Micro Comprehensive Exam 2020

Part A

(Please answer BOTH questions from this part.)

1. *Preferences* (Rubinstein, C5)

Identify a professor's lifetime with the interval $[0,1]$. There are $K+1$ academic ranks, $0,..,K$. All professors start at rank 0 and eventually reach rank K. Define a career as a sequence $t=(t_1,...,t_K)$ where $t_0=0\le t_1\le t_2\le ...\le t_K\le 1$ with the interpretation that t_k is the time it takes to get the k'th promotion. (Note that a professor can receive multiple promotions at the same time.) Denote by $\succsim$ the professor's (rational) preferences on the set of all possible careers. For any $\epsilon>0$ and for any career t such that $t_K\le 1-\epsilon$, define $t+\epsilon$ to be the career $(t+\epsilon)_k=t_k+\epsilon$ for all k (i.e. all promotions are delayed by ϵ). Following are two properties of the professor's preferences:

Monotonicity: For any two careers t and s, if $t_k\le s_k$ for all k then $t\succsim s$ and if $t_k<s_k$ for all k, then $t\succ s$.

Invariance: For every $\epsilon>0$ and every two careers t and s for which $t+\epsilon$ and $s+\epsilon$ are well defined, $t\succsim s$ if and only if $t+\epsilon\succsim s+\epsilon$.

1. Formulate the set L of careers in which a professor receives all K promotions at the same time. Show that if $\succsim$ satisfies continuity and monotonicity, then for every career t there is exactly one career $s\in L$ such that $s\sim t$.

2. Show that any preference which is represented by the function $U(t)=-\sum\Delta_k t_k$ (for some $\Delta_k>0$) satisfies Monotonicity, Invariance and Continuity.

3. One professor evaluates a career by the maximum length of time one has to wait for a promotion (i.e. the maximum t_k-t_{k-1}) and the smaller this number the better. Show that these preferences cannot be represented by the utility function described in (2).

2. *General equilibrium*

Consider a market with a set N of n individuals and a set H of n houses. Each house can be owned by/allocated to at most one individual. An *allocation* in this market is a distribution of houses such that each house $h\in H$ is allocated to exactly one individual. Initially, each house is owned by exactly one individual and e denotes this initial allocation of houses. Every individual i strictly prefers owning a house over not owning a house and each $i\in N$ has a **strict** (rational) preference relation $\succsim_i$ over H, that is, no individual is indifferent between any two different houses. An equilibrium of the market is a pair (p,a) where

- p, a price system, is a function that attaches a number $p(h)\ge 0$ (a price) to each house $h\in H$ and

- a is an allocation of houses

such that

for every individual i, the house $a(i)$ maximizes i's preference relation $\succsim_i$ over her budget set $\{h \in H : p(h) \leq p(e(i))\}$:

$$p(a(i)) \leq p(e(i)) \text{ and } a(i) \succsim_i h \text{ for all } h \in H \text{ with } p(h) \leq p(e(i))$$

1. Show that every equilibrium allocation a is strongly Pareto efficient; that is, for any equilibrium allocation a, there is no allocation $b \neq a$ such that $b(i) \succsim_i a(i)$ for all $i \in N$ and $b(i) \succ_i a(i)$ for some $i \in N$.

2. Show that if the initial distribution e is strongly Pareto efficient, then for every equilibrium (p, a) we have $a = e$.

Part B

(Please answer BOTH questions from this part.)

3. *Finitely Repeated Games*

Consider the following stage game played between Benoit and Krishna with common discount factor $\delta = 3/4$.

	L	M	R
U	$8, 2$	$1, 1$	$5, 3$
C	$7, 1$	$3, 3$	$6, 4$
D	$4, 4$	$4, 5$	$1, 3$

Suppose the game is repeated once (therefore played twice).

a) State *a* pure strategy subgame perfect equilibrium.

b) Is there a pure strategy subgame perfect equilibrium in which the first period outcome does *not* correspond to a Nash equilibrium of the stage game? If so, state it. Otherwise, explain why not.

4. *VCG*

Consider the allocation of three identical and indivisible objects to three agents, $N = \{1, 2, 3\}$, each of whom can consume multiple units. Agent i values the consumption of k units of the object at v_k^i. Her valuation profile $\theta^i = (v_1^i, v_2^i, v_3^i)$ is her private information. All that is publicly known is that for each player i, $0 \leq v_1^i \leq v_2^i \leq v_3^i$. Suppose the true valuations of the three players are as follows,

	θ^1	θ^2	θ^3
v_1	5	3	6
v_2	10	9	10
v_3	13	14	12

The table above shows, for instance, that player 1 has valuations $v_1^1 = 5$, $v_2^1 = 10$ and $v_3^1 = 13$.

Suppose the three players report their respective valuation profile truthfully.

a) What is the allocation of the three objects that maximizes the sum of values (utilities) across all players, for such a profile? (Who gets what?)

b) What are the transfers according to the VCG mechanism for this valuation profile?

(Note: You are only required to state the allocation and transfers for *this particular* valuation profile and not for any general valuation profile. The correct answer involves explicit numbers and not algebraic expressions.)

Now suppose instead that Players 2 and 3 collude behind Player 1's back and both lie about their valuations, while Player 1 reports truthfully.

c) Is there an untruthful valuation profile for players 2 and 3 that make both these players strictly better off? If yes, provide an example. If no, explain why not.