Macroeconomic Theory
Comprehensive Exam May 2021

Exam committee: Professors Poschke and Ruge-Murcia

May 21, 2021
9am-12.30pm

General instructions: You have three and a half hours for this exam. Neither books nor class notes are permitted. If possible, print this exam paper.

The exam consists of 4 pages. There are 180 points in total. Each short question accounts for 15 points and each long question for 60 points. Answer all questions.

Please read the whole exam before starting. Wherever you do maths, explain briefly what you are doing.

Technical instructions: You must be connected to the Zoom meeting for the exam for the entire time of the exam. You must have your camera on all the time, and be muted. You must be in front of the camera all the time, except when strictly necessary (e.g. bathroom breaks). The Zoom session is not recorded.

The exam is closed-book. You cannot access your computer during the exam time, except to (i) print the exam if you wish to, and (ii) communicate with me as described in the next point.

All communication with me must be made via the chat feature in Zoom. Communication related to the content of the exam (e.g. clarifying questions) must go through the public chat. You must also write down on the private chat whenever you are about to leave the camera (e.g. "leaving now for a bathroom break"), and whenever you are returning (e.g. "coming back now from a bathroom break").

Please email me at markus.poschke@mcgill.ca asap (e.g. using your phone) if you happen to experience a serious technical issue during the exam, or the submission process.

I will announce when the exam is over. At this time, you must stop writing and prepare the submission. You have up to 30 minutes to do this, and you need to stay on camera (and muted) throughout the submission process. Please:

1. make sure you identify clearly which question you're answering,
2. number your pages before scanning them, and submit them in order,
3. choose the PDF format for your submission. Please ensure that the file size is not too big, so that you're able to send it by email.
4. You're responsible for submitting a legible version of your answers. If part of your answer is not legible, or cropped, you won't be asked for a clarification and that part won't count. Also, please make sure your file is not corrupted, before you send it. Your submission by the end of the exam is final.

Short Questions (15 points each, answer all four questions)

1. Explain the concept of *self-insurance*. In what circumstances may households want to self-insure? Briefly discuss if and why (or why not) self-insurance arises under the following utility specifications:

   (a) $u(c) = c$,

   (b) $u(c) = -\frac{1}{2}(\bar{c} - c)^2$, where $\bar{c}$ is a constant greater than the household's maximum possible lifetime income,

   (c) $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$.

   (A graphical illustration may help.)

2. Use a simple model to show how a tax on wealth affects saving behavior.

3. What is the main difference between the perturbation and projection methods used to solve dynamic equilibrium models?

4. What is the nominal and the real return of money?

Long question 1 (60 points)

Consider an economy populated by identical, infinitely-lived traders whose total number is normalized to be one. The representative trader solves the problem

$$\underset{\{c_t, s_t, b_t\}_{t=\tau}^{\infty}}{Max} \quad E_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau} (c_t)^{1-\eta}/(1-\eta), \tag{1}$$

where E_τ denotes the expectation conditional on information known at time τ, $\beta \in (0,1)$ is the discount rate, η is a positive parameter, c_t is consumption, s_t is the number of shares, and b_t are holdings of a real bond that pays one unit of consumption with certainty at maturity. The maximization is subject to the budget constraint

$$c_t + p_t b_t + q_t s_t = b_{t-1} + (d_t + q_t) s_{t-1}, \tag{2}$$

where p_t is the bond price, q_t is the price of a share, and d_t is the per-share payoff received by the trader at time t. The payoff is generated by an infinitely-lived three that produces all the output in this economy. The output is stochastic and follows the process

$$\ln d_t = \rho \ln d_{t-1} + \epsilon_t, \tag{3}$$

where $\rho \in (-1,1)$ is the autocorrelation coefficient and ϵ_t is an independent and identically distributed (i.i.d.) innovation with mean zero and standard deviation σ.

a) Write the problem of the representative trader in recursive form and justify your choice of state variable(s). (12 points).

b) Derive the Euler equation for bonds and shares in this economy. Interpret you result. (12 points).

c) Does this model imply that the expected return of bonds and shares is the same? Explain. (12 points).

d) Define $r_t = 1/p_t$ to be the bond return and $R_t = (d_{t+1} + q_{t+1})/q_t$ to be the return on shares. Derive an expression for the (expected) excess return of shares compared with bonds. Explain. (12 points).

e) Assume that $Cov(c_{t+1}, R_{t+1}) > 0$. Is the excess return of shares compared with bonds positive and negative? Explain why. (12 points).

Long Question 2 (60 points)

Consider a neoclassical growth model with the following structure. Each consumer has preferences of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(c_t + B\frac{(1-n_t)^{1-\nu}-1}{1-\nu}\right)^{1-\sigma} - 1}{1-\sigma},$$

where labor supply $n_t < 1$. The economy's technology is described by

$$c_t + k_{t+1} - (1-\delta)k_t = k_t^{\alpha} n_t^{1-\alpha}.$$

1. State the consumer's problem. (10 points)
2. Carefully define a recursive competitive equilibrium for this economy. (10 points)
3. Derive the Euler equation for the savings decision. (10 points)
4. Derive the first-order condition for the labor-leisure decision. (6 points)
5. Show that the first-order condition for the labor-leisure decision can be written as a simple function relating $\ln(1-n)$ to $\ln w$, where w is the wage rate. (4 points)
6. What does optimal labor supply depend on in this setting? What does it *not* depend on? Which economic effects affect optimal labor supply choices here? (4 points)
7. Use the following facts from the hypothetical economy Caliland to calibrate all of the model's parameters. (16 points)

    - The average value of the capital-output ratio (in annual terms) is 2.
    - The inhabitants of Caliland work (on average) one-half of their total available time.
    - Experimental evidence on their attitudes towards risk shows that they all have a coefficient of relative risk aversion equal to 2.
    - Capital's share of income is one-third.
    - The average value of the investment-to-output ratio is 0.2.
    - Labor economists in Caliland have found that, in a log-log regression of a typical inhabitant's hours of leisure on the wage, the coefficient on the log of the wage is -0.5: if wages go up by 1%, a typical Calilander works 0.5% more hours.

    You do not need to compute all of the parameters numerically, but you do need to describe how you would compute them.