Macroeconomic Theory
Comprehensive Exam 2010

May 26, 2010

You have three and a half hours for this exam. Neither books nor class notes are permitted. No electronic devices are permitted. There are 180 points in total, with points per question shown below. The exam consist of 5 pages. Please read the whole exam before starting. Wherever you do maths, explain briefly what you are doing.

1 Short Questions (10 points each)

1. The income elasticity of demand. At early stages of development most people are employed in traditional agricultural production and as economies grow, labor reallocates to modern manufacturing activities. This process is known as structural change. One of the potential drivers behind this reallocation of resources is the income elasticity of demand. Let's consider a simple static endowment economy. Preferences are defined over two non-storable goods a and c according to the following additive specification,

$$\frac{a^{1-\sigma}}{1-\sigma}+\frac{c^{1-\phi}}{1-\phi}, \quad \sigma,\phi>0.$$

   Each agent receives an identical endowment, y, that is allocated between the two goods that for simplicity are assumed to have the same price, so $y = c + a$. If c were modern manufactures and a were agricultural goods, what restrictions do we need to impose on σ and ϕ to be consistent with the observed patterns of structural change?

2. The cyclical behavior of real wages I. Most evidence suggests that real wages do not vary much through the business cycle. During recessions we observe important increases in unemployment with very limited reductions in real wages. What does the standard RBC model imply for the cyclical behavior of wages? What features could one introduce in the standard RBC framework to improve its predictions along this dimension?

3. The cyclical behavior of real wages II. Most evidence suggests that real wages do not vary much through the business cycle. During recessions we observe important increases in unemployment with very limited reductions in real wages. Some authors claim that these patterns result from wages affecting the level of effort exerted by workers. Lets assume there are N workers, each of them exerting effort according to $e(w) = (w - \eta)^{\beta}$, where w is the wage and $\eta > 0$ and $0 < \beta < 1$ are parameters. Firms have access to the following technology, $Y = A(e(w)L)^{\alpha}$, where L is employment, A some constant measure of productivity and $0 < \alpha < 1$ is the elasticity of output to effective labor. What are the effects of changes in A on employment and wages?

4. Explain why we cannot directly apply recursive methods (dynamic programming) to some problems that can be solved in sequence form. Give two examples.

5. Consider an unemployed worker who each period receives n wage offers. The probability of receiving n offers follows a Markov process. Let it be $\pi(n|n_-)$, with $\pi(n|n_-) \geq 0$ for $1 \leq n \leq N$ and $\sum_{n=1}^{N} \pi(n|n_-) = 1$ for $N < \infty$. Each offer consists in an independent wage draw from $F(w)$. If the worker accepts an offer, he/she can work forever at that wage. While unemployed, he/she receives unemployment compensation c. The worker maximizes $\mathbb{E} \sum_{t=0}^{\infty} \beta^t y_t$, where $y_t = w$ if employed and $y_t = c$ otherwise, and $\beta \in (0, 1)$.

   Formulate the Bellman equation for the worker's problem. (Hint: Note that the event $\max(w_1, w_2) \leq w$ is the same as the event $(w_1 \leq w) \cap (w_2 \leq w)$, so that $prob(\max(w_1, w_2) \leq w) = F(w)^2$.) Explain what kind of policy is optimal and show why. Give an intuitive explanation of why it is better to receive more offers.

6. Consider an agent who lives two periods. He/she consumes in both periods, valuing consumption with an increasing and concave period utility function $u(c)$ and discounting future utility using a discount factor $\beta \in (0, 1)$. This is financed by labor income $y = nh$, where n are hours worked and h is human capital. Total hours each period are normalized to 1. In the first period, the agent starts with $h = 1$, but can spend time to accumulate more human capital. Concretely, assume that $h' = (2 - n)^{\gamma} h$, with $\gamma \in (0, 1)$.

   Write down the agent's problem. Derive the first order condition for accumulating human capital. How would the choice of h' be affected by an equal tax τ on labor income levied in both periods? Explain.

2 Long Questions

2.1 Capital adjustment costs (60 points)

Consider an industry with N identical firms. Profits at time t of the i-th firm, neglecting any costs of acquiring and installing capital, are given by,

$$\pi(K_t)K_{it} \qquad \pi' < 0 \qquad (1)$$

Firms face costs of *acquiring* and installing/uninstalling capital given by the quadratic adjustment cost function (where the price of capital has been normalized to 1),

$$C(I_i) = I_i + \frac{h}{2}I_i^2 \qquad (2)$$

Assume that the firm invests so as to maximize the present value of its profits, where it is free to borrow as much as it desires at a fixed interest rate, r. Capital evolves according to the following law of motion,

$$\dot{K}_i = I_i$$

a) Write the intertemporal profit function of the firm.
b) Derive and interpret the optimality conditions. Derive the firm's investment demand.
c) Draw the phase diagram and assess the stability of the resulting equilibrium (graphically or analytically).
d) Some people believe that monetary policy has real effects on economic activity. Evaluate the impact on investment of a temporary (unanticipated) decrease in the real interest rate. Discuss the transition and draw the time paths of investment and capital (before the shock, at the time of the shock, along the transition and in the new steady state).
Some people advocate for the use of investment tax credits as a quick way to stimulate economic activity. Let's model the ITC as a direct rebate to the firm of a fraction $\theta > 0$ of the price of capital, so (2) becomes

$$C(I_i) = I_i(1-\theta) + \frac{h}{2}I_i^2 \qquad (2')$$

Notice that for the analysis that follows you only need to modify one of the first order conditions and as a result one of the dynamic equations. Assume the industry begins in a steady state associated with $\theta = 0$.

e) Trace the effects of a permanent (unanticipated) ITC (i.e. $\theta > 0$) in a phase diagram.

f) Trace the effects of a temporary (unanticipated) ITC (i.e. $\theta > 0$) in a phase diagram.

g) Draw in the same graph the path of q in f) and f). Draw in the same graph the time path of I in f) and f).

h) Use f), g) and h) to explain in words the response of the economy in both scenarios. Which one of the two types of ITC provides a stronger short term stimulus?

2.2 Consumption, Savings and Wealth (60 points)

Consider a consumer's consumption-savings problem. The consumer values consumption at time t according to the CEIS utility function $u(c_t) = \frac{c_t^{\sigma-1}-1}{\sigma-1}, \sigma > 0$. Lifetime utility is $U = \sum_{t=0}^{\infty} \beta^t u(c_t), \beta \in (0,1)$. The consumer starts the first period with zero assets. He/she can only invest in a bond that pays a net interest rate of r_t. Denote assets invested by a_{t+1}.

1. Consider first a two-period problem. The consumer receives income y in period 1. At the end of the period, he/she retires and uses savings to finance second-period consumption.

   (a) State the consumer's problem. Derive the Euler equation.

   (b) Show how the consumer's savings rate $s_t = a_{t+1}/y_t$ varies with the interest rate. Show how the direction of this effect depends on σ. Explain which two countervailing effects of an increase in r_t are at work here. Use the Euler equation to illustrate how they affect the consumption-savings decision.

2. Now consider an infinite-horizon economy consisting of two such consumers A and B, who have different elasticities of intertemporal substitution σ_i, $i = A, B$. They have the same endowment stream $\{y_t\}_{t=0}^{\infty}$. They again trade a risk-free bond each period and begin with zero asset holdings.

   (a) Carefully define a sequential competitive equilibrium for this economy.

   (b) Suppose that the endowment stream is constant: $y_t = y$ for all t. Show that, in this case, the economy has a steady state. Express the steady-state interest rate in terms of primitives.

   (c) Now suppose that the endowment grows over time: $y_{t+1} = (1+g)y_t, g > 0$, for all t. In this case, is there a steady state, that is, an equilibrium in which the consumption of both consumers grows at the same rate? Explain why or why not.

   (d) Explain in words what an equilibrium could look like.

3. Now suppose that A and B have equal elasticity of intertemporal substitution, but different discount factors $\beta_i \in (0,1), \beta_1 > \beta_2, i = A, B$.

   (a) What do the two agents' consumption profiles look like? Compare them.

   (b) What can you say about the market-clearing interest rate? Argue. Draw an illustrative graph.

   (c) What can you say about the pattern of asset ownership?

   (d) Now suppose that there is a continuum of agents. Each agent has a discount factor β_1 or β_2, and faces a constant probability π that this discount rate changes. Explain intuitively: What is the distribution of discount rates in the population? What does this imply for the distribution of assets? What is the difference between this case and the case with constant discount rates? How does π affect the asset distribution?