McGill University
Department of Economics
Comprehensive Examination

Microeconomic Theory

Examiners: Prof. Takashi Kunimoto and Prof. Licun Xue
Location: Leacock 424
Date and Time: Friday, June 4th, 2010, 12:00pm-3:30pm.

Instructions:

- Answer all 4 questions.
- All questions have equal weights.
- Calculators are allowed.
- No notes or texts are allowed.
- This exam comprises 4 pages, including this cover page.

Good luck!

Question I. There are only two commodities, commodity 1 and 2. Consider the expenditure function

$$e(p, u) = \exp\left[\alpha_1 \ln p_1 + \alpha_2 \ln p_2 + \left(p_1^{\beta_1} + p_2^{\beta_2}\right) u\right].$$

Answer the following questions.

1. If the expenditure function $e(\cdot)$ is nondecreasing in p_i for each $i = 1, 2$, what restrictions on $\alpha_1, \alpha_2, \beta_1$, and β_2 are necessary to be imposed?

2. If the expenditure function $e(\cdot)$ is homogeneous of degree one in prices, what restrictions on $\alpha_1, \alpha_2, \beta_1$, and β_2 are necessary to be imposed?

3. Let us define $S(p, u)$ as follows:

$$S(p, u) = \begin{bmatrix} \frac{\partial^2 e(p,u)}{\partial p_1^2} & \frac{\partial^2 e(p,u)}{\partial p_2 \partial p_1} \\ \frac{\partial^2 e(p,u)}{\partial p_1 \partial p_2} & \frac{\partial^2 e(p,u)}{\partial p_2^2} \end{bmatrix}.$$

   Taking into account all restrictions on $\alpha_1, \alpha_2, \beta_1$, and β_2 you deduced from the previous questions, derive $S(p, u)$.

4. Find the indirect utility function that corresponds to it.

5. Verify Roy's Identity.

Question II. Consider an exchange economy $\mathcal{E} = (X^i = \mathbb{R}^n_+, e^i, u^i(\cdot))_{i \in \mathcal{I}}$ where each consumer i's utility function $u^i(\cdot)$ is increasing. Suppose that, for any redistribution of the initial endowments among the consumers, there always exists an associated Walrasian equilibrium. Show that if $\hat{x}$ is Pareto efficient, it can be supported as a Walrasian equilibrium allocation, i.e., there exists $p \in \mathbb{R}^n_+ \backslash \{\mathbf{0}\}$ such that $(p, \hat{x})$ is a Walrasian equilibrium.

Question III. Consider the following Principal-Agent problem: The principal is risk-neutral, but the agent is risk-averse with the utility function of $U(w, e) = \sqrt{2w} - e^2$, where w is the wage and e is the effort. The agent can choose between putting low effort $e = 1$ or high effort $e = 3$. The agent's effort impacts the work result and there are three possible results, valued respectively at $V = 0$, $V = 2000$, and $V = 3000$ to the principal. The probability of each result conditional on any of the two effort levels is given in the following table:

effort\result	0	2000	3000
$e = 1$	0.6	0.2	0.2
$e = 3$	0.2	0.3	0.5

The agent's reservation utility is 17.

1. What is the optimal contract when agent's effort is observable (i.e., the *first best* contract)?

2. Set up the owner's problem of designing optimal contract when efforts are hidden. What is the optimal contract (i.e., the *second best* contract)? Compare your results (the feature of the contract, the action induced by this contract, the expected utility of the manager, and the expected profit of the owner) with those for the first best contract.

Question IV. Consider two firms, an *incumbent* and a *potential entrant.* Assume that both firms have constant unit costs, that the entrant faces an entry fee of $K = 100$, and that the market inverse demand function is given by $P(Q) = 24 - \frac{1}{3}Q$, where Q is the (aggregate) quantity on the market. Assume further that, according to the entrant's assessment, the incumbent's unit cost is $c^h = 9$ with probability $\frac{1}{3}$ and $c^\ell = 3$ with probability $\frac{2}{3}$. The entrant has a unit cost $c = 6$. All of the above information is common knowledge.

1. Suppose that only the incumbent is in the market. Determine the profit-maximizing output and profit for each of type.

2. Next, assume that both firms are in the market. Derive the Bayesian Cournot (Nash) equilibrium.

Now, consider a *two-period* model. In the first period, only the incumbent is in the market. The entrant can enter the market in the second period, after observing the behavior (i.e., quantity choice) of the incumbent in the first period. The entrant does not know the cost of the incumbent when the entry decision has to be made; therefore, the behavior of the incumbent in the first period may provide crucial information about the incumbent's cost. After the entrant enters, both firms engage in a *Cournot* type (i.e., quantity) competition. The incumbent's total profit is the sum of his profits in both periods.

3. Does there exist a pooling perfect Bayesian equilibrium? If so, fully specify such an equilibrium; if not, show why not.

4. Is there a separating perfect Bayesian equilibrium? If so, fully specify such an equilibrium; if not, show why not.