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McGill University
Department of Economics
Comprehensive Examination

Microeconomic Theory

Examiners: Profs Jian Li and Licun Xue
Date and Time: Thursday, May 23, 2019

Instructions:

- Calculators are allowed.
- No notes or texts are allowed.
- This exam comprises 4 pages, including this cover page.

Part A

(Please answer BOTH questions from this part.)

1.

Suppose outcomes X and states Ω are finite. **Directly** prove the necessity ($\Leftarrow$) part of the Anscombe-Aumann expected utility theorem: if a preference relation $\succsim$ on the space of Anscombe-Aumann acts $H = (\Delta X)^{\Omega}$ admits a utility representation:

$$U(h) = \sum_{s \in \Omega} \mu(s) \left[\sum_{x \in X} h_s(x) v(x) \right],$$

where μ is a strictly positive probability measure on Ω and $v : X \mapsto \mathbb{R}$ is a vNM utility index on X, then $\succsim$ is independent, Archimedean, and state-independent. [To avoid losing points, you have to provide a direct proof; you may not invoke the mixture space theorem in any way.]

Definition: A binary relation $\succsim$ on H is *independent* if, for all $f, g, h \in H$ and $\alpha \in (0, 1)$,

$$f \succsim g \Longleftrightarrow \alpha f + (1 - \alpha)h \succsim \alpha g + (1 - \alpha)h.$$

Definition: A binary relation $\succsim$ on H is *Archimedean* if, for all $f, g, h \in H$,

$$f \succ g \succ h \Longrightarrow \begin{cases} \exists\, \alpha \in (0,1) \text{ such that } \alpha f + (1-\alpha)h \succ g \\ \text{and} \\ \exists\, \beta \in (0,1) \text{ such that } g \succ \beta f + (1-\beta)h \end{cases}$$

Definition: A state $s \in \Omega$ is *null* if, for all $h \in (\Delta X)^{\Omega}$ and $\pi, \rho \in \Delta X$, $(h_{-s}, \pi) \sim (h_{-s}, \rho)$. A state $s \in \Omega$ is *non-null* if is is not null, i.e. if there exist $h \in (\Delta X)^{\Omega}$ and $\pi, \rho \in \Delta X$ such that $(h_{-s}, \pi) \succ (h_{-s}, \rho)$.

Definition: The binary relation $\succsim$ on H is *state-independent* if, for all non-null states $s, t \in \Omega$, acts $h, g \in H$, and lotteries $\pi, \rho \in \Delta X$,

$$(h_{-s}, \pi) \succsim (h_{-s}, \rho) \implies (g_{-t}, \pi) \succsim (g_{-t}, \rho).$$

2.

Consider a standard private ownership production economy with L goods, a single consumer and a single firm. The firm has a production set Y that is closed and convex, and $0 \in Y$. The representative consumer has a utility function $U : \mathbb{R}^L_+ \mapsto \mathbb{R}$ that is continuous, quasiconcave, and strongly monotone, an initial endowment $\omega \gg 0$, and owns the firm. Assume that the consumer's utility function satisfies the following strong desirability property: for every $\bar{x} \gg 0, \{x \in \mathbb{R}^L_+ : U(x) \geq U(\bar{x})\} \subset \mathbb{R}^L_{++}$.

(a). Prove that every Pareto optimal allocation is a competitive equilibrium allocation.

(b). Now suppose there are two goods — the labor (or leisure) of the consumer, denoted l, and the output of the firm. Suppose that production of the output good requires a fixed cost of $\bar{l}$ units of labor, and the firm's production set is

$$Y := \{(-l, y) \in \mathbb{R}^2 : l \geq 0, y \leq F(l)\},$$

defined by a production function of the form

$$F(l) = \begin{cases} 0, & \text{if } l \leq \bar{l}; \\ f(l - \bar{l}), & \text{if } l > \bar{l}; \end{cases}$$

where $f : \mathbb{R}_+ \mapsto \mathbb{R}$ is strictly concave, continuous, strictly monotone, and $f(0) = 0$. Assume the consumer's endowment is $(\hat{l}, 0)$ for some $\hat{l} > \bar{l} > 0$. Does the result in (a) still hold? Either prove that it does or provide a counterexample. [Hint: draw a picture.]

Part B

Answer both questions.

3. Two roommates who share an apartment are planning to jointly purchase an espresso machine that costs \$150. The espresso machine can be considered an example of *public good*. Each roommate's willingness to pay for the machine is private information and is assumed to be drawn *independently* from the interval $[0, 100]$ with uniform distribution. Each roommate is assumed to have quasi-linear utility. Thus, a roommate with willingness to pay of v gets a utility of $v - p$ if the machine is purchased and his contribution is p.

(1) Specify the VCG mechanism for this problem and show that truth revealing is a weakly dominant strategy for each agent.

Now consider the following Bayesian mechanism in which each roommate simultaneously makes a contribution towards the purchase of the espresso machine. If the total contribution is less than \$150, the espresso machine is not purchased and the money contributed by each roommate is refunded. If total contribution is at least \$150, the machine is purchased and "budget surplus" is shared equally between the roommates.

(2) Find the Bayesian Nash equilibrium in which each roommate's strategy is a linear function of his willingness to pay.

4. The productivity of a worker with education level e is $\theta(1+\alpha e)$ where θ is his productivity type and α is a parameter. The worker knows both α and θ while a potential employer knows α but *not* θ. Assumes that θ takes the value of either θ^L or $\theta^H > \theta^L$ and the probabilities of these values are p^L and p^H, respectively. Assume that $\theta^H = 4, \theta^L = 2, \alpha = 1$, and $p^H = 0.5$. There are two prospective employers who cannot tell the two types of workers apart. The value of the worker to an employer is the expected productivity. The employers observe the level of education e that a job applicant has undertaken and each simultaneously offers a wage. A worker observes the wage offers and then decides whether to accept one of them or reject both. If a worker with education level e accepts a wage offer of w, his payoff is $w - e^2/\theta$ (the second term reflects the disutility that a type-θ worker incurs to acquire education level e).

(1) What would be the equilibrium under symmetric information (i.e., *if* the employers knew θ)?

(2) Under asymmetric information, what is the minimum level of education that θ^H-type worker acquires in a separating (perfect Bayesian) equilibrium in pure strategies? What is the maximum? What are the levels of education that θ^L-type worker acquires in separating equilibria? Compare your result*s* with (1). Fully specify a separating equilibrium.

(3) Determine and fully specify a pooling (perfect Bayesian) equilibrium.