McGill University
Department of Economics
Comprehensive Examination

Microeconomics

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Location: Leacock 424
Time: May 26, 2016, 9:00 am—12:30 pm.

Instructions:

• This exam has three parts, A, B, and C. **Please answer ALL questions from part A and B, and ONE question from part C.** The FOUR questions you answer have equal weight.

• Calculators are allowed.

• No notes or texts are allowed.

• This exam consists of 6 pages, including this cover page.

Good luck!

Part A

(Please answer BOTH questions from this part.)

1.

[Preference and Utility]

Suppose a consumer has some (complete and transitive) preference relation $\succcurlyeq$ over the set $X = \mathbb{R}_+ \times \{0, 1, 2, \ldots\}$, where an element (x, t) is interpreted as receiving \$ x in period t. Consider the following five properties of $\succcurlyeq$:

(1) The consumer is indifferent between receiving \$0 in any two periods: $(0, t) \sim (0, s)$ for all s, t.

(2) The consumer prefers to receive a positive amount of money earlier than later: for all $x > 0$ and $s > t$, $(x, t) \succ (x, s)$.

(3) More money is better: for all t and $y > x$, $(y, t) \succ (x, t)$.

(4) The consumer's preference between (x, t) and $(y, t + 1)$ is independent of t: for all x, y and s, t,

$$(x, t) \succcurlyeq (y, t + 1) \quad \Leftrightarrow \quad (x, s) \succcurlyeq (y, s + 1)$$

(5) Continuity.

Answer the following questions:

(a). Show that if a consumer's preference relation $\succcurlyeq$ satisfies properties (1)-(5), then it admits a utility representation.

(b). Conversely, suppose the consumer's preference relation $\succcurlyeq$ can be represented by the following utility function

$$U(x, t) = \delta^t u(x),$$

where $\delta \in (0, 1)$ and $u(\cdot)$ is a continuous and increasing function with $u(0) = 0$. Show that $\succcurlyeq$ satisfies properties (1)-(5).

(c). Consider the statment "consumer 1 is more impatient than consumer 2". Formalize this comparison as properties of $\succcurlyeq_1$ and $\succcurlyeq_2$.

(d). Now consider a third consumer, who has a preference relation $\succcurlyeq_3$ that can be represented by utility function

$$V(x, t) = \begin{cases} u(x) & \text{if } t = 0 \\ \beta\delta^t u(x) & \text{if } t \geq 1 \end{cases},$$

where δ and u are the same as those in part (b), and $\beta \in (0, 1)$. Interpret this consumer's time preference, and compare it to that of the consumer in part (b). Does $\succcurlyeq_3$ satisfy properties (1)-(5)? Either prove or give a counterexample to each of your answer.

2.

[Second Welfare Theorem for Diffferentiable Utility Functions.]

Consider a pure exchange economy with L commodities and m agents. Suppose each agent $i = 1, \dots, m$ has a utility function $u_i : \mathbb{R}^L_+ \mapsto \mathbb{R}$ that is continuous and strongly monotone, and some initial endowment $\omega_i \in \mathbb{R}^L_+$.

In class, we proved the following proposition: An interior allocation $x^* = (x_1^*, \dots, x_m^*)$ is Pareto optimal if and only if it is a solution to the social planner's problem (P) for some $\{\bar{u}_i\}_{i \neq j}$, that is, x^* solves

$$\begin{aligned} \max_{x_1,\dots,x_m \in \mathbb{R}^L_+} \quad & u_j(x_j) \\ s.t. \quad & u_i(x_i) \geq \bar{u}_i \text{ for all } i \neq j \\ & \textstyle\sum_{i=1}^m x_i = \sum_{i=1}^m \omega_i \end{aligned}$$

Now suppose u_i is also quasiconcave on $\mathbb{R}^L_+$. Fix some interior Pareto optimal allocation $x^* = (x_1^*, \dots, x_m^*)$, suppose that each u_i is differentiable at x_i^*, and that the gradient vector $\nabla u_i(x_i^*) = \left(\frac{\partial u_i(x_i^*)}{\partial x_{i1}}, \dots, \frac{\partial u_i(x_i^*)}{\partial x_{iL}}\right) >> 0$. Follow the steps below to derive an alternative proof of the second welfare theorem.

(a). Use the proposition stated above, show that for any two consumers i and j, the gradient vectors $\nabla u_i(x_i^*)$ and $\nabla u_j(x_j^*)$ must be proportional. That is, there must exists some $\alpha > 0$ (depending on i and j) such that $\nabla u_i(x_i^*) = \alpha \nabla u_j(x_j^*)$. Interpret this condition in the Edgeworth box economy (when $m = L = 2$).

(b). Define $\mathbf{p} = \nabla u_1(x_1^*) >> 0$. Verify that for every consumer i, there exists some $\lambda_i > 0$ such that $\nabla u_i(x_i^*) = \lambda_i \mathbf{p}$.

(c). Show that for every consumer i, x_i^* solves

$$\begin{aligned} \max_{x_i \in \mathbb{R}^L_+} \quad & u_i(x_i) \\ s.t. \quad & x_i \cdot \mathbf{p} \leq x_i^* \cdot \mathbf{p} \end{aligned}$$

That is, x_i^* is agent i's Walrasian demand at price $\mathbf{p}$ and income $x_i^* \cdot \mathbf{p}$.

Part B

Question 1

Definition: A coalitional game with transferable payoff $\langle N, v \rangle$ *is monotonic if for any pair of coalitions* S *and* T *such that* $S \subseteq T$

$$v(S) \leq v(T)$$

Prove or provide a counter-example for each of the following statements:
a) The core of a coalitional game with transferable payoff $\langle N, v \rangle$ is convex.
b) The core of a monotonic coalitional game with transferable payoff is non-empty.

Part C

(Please answer ONE of the two questions from this part.)

Q1. There are 2 individuals with private independent values (willingness to pay) for a public good. Each individual $i \in \{1, 2\}$ knows his/her own value v_i but only the distribution of his/her opponent's value. Values are assumed to be independently drawn from the uniform distribution on $[0, 40]$. The cost of providing the public good is $c = 50$. Thus, the project cost is such that an individual alone will not provide the public good. Individual i's utility is $v_i - b_i$ if he/she contributes b_i and the public good is provided.

(1) Consider the following "subscription mechanism": each individual makes a contribution for the provision of the public good. If total contribution is at least c, the public good is provided; otherwise, it is not provided and the money contributed by each individual is refunded. Find a Bayesian Nash equilibrium in which the public good is provided with positive probability.[Hint: consider strategies that are linear in values.]

(2) Consider the following mechanism in which each individual makes a contribution for the provision of the public good. If total contribution is at least c, the public good is provided and *budget surplus is equally shared by the two individuals*; otherwise, it is not provided and the money contributed by each individual is refunded. Find a Bayesian Nash equilibrium in which the public good is provided with positive probability. [Hint: consider strategies that are linear in values.]

Q2. Consider a principal-agent relationship with only two outcomes, valued at $x_1 = 25,000$ and $x_2 = 50,000$, respectively. The agent must choose among three effort leves, a_1, a_2, and a_3. The probability of each outmcome contingent on the effort levels are give below:

		Outcomes x_1	x_2
	a_1	0.25	0.75
Efforts	a_2	0.50	0.50
	a_3	0.75	0.25

The principal's utility function is given by $U_P(x, w) = x - w$ and the agent's utility function $U_A(w, a) = \sqrt{w} - d(a)$, where x is the value of the outcome, w represents the wage that the worker receives, and $d(a)$ is the agent's disutility from effort a with $d(a_1) = 40$, $d(a_2) = 20$, and $d(a_3) = 5$. The agent's reservation utility is $\underline{U} = 120$. The principal and the agent maximize their respective expected utilities.

(1) Consider first the symmetric information setting (in which the principal can monitor the agent's effort level). Find, for each level of effort,

the contract that implements (induces) that level of effort. What is the optimal effort level for the principal?

(2) Suppose that the effort is unobservable. Find, for each level of effort, the contract that implements (induces) that level of effort. What is the optimal effort level for the principal?