ECON 761
Time Series Analysis

Department of Economics
McGill University
Winter 2023

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Office hours: Monday 10:30-11:30am.
Regular classes: Monday, 11.35am-2.25pm.

Course Overview and Learning Outcomes

This course covers topics in time series analysis at the graduate level. The goal of the course is to provide a foundation in core time series methods that will allow students to undertake serious empirical work or pursue more advanced theoretical modeling.

In the first part of this course, we will study univariate and multivariate stationary time series models (ARMA and VAR models). The focus will be on discussing the properties of the models, estimators and test statistics, but we will provide proofs whenever they are helpful for the understanding of a topic. The main reference for this part of the course is the standard graduate time series textbook "Time Series Analysis" by James Hamilton (1994, Princeton University Press).

We will then study several additional topics in time series analysis. This includes bootstrap methods applied to time series data, forecasting methods and forecast evaluation, and large factor models. Standard asymptotic theory (laws of large numbers, central limit theorems, etc.) will be used throughout the course and I will assume that students have a working knowledge of probability, statistics and econometrics.

Course Materials

The recommended textbook for this class is Time Series Analysis by James Hamilton (1994, Princeton University Press), but the standard graduate text by Fumio Hayashi (Econometrics, Princeton University Press, 2000) contains several chapters on time series analysis that are useful for this class. See in particular Chapter 6 on serial correlation and Chapters 10 and 11 on unit root econometrics and cointegration. Additional references for the more specialized topics appear at the end of this outline.

I will provide a set of slides after each class that will help understanding the material. More references will appear on these slides. I will also provide a set of practice exercises to help students prepare for the midterm.

Course outline (chapters refer to Hamilton's book)

1. Univariate Stationary Time Series (Chapters 3 and 4)

   (a) Mean and autocovariance function, stationarity, ergodicity

   (b) Causality and invertibility

   (c) Yule-Walker equations for general ARMA processes.

   (d) Principles of forecasting:

   - Wold's decomposition, Wiener-Kolmogorov formula
   - Iterated forecasts versus direct forecasts.
   - Forecasting variables with unit roots.

   (e) Box-Jenkins modeling

2. Estimation and Inference in Stationary ARMA Models (Chapter 5)

   (a) Derivation of exact and conditional likelihood functions.

   (b) MLE and Conditional MLE

   (c) Autocovariance generating function

3. Spectral analysis (Chapter 6)

   (a) The spectral representation theorem

   (b) Definition of spectrum and spectral density function

   (c) Examples of spectrum

4. Asymptotic distribution of the sample mean of serially dependent processes

   (a) Long run variance and its relationship to the spectral density function at zero

   (b) Approaches to long run variance estimation

   i. Parametric VAR-HAC estimators

   ii. Kernel-based HAC estimators

iii. Weighted averages of periodogram estimators

(c) Alternative notions of weak dependence: ergodic stationary processes, strong and uniform mixing processes

(d) Alternative notions of absence of serial dependence: white noise, martingale difference sequence, independent white noise

5. Asymptotic theory for the OLS estimator

   (a) general time series regression with m.d.s. scores.

   (b) special case: AR(1) model with conditional heteroskedasticity of unknown form

6. Multivariate Stationary Time Series (Chapters 10 and 11)

   (a) Stationary VAR model: companion form, stationarity conditions, VMA(∞) representation

   (b) Estimation of unrestricted VAR models

      i. Conditional MLE equals OLS equation-by-equation

      ii. VAR as a special case of a SUR model

   (c) AR and VAR lag order selection: specific to general approach, general to specific approach and information criteria

   (d) Granger causality and tests of Granger causality

   (e) Impulse response analysis: reduced-form and orthogonalized IRFs

   (f) Structural VAR: recursive SVAR and non-recursive SVAR

7. Forecast evaluation

   (a) Loss functions and optimal forecasts

   (b) Forecast rationality and optimality tests: forecast unbiasedness and absence of serial correlation of forecast errors under MSE loss; complications arising from non-quadratic loss functions

   (c) Out-of-sample forecast evaluation: sampling distribution of average losses

   (d) Forecast comparisons between two forecasting models

      i. non-nested models: Diebold and Mariano (1995) and West (1996)

      ii. nested models: McCracken (2007)

8. Factor models
    (a) Estimation by principal components (PC)
    (b) Asymptotic properties of PC: Stock and Watson (JASA, 2002) and Bai (1993, Econometrica).
    (c) How to select the number of factors: Bai and Ng (Econometrica, 2002)
    (d) Factor-augmented regression models (Bai and Ng (2006) and Goncalves and Perron (2014)):
        i. Asymptotic bias, asymptotic variance and asymptotic distribution of OLS estimator with estimated factors
        ii. Bias-corrected estimator and variance estimation.
    (e) Prediction intervals based on factor augmented regression models:
        i. prediction intervals for regression models without factors: how to account for estimation parameter uncertainty
        ii. prediction intervals for factor augmented regression models: how to account for estimation parameter uncertainty and factors estimation uncertainty

9. Bootstrap methods
    (a) Bootstrap bias correction and variance estimation for the sample mean of i.i.d. data
    (b) Bootstrap confidence intervals: equal-tailed versus symmetric intervals, percentile versus percentile-t, Efron's percentile interval
    (c) Asymptotic validity of bootstrap confidence intervals
    (d) Asymptotic refinements of the bootstrap
    (e) Bootstrap for linear regression models: pairs bootstrap, residual-based bootstrap and wild bootstrap
    (f) Bootstrapping a time series regression: block bootstrap methods and residual-based bootstrap methods for AR/VAR models
    (g) Bootstrapping factor-augmented regression models

Evaluation

The final grade for the course will be based on:

1. Midterm exam (50%) on **March 6, 2023** during regular class time, based upon the material covered until then.

2. Term paper (40%) and presentation (10%).

The term paper involves applying a time series technique to data. Although most papers will focus on empirical applications to actual data, some students might choose to run a Monte Carlo simulation. After identifying a particular paper of interest, students should aim at first trying to replicate the existing findings and then providing an interesting extension of these results. The analysis should be concisely written and clearly spell out the question of interest and the findings. All topics are subject to my approval and the deadline for submitting a written description of the research idea is **February 13, 2023**. The course paper should not normally exceed 15-20 pages in length and is due at the end of the course without fail. The format of the papers should adhere to the standards required for submission to a journal. Papers must not be coauthored. The paper is due on **April 24**, **2023**, by email. An oral presentation of the main findings (which will be a maximum of 10-15 minutes long) is scheduled for **April 17, 2023**.

Additional references

For more specialized topics here is a partial list of useful references.

1. Bootstrap methods for time series.

   - Hansen, B. 2015. *Econometrics*, ch. 10.
   - Härdle, W., J.L. Horowitz, and J.-P. Kreiss (2003). "Bootstrap methods for time series," *International Statistical Review*, 71, 435-459.
   - Politis, D. (2003). "The Impact of Bootstrap Methods on Time Series Analysis," *Statistical Science*, Vol. 18, No. 2, 219–230.
   - Kreiss, J.-P., and E. Paparoditis (2011). "Bootstrap methods for dependent data: A review", *Journal of the Korean Statistical Society*, Vol. 40, 357-378.
   - Kreiss, J.-P. and S. Lahiri (2012). "Bootstrap Methods for Time Series", *Time Series Analysis: Methods and Applications*, Vol. 30, 1-24.
   - Lahiri, S.N., 2003. Resampling Methods for Dependent Data, Springer, New York.
   - Politis, D. J. Romano, M. Wolf, 1999. *Subsampling*, Springer, New York.

- Shao, J. and D. Tu, 1996. The Jackknife and Bootstrap, Springer, New York.

2. Forecasting methods

   - Diebold, F.X. (2006), Elements of Forecasting, Cincinnati, 4th ed., South-Western College Publishing.
   - Elliott, G., and A. Timmermann (2016), Economic Forecasting, Princeton University Press.

3. Large factor models.

   - Bai, J. and S. Ng, 2008. "Large Dimensional Factor Analysis", in Foundations and Trends in Econometrics , 3:2, 89-163.
   - Bai, J. 2003. "Inferential Theory for Factor Models of Large Dimensions," *Econometrica* 71, 135-171.
   - Bai, J. and S. Ng, 2002. "Determining the Number of Factors in Approximate Factor Models", *Econometrica*, 70,191-221.
   - Stock, J. H. and M. Watson, 2002. "Forecasting using Principal Components from a Large Number of Predictors," *Journal of the American Statistical Association*, 97, 1167-1179.
   - Bai, J. and S. Ng, 2006. "Confidence intervals for diffusion index forecast and inference with factor-augmented regressions", *Econometrica*,74, 1133-1155.
   - Bai, J. 2013. "Panel data models and factor analysis," Advances in Economics and Econometrics, Theory and Applications, The Tenth World Congress of the Econometric Society, edited by D. Acemoglu, M. Arellano, and E. Dekel, Vol. III, 437-484.

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