Macroeconomic Theory
Comprehensive Exam 2017

May 11, 2017

You have three and a half hours for this exam. Neither books nor class notes are permitted. No electronic devices are permitted.

The exam consists of 4 pages. There are 180 points in total. The short questions differ slightly in weight and account for a total of 50 points. The long questions account for a total of 130 points. Answer all questions.

Please read the whole exam before starting. Wherever you do maths, explain briefly what you are doing.

Short Questions (50 points in total)

1. "All unemployment is involuntary." Comment. (10 points.)

2. What is the risk involved in holding a risk-free bond in an economy with aggregate risk? How does it affect the bond's price? Give a formula that illustrates the point (derivation not needed) and explain the intuition clearly. (10 points.)

3. Consider the stochastic version of the Solow growth model described by

$$\begin{aligned} y_t &= c_t + i_t, \\ k_{t+1} &= i_t + (1-\delta)k_t, \\ y_t &= A(k_t)^\alpha, \\ c_t &= (1-s)y_t, \end{aligned}$$

where y_t is output, c_t is consumption, i_t is investment, k_t is capital, A_t is the technology shock, δ is the depreciation rate and $\alpha, s \in (0,1)$. The equations above are, respectively, the national accounting identity, the law of motion for capital, the production function, and the decision rule for consumption. The technology shock A_t can take values $\{\underline{A}, \overline{A}\}$ and follows a Markov chain with transition probabilites

$$\begin{bmatrix} p & 1-q \\ 1-p & q \end{bmatrix},$$

where $q = Prob(A_{t+1} = \underline{A} | A_t = \underline{A})$ and $p = Prob(A_{t+1} = \overline{A} | A_t = \overline{A})$. In what follows, assume that $\delta = 0.02, \alpha = 0.3, s = 0.2, \underline{A} = 0.8$, $\overline{A} = 1.2$, and the initial capital stock $k_0 = 0.001$. Assume that $p = 0.5$ and $q = 0.5$.

Compute analytically the boundaries of the ergodic distribution of the capital stock. (15 points.)

4. Discuss briefly the implications of stochastic singularity for the estimation of linearized (first-order) dynamic equilibrium models. Use a maximum number of 10 lines. (15 points.)

Long question 1 (70 points)

This is a single question, so you need to do both parts.

Part A: Income risk (40 points)

Consider an economy with two types of consumers. Both types are infinitely lived, discount future utility with a common, constant discount factor $\beta \in (0, 1)$, and have identical period utility functions with standard properties. Both types receive an exogenous endowment every period. They can consume it right away, or can save it by investing in a bond that pays a common, constant return r. Assume for now that $\beta(1 + r) = 1$.

The difference between the two types is the level of their endowment: It is ω_i per period for type i, $i \in \{H, L\}$, $\omega_H > \omega_L$.

1. State the consumers' problem and derive a condition describing optimal behavior. Explain. Make sure to show how much each type consumes.

Suppose now and in the following that every period, there is some probability p that consumers switch to the other type. They remain of the same type with probability $1 - p$. p is constant and common.

2. Consider type H. State the problem for a type H consumer, and derive a condition describing optimal behavior. Explain. How does consumption for type H change compared to the situation with a constant endowment? What kinds of saving motives does this consumer have? Be precise. A graph may help. [Hint: it helps to compare behavior to that of a consumer who faces a certain decline of the endowment to $E_H\omega' \equiv p\omega_L + (1 - p)\omega_H$.]

3. Now consider type L, who faces a potential increase in the endowment. How is consumption for this type affected? Is there similar behavior? Be precise.

4. Now reconsider the assumption that $\beta(1 + r) = 1$. Can this be an equilibrium outcome in the economy with uncertainty if the two types of consumer described here are the only credit market participants? Argue in words and using a graph.

Part B: Work and search in a seasonal labor market (30 points)

Consider the following problem confronting an unemployed worker. The worker wants to maximize

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^s y_t, \beta \in (0, 1),$$

where $y_t = w_t$ in periods in which the worker is employed and $y_t = c$ in periods in which the worker is unemployed, where w_t is a wage rate and c is a constant level of unemployment compensation. At the start of each period, an unemployed worker receives one and only one offer to work at a wage w drawn from a *c.d.f.* $F(W)$, where $F(0) = 0, F(B) = 1$ for some finite $B > c$. Successive draws from F are identically and independently distributed. There is no recall of past offers. Only unemployed workers receive wage offers. The wage is fixed as long as the worker remains in the job. The only way a worker can leave a job is if she is fired. At the beginning of each odd period ($t = 1, 3, ...$), a previously employed worker faces the probability of $\pi \in (0, 1)$ of being fired. If a worker is fired, she immediately receives a new draw of an offer to work at wage w. At each even period ($t = 0, 2, ...$), there is no chance of being fired.

1. Formulate a Bellman equation for the worker's problem.

2. Demonstrate what form the worker's optimal policy takes.

Long question 2 (60 points)

Consider a central planner that solves the problem

$$\underset{\{c_t, M_t, \ell_t, k_t\}_{t=s}^{\infty}}{Max} \qquad E_s \sum_{t=s}^{\infty} \beta^{t-s} \ln(c_t) + a \ln(m_t) + b \ln(\ell_t),$$

where $\beta \in (0, 1)$ is the discount rate, c_t is consumption, $m_t = M_t/P_t$ is the real money stock, M_t is the nominal money stock, P_t is the price level, ℓ_t is leisure, and $a, b > 0$ are parameters. The maximization is subject to the constraints

$$\begin{aligned} n_t + \ell_t &= 1, \\ c_t + m_t + k_t &= y_t + \frac{m_{t-1}}{1 + \pi_t} + (1 - \delta)k_{t-1} + \tau_t, \end{aligned}$$

where n_t is the number of hours worked, π_t is the inflation rate, k_t is the capital stock, and τ_t is a lump-sum transfert. The production function is

$$y_t = z_t (k_{t-1})^{\alpha} (n_t)^{1-\alpha},$$

where $0 < \alpha < 1$ and z_t is a productivity shock. The productivity shock follows the process

$$\ln z_t = \rho \ln z_{t-1} + \epsilon_t,$$

where $\rho \in (-1, 1)$ and ϵ_t is identically and independently distributed with mean zero and constant variance. Money is introduced in the economy using the lump-sum transfer τ_t meaning that

$$\tau_t = m_t - \frac{m_{t-1}}{1 + \pi_t}.$$

The rate of money growth is exogenous and follows the process

$$\ln \theta_t = \gamma \ln \theta_{t-1} + \xi_t,$$

where $\gamma \in (-1, 1)$ and ξ_t is identically and independently distributed with mean zero and constant variance.

1) Write the problem of the central planner in recursive form. (6 points).

2) Justify your choice of state variables. (6 points).

3) Find and interpret the first-order necessary conditions of this problem. (24 points)

4) Impose equilibrium conditions and derive the national accounting identity. (6 points)

5) Is money super-neutral in the steady state? (6 points)

6) If the central planner were able to choose the rate of money growth optimally, what would the optimal rate be? (Hint: The optimal rate would eliminate inflation distortions.) (12 points)