, as demonstrated by our Theorem 4. Our results show that as office benefit increases, equilibria exist in mixed strategies, that the moderate politician type chooses a left-wing policy with probability one, and that the right-wing type mixes between a left-wing policy (going for broke) and a policy near its ideal point (taking it easy), with the probability of the latter going to zero. Moreover, the left-wing policies of the politicians become *arbitrarily populist* as office benefit increases. This conclusion does not depend on functional form assumptions on the noise density or voter and politician utilities, and if all politician types have ideal policies at or to the right of the median (e.g., voters can infer from the incumbent's party affiliation that the politician is not liberal), then the conclusion holds regardless of the number of politician types, showing that the insights of Acemoglu et al. (2013) are robust to numerous extensions of their model.

Chapter 3 of Besley (2006) presents a two-period, two-type model in which the incumbent politician observes the values of a binary state of the world and a preference shock, and then makes a binary policy choice. Closer to the model of our paper, Chapter 4 (coauthored with Smart) of the book investigates a two-type model in which an incumbent essentially chooses a level x of shirking, and voters observe this with noise, $x+\varepsilon$. Besley and Smart assume, however, that the incumbent politician observes the policy shock ε before her choice of x; in addition, the policy choice of the good type of politician is fixed exogenously. In these models, the politician's policy choice is either explicitly binary (between two possible policies), or it reduces to a finite number of policies, so that equilibria in mixed strategies are assured to exist. Interestingly, Theorem 4 reveals an analogy between equilibria of our model with high office benefit and the two-type model, as all types $j \geqslant 2$ go for broke, choosing similar policies, while the type 1 politician mixes between her ideal policy and going for broke.

Chapter 4 of Persson and Tabellini (2000) contains a simplified, two-period model of symmetric learning, i.e., politicians are parameterized by a skill level that is unobserved by voters and the politicians themselves. In this setting, voters and politicians update their beliefs symmetrically along the equilibrium path, and signaling cannot occur. Moreover, voters are assumed to be risk neutral. Ashworth (2005) considers a three-period model of symmetric learning that further differs

[13] The assumption that the noise density is sufficiently dispersed, Acemoglu et al.'s (2013) Assumption 1, allows the authors to focus on pure strategy equilibria .

from ours in that the skill level of a politician evolves over time according to a random walk. Although the model assumes three periods, the first-term office holder has private information about her ability only in the second and third terms, as her actions in office are hidden from voters. Ashworth assumes that office benefit is small relative to the variance of policy outcomes in order to guarantee existence of equilibrium in pure strategies. Ashworth and Bueno de Mesquita (2008) use a variant of the model, one in which the voter has quadratic policy utility and a stochastic partisan preference, to establish existence and comparative statics of incumbency advantage; and Ashworth et al. (2017) use the symmetric learning model to study the trade off between sanctioning and selection.

Other work, including Barganza (2000) and Canes-Wrone et al. (2001), studies a two-type model in which politicians differ in ability. In the latter papers, the voter's desired policy depends on the realization of a state of the world, about which politicians are better informed than voters, and politicians may have an incentive to pander to voters by knowingly choosing policies that are not in the voter's best interest. Maskin and Tirole (2004) study pandering in a two-type model in which politicians differ in preferences. Austen-Smith and Banks (1989) investigate the voter's ability to discipline politicians when all politicians have the same preferences, so that the model is one of pure moral hazard. In a two-period model of pure adverse selection, where politicians' policy choices are directly observed by voters, Duggan and Martinelli (forthcoming) show that responsive democracy can arise due to the incentive of all politician types to pool with the median type. Theorem 2 in the current paper establishes that a form of this incentive holds for all above average types when office benefit is high, delivering the responsiveness result even when policy choices are observed by voters with noise.

Due to difficult theoretical issues, the literature going beyond two periods is small. In an infinite-horizon version of our model, Banks and Sundaram (1993) find existence of (a continuum of) perfect Bayesian equilibria in which the voter uses a trigger strategy: if the observed policy outcome ever falls below a given level, then the voter resolves to replace the incumbent with a challenger, and the incumbent shirks for all remaining terms of office. The difficulty with such equilibria is that even if the voter believes that the incumbent is a good type with very high probability, there is a chance that the "trigger" is pulled, and then equilibrium strategies dictate that the otherwise desirable incumbent politician is replaced. Banks and Sundaram (1998) study the infinite-horizon model with a two-period term limit, and Duggan (2016) establishes equilibrium existence and limits on the possibility of responsive democracy in the infinite-horizon model: because voters cannot commit to replacing a politician after her first term of office, the voter's expected payoff from a first-term office holder is bounded strictly above by the expected utility from the ideal policy of the best politician type. Thus, the commitment problem of voters implies a qualitative difference between the two-period model and the infinite-horizon model with a two-period term limit. Kartik and Van Weelden (2016) consider a simplified version of the model with a myopic voter and binary policy space, and they examine the consequences of good and bad signaling technology.

9 Conclusion

The two-period model of elections provides a tractable setting for analysis of the interplay between short-term opportunistic incentives and long-term re-election incentives in determining politicians' behavior. We consider a natural, but analytically non-trivial, environment in which voters are imperfectly informed about both the preferences and the actions of politicians, and we allow for an arbitrary finite set of politician types and general preferences. In line with the extant literature on electoral accountability, we assume that politicians and voters cannot commit to future actions, opening the scope for opportunistic behavior and creating potential difficulties for the success of democratic electoral mechanisms. The two-period accountability model provides a canonical framework in which to approach these issues, but despite this, foundational questions of equilibrium existence and responsiveness of policy to voter preferences have remained open.

We address these questions by showing that office holders mix over at most two policy choices—"taking it easy" and "going for broke"—and establishing existence of electoral equilibrium. We then establish the possibility of responsive democracy: as politicians become more office motivated, voters become more demanding; policy choices of above average politicians in the first period increase without bound; and incentive effects of elections shift from selection to sanctioning. Our results suggest that when voter preferences are monotonic in policy (or effort), the incentives present in democratic elections have the potential to discipline the actions of elected representatives and to overcome the difficulties inherent in voters' sparse information and limited ability to sanction politicians. However, this positive welfare result can be overturned when voter preferences are single-peaked, indicating that the effectiveness of elections depends in a nuanced way on the policy making environment.

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