Macroeconomic Theory
Comprehensive Exam August 2020

August 13, 2020

General instructions: You have three and a half hours for this exam. Neither books nor class notes are permitted. If possible, print this exam paper.

The exam consists of 4 pages. There are 180 points in total. Each short question accounts for 15 points and each long question for 60 points. Answer all questions.

Please read the whole exam before starting. Wherever you do maths, explain briefly what you are doing.

Technical instructions: You must be connected to the Zoom meeting for the exam for the entire time of the exam. You must have your camera on all the time, and be muted. You must be in front of the camera all the time, except when strictly necessary (e.g. bathroom breaks).

All communication with me must be made via the chat feature in Zoom. Communication related to the content of the exam (e.g. clarifying questions) must go through the public chat. You must also write down on the private chat whenever you are about to leave the camera (e.g. "leaving now for a bathroom break"), and whenever you are returning (e.g. "coming back now from a bathroom break").

Please email me at markus.poschke@mcgill.ca asap (e.g. using your phone) if you happen to experience a serious technical issue during the exam, or the submission process.

I will announce when the exam is over. At this time, you must stop writing and prepare the submission. You have up to 30 minutes to do this, and you need to stay on camera (and muted) throughout the submission process. Please:

1. make sure you identify clearly which question you're answering,
2. number your pages before scanning them, and submit them in order,
3. choose the PDF format for your submission. Please ensure that the file size is not too big, so that you're able to send it by email.
4. You're responsible for submitting a legible version of your answers. If part of your answer is not legible, or cropped, you won't be asked for a clarification and that part won't count. Also, please make sure your file is not corrupted, before you send it. Your submission by the end of the exam is final.

Short Questions (15 points each)

1. Explain the concept of precautionary saving. Use an appropriate graph and one appropriate equation (do not derive it).

2. In the Diamond-Mortensen-Pissarides model, wages are set by bargaining. Explain why there is scope for bargaining over the wage in the model.

3. What is the key difference between the perturbation and projection methods used to solve dynamic equilibrium models?

4. What is the curse of dimensionality? How does it affect methods (like value function iteration) used to solve dynamic stochastic models? Use a maximum number of 10 lines.

Long question 1 (60 points)

Consider a consumer's consumption-savings problem. The consumer values consumption at time t according to the CEIS utility function $u(c_t) = \frac{c_t^{1-\sigma}-1}{1-\sigma}, \sigma > 0$. Lifetime utility is $U = \sum_{t=0}^{\infty} \beta^t u(c_t), \beta \in (0,1)$. The consumer starts the first period with zero assets. He/she can only invest in a bond that pays a net interest rate of r_t. Denote assets invested by a_{t+1}.

1. Consider first a two-period problem. The consumer receives income y_1 in period 1. At the end of the period, he/she retires and uses savings to finance second-period consumption.

   (a) State the consumer's problem. Derive the Euler equation.

   (b) Show how the consumer's savings rate $s_t = a_{t+1}/y_t$ varies with the interest rate. Show how the direction of this effect depends on σ. Explain which two countervailing effects of an increase in r_t are at work here. Use the Euler equation to illustrate how they affect the consumption-savings decision.

   (c) Analyze whether this changes when in the second period, the consumer has income y_2.

2. Now consider an infinite-horizon economy consisting of two such consumers A and B, who have different elasticities of intertemporal substitution σ_i, $i = A, B$. They have the same endowment stream $\{y_t\}_{t=0}^{\infty}$. They again trade a risk-free bond each period and begin with zero asset holdings.

   (a) Carefully define a sequential competitive equilibrium for this economy.

   (b) Suppose that the endowment stream is constant: $y_t = y$ for all t. Show that, in this case, the economy has a steady state. Express the steady-state interest rate in terms of primitives.

   (c) Now suppose that the endowment grows over time: $y_{t+1} = (1+g)y_t, g > 0$, for all t. In this case, is there a steady state, that is, an equilibrium in which the consumption of both consumers grows at the same rate? Explain why or why not.

   (d) Explain in words what an equilibrium could look like.

3. Now suppose that A and B have equal elasticity of intertemporal substitution, but different discount factors $\beta_i \in (0,1), \beta_1 > \beta_2, i = A, B$.

   (a) What do the two agents' consumption profiles look like? Compare them.

   (b) What can you say about the market-clearing interest rate? Argue. Draw an illustrative graph.

   (c) What can you say about the pattern of asset ownership?

   (d) Now suppose that there is a continuum of agents. Each agent has a discount factor β_1 or β_2, and faces a constant probability π that this discount rate changes. Explain intuitively: What is the distribution of discount rates in the population? What does this imply for the distribution of assets? What is the difference between this case and the case with constant discount rates? How does π affect the asset distribution?

Long question 2 (60 points)

Consider an economy populated by a continuum of household-producers in $[0,1]$ with preferences

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $\beta \in (0,1)$ is the discount factor, c_t is consumption, and $u(\cdot)$ is an instantaneous utility function assumed to be strictly increasing and strictly concave, and which satisfies the Inada conditions. The only consumption good in this economy is produced according to

$$y_t = z_t f(k_{t-1}),$$

where y_t is output, z_t is the stochastic level of productivity, k_t is the capital stock, and $f(\cdot)$ is a production function assumed to be strictly increasing and strictly concave, and which satisfies the Inada conditions. Assume that z_t is identically and independently distributed with mean zero and variance σ^2. The budget constraint of the representative household-producer (expressed in real terms) is

$$(1 + h(m_t))c_t + k_t + m_t = y_t + m_{t-1}/\pi_t + (1 - \delta)k_{t-1} + \tau_t,$$

where m_t is the real money stock, π_t is the gross inflation rate between periods $t-1$ and t, τ_t is a lump-sum transfer from the government, and $\delta \in (0,1)$ is the rate of depreciation. The initial stock of money and capital is given. The government is subject to the budget constraint

$$\tau_t = m_t - m_{t-1}/\pi_t.$$

Assume that the nominal money stock grows at the exogenous (gross) rate θ_t.

a) Interpret the term $1 + h(m_t)$ in the budget constraint of the household-producer. **(9 points)**

b) What are reasonable assumptions about the function $h(m_t)$? **(9 points)**

c) Write the optimization problem of the household-producer in recursive form. **(9 points)**

d) Find and interpret the first-order conditions for utility maximization. **(12 points)**

e) Impose general equilibrium conditions and derive the national accounting identity in this economy. **(9 points)**

f) Is money super-neutral in the deterministic steady state of this economy? **(12 points)**