Part A

(Please answer BOTH questions from this part.)

1.

Let $X \subset \mathbb{R}$ be a finite set of monetary outcomes. Suppose the DM has some preferences $\succsim$ over monetary lotteries ΔX that admit the following utility representation:

$$W(\pi) = \sum_{k=1}^{K} f(G_\pi(x_k)) \cdot (x_k - x_{k-1}),$$

where outcomes in the support of lottery π are ranked by $x_0 < x_1 < \cdots < x_K$, $G_\pi(x_k) = \sum_{j \geq k} \pi(x_j)$, and $f : [0,1] \mapsto [0,1]$ is a continuous and increasing function.

Intuitively, outcomes in the support of the lottery are ranked from the lowest to the highest. The DM counts every increment $(x_k - x_{k-1})$ by some weight. The weight is determined by first assessing $(G_\pi(x_k))$ the probability of the outcome of the lottery being no worse than x_k, and then "reweigh" this probability by an increasing transformation (f).

It might be useful to know that W generalizes the standard expectation functional $E(\pi) = \sum_{k=1}^{K} \pi_k x_k = \sum_{k=1}^{K} G_\pi(x_k)(x_k - x_{k-1})$.

Either prove or provide a counterexample to the following TRUE OR FALSE statements:

(a). $\succsim$ is Independent.

(b). $\succsim$ is Archimedean.

(c). $\succsim$ is Monotone: If π first order stochastically dominates ρ, then $\pi \succsim \rho$ for all $\pi, \rho \in \Delta X$.

2.

Suppose there are two consumers, two states, and a single consumption good. Both consumers have expected utility functions as follows

$$\begin{aligned} U_1(x_{11}, x_{21}) &= \pi_{11} u_1(x_{11}) + \pi_{21} u_1(x_{21}), \\ U_2(x_{12}, x_{22}) &= \pi_{12} u_2(x_{12}) + \pi_{22} u_2(x_{22}), \end{aligned}$$

where x_{si} is i's consumption in state s and π_{si} is the subjective probability of state s by consumer i. Assume further that each u_i is concave and differentiable. In particular, suppose consumer 1 is risk neutral and consumer 2 is strictly risk averse. Let $\bar{\omega} = (\omega_1, \omega_2)$ be the aggregate endownment of contingent commodities for these two states. Assume each consumer is endowed with half of the total resources, that is, $(\omega_{1i}, \omega_{2i}) = \frac{1}{2}\bar{\omega}$.

(a). Suppose the two consumers have the same subjective probability so $\pi_{s1} = \pi_{s2}$ for all s. Prove that consumer 2 will fully insure his consumption at any interior Arrow-Debreau equilibrium. Provide an intuition for why the equilibrium allocation of risk is efficient.

(b). Now suppose the two consumers have different subjective probabilities about the states so $\pi_{s1} \neq \pi_{s2}$ for some s. Will consumer 2 fully insure at an interior Arrow-Debreau equilibrium? Prove your answer. Which is the direction of consumer 2's bias (relative to full insurance) in terms of the differences of the subjective probabilities?

(c). Argue that the risk-neutral consumer 1 will not gain from trade.

Part B

(Please answer TWO of the three questions from this part.)

1. Consider the following job market signaling model: A work knows his talent θ but his potential employer does not. The worker's talent θ takes two possible values

$$\theta = \begin{cases} \theta^L & \textit{with probability } 1-\lambda \\ \theta^H & \textit{with probability } \lambda \end{cases}$$

where $\theta^H > \theta^L > 0$. The value of the worker to the employer is $E(\theta)$, the expected value of θ given the information the employer has about θ. We assume that the employer pays the worker a wage w that is equal to this expectation. (Implicitly, we are assuming that the job market is competitive.)

The worker can acquire education (which the employer observes) to signal his talent. However, acquiring education is costly. The cost of obtaining education level e is e/θ. The utility of the worker is $w - e/\theta$ where w is the wage he receives.

   (a) What would be the equilibrium under symmetric information (i.e., *if* the employer knew θ)?

   (b) Under asymmetric information, what is the minimum level of education $\underline{e}$ that θ^H-type (productive) worker acquires in separating (perfect Bayesian) equilibria in pure strategies? What is the maximum? What is the level of education that θ^L-type (less productive) worker acquires in separating equilibria? Compare your result*s* with (a). Specify *fully* a separating equilibrium where θ^H-type (productive) worker acquires the level of education $\underline{e}$.

   (c) What is the maximum level of education that can be supported by a pooling equilibrium?

2. Consider the following simultaneous move game played between the row player Ann and the column player Bob.

	S	A
S	$5,6$	$3,7$
A	$6,4$	$1,2$

   (a) Suppose $\hat{p}$ and $\tilde{p}$ constitute two *correlated equilibrium distributions over outcomes* of the game above. Let $p = \alpha\hat{p} + (1-\alpha)\tilde{p}$. Is p a correlated equilibrium distribution of the game too? Explain why or why not. (The question is essentially asking if the set of correlated equilibrium distributions over outcomes is a convex set or not.)

(b) Find the correlated equilibrium of this game that generates the *lowest sum of payoffs* to the players.

3. Consider a random proposer version of the Rubinstein bargaining game played between Ann and Bob with an exogenous risk of *breakdown.* In each period Ann is selected as the proposer with probability 1/3 and Bob with probability 1/2. With probability 1/6 the game ends with both players getting 0. The proposer (if and when there is one) makes an offer (x_A, x_B) such that $x_A + x_B \leq 1$. The responder may then accept the offer or reject it. Accepting the offer ends the game and Ann and Bob receive x_A and x_B respectively, in that period. Rejecting the offer leads the bargaining to continue to the next period. Ann and Bob have the same discount factor, δ.

   Write down a subgame perfect equilibrium of this game.