Macroeconomic Theory
Comprehensive Exam 2016

May 12, 2016

You have three and a half hours for this exam. Neither books nor class notes are permitted. No electronic devices are permitted.

The exam consists of 4 pages. There are 180 points in total. Each short question accounts for 15 points and each long question for 60 points. Answer all questions.

Please read the whole exam before starting. Wherever you do maths, explain briefly what you are doing.

Short Questions

1. Explain why we cannot directly apply recursive methods (dynamic programming) to some problems that can be solved in sequence form. Give two examples.

2. Consider an economy with two agents who value consumption and each receive a stochastic, strictly positive endowment every period. Give an example of a case where the equilibrium features full insurance against idiosyncratic risk although markets are not complete. Explain. In what context of analysis would such a setting be useful?

3. Consider an economy populated by a continuum of identical agents with preferences
$$U = E_0 \sum_{t=0}^{\infty} \beta^t \ln(c_t),$$
where $\beta \in (0,1)$ is the discount rate and c_t is consumption. The aggregate production function is
$$y_t = k_t^{\alpha},$$
where $\alpha \in (0,1)$ is a parameter, k_t is the capital stock, and y_t is output. The aggregate resource constraint is
$$c_t + k_{t+1} = y_t.$$
The initial capital stock is given. Find the exact value function that solves this problem using the method of guess-and-verify.

4. Explain (in less than 5 lines) why dynamic general equilibrium models solved using a second-order perturbation feature (or not) certainty equivalence.

Long question 1 (60 points)

(This is a single question, so you need to do both parts. Parts A and B are each worth 30 points.)

A. Labor supply and balanced growth (30 points)

Consider the problem

$$\begin{aligned}&\max \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)\\ &\text{s.t. } c_t + a_{t+1} = (1 + r_t(1-\tau_k))a_t + \lambda(w_t l_t)^{1-\tau_l}.\end{aligned}$$

λ, τ_k and τ_l are parameters of the tax system. Take them to be constant. Consider the following three commonly used specifications of preferences:

$$u(c_t, l_t) = \frac{c_t^{1-\sigma}}{1-\sigma} + \theta\frac{(1-l_t)^{1-\epsilon}}{1-\epsilon}, \tag{a}$$

$$u(c_t, l_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \theta\frac{l_t^{1+\epsilon}}{1+\epsilon} \tag{b}$$

$$u(c_t, l_t) = \frac{[c_t^{\rho}(1-l_t)^{1-\rho}]^{1-\sigma}}{1-\sigma}, \tag{c}$$

$\sigma, \varepsilon > 0$, each with logs if $\sigma = 1$, and a) with logs if $\varepsilon = 1$.

1. Give an intuitive account of why each of these specifications may be plausible. (No derivations; argue just by "staring" at the utility functions.) In each case, what do θ and ρ control?

2. Derive the first order condition for labor supply for the general case and for each of the three special cases. Point out any similarity across cases.

3. Consider a situation with $\lambda = 1, \tau_l = 0, \tau_k = 0$. Define a balanced growth path as a situation in which consumption and wages grow at the same, constant rate, and employment is constant. Suppose that we are dealing with a representative agent problem here.

   Are all three preference specifications consistent with balanced growth (in general/under some parameter restriction)? Give an economic intuition for the result you find.

4. Suppose that preferences are such that they are consistent with balanced growth under the conditions in the previous question. Now consider a situation where $\tau_l, \tau_k > 0$. Can there still be balanced growth? If not, can you suggest a simple way of fixing the problem?

B. Consumption and Saving (30 points)

Consider a consumer's consumption-savings problem. The consumer values consumption at time t according to the CEIS utility function $u(c_t) = \frac{c_t^{1-\sigma}-1}{1-\sigma}, \sigma > 0$. Lifetime utility is $U = \sum_{t=0}^{\infty} \beta^t u(c_t), \beta \in (0,1)$. The consumer starts the first period with zero assets. He/she can only invest in a bond that pays a net interest rate of r_t. Denote assets invested by a_{t+1}.

1. Consider first a two-period problem. The consumer receives income y in period 1. At the end of the period, he/she retires and uses savings to finance second-period consumption.

   (a) State the consumer's problem. Derive the Euler equation.

   (b) Show how the consumer's savings rate $s_t = a_{t+1}/y_t$ varies with the interest rate. Show how the direction of this effect depends on σ. Explain which two countervailing effects of an increase in r_t are at work here. Use the Euler equation to illustrate how they affect the consumption-savings decision.

2. Now consider an infinite-horizon economy consisting of two such consumers A and B, who have different elasticities of intertemporal substitution $1/\sigma_i$, $i = A, B$. They have the same endowment stream $\{y_t\}_{t=0}^{\infty}$. They again trade a risk-free bond each period and begin with zero asset holdings.

   (a) Carefully define a sequential competitive equilibrium for this economy.

   (b) Suppose that the endowment stream is constant: $y_t = y$ for all t. Show that, in this case, there is an interest rate supporting a steady state equilibrium. Express the steady-state interest rate in terms of primitives.

   (c) Now suppose that the endowment grows over time: $y_{t+1} = (1+g)y_t, g > 0$, for all t. Show why in this case, there is no steady state (defined as an equilibrium in which the consumption of both consumers grows at the same rate). Explain.

   (d) Explain in words what a (non-steady state) equilibrium could look like.

   (e) Now suppose that there is a continuum of agents. Each agent has an EIS $1/\sigma_1$ or $1/\sigma_2$, and faces a constant probability π that this elasticity changes. Explain intuitively: What is the distribution of elasticities in the population? What does this imply for the distribution of assets? What is the difference between this case and the case with constant elasticities? How does π affect the asset distribution?

Long question 2 (60 points)

Consider an economy populated by a continuum of household-producers in $[0, 1]$ with preferences

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $\beta \in (0, 1)$ is the discount factor, c_t is consumption, and $u(\cdot)$ is an instantaneous utility function assumed to be strictly increasing and strictly concave, and which satisfies the Inada conditions. The only consumption good in this economy is produced according to

$$y_t = z_t f(k_{t-1}),$$

where y_t is output, z_t is the stochastic level of productivity, k_t is the capital stock, and $f(\cdot)$ is a production function assumed to be strictly increasing and strictly concave, and which satisfies the Inada conditions. Assume that z_t is identically and independently distributed with mean zero and variance σ^2. The budget constraint of the representative household-producer (expressed in real terms) is

$$(1 + h(m_t))c_t + k_t + m_t = y_t + m_{t-1}/\pi_t + (1 - \delta)k_{t-1} + \tau_t,$$

where m_t is the real money stock, π_t is the gross inflation rate between periods $t - 1$ and t, τ_t is a lump-sum transfer from the government, and $\delta \in (0, 1)$ is the rate of depreciation. The initial stock of money and capital is given. The government is subject to the budget constraint

$$\tau_t = m_t - m_{t-1}/\pi_t.$$

Assume that the nominal money stock grows at the exogenous (gross) rate θ_t.

a) Interpret the term $1 + h(m_t)$ in the budget constraint of the household-producer. **(9 points)**

b) What are reasonable assumptions about the function $h(m_t)$? **(9 points)**

c) Write the optimization problem of the household-producer in recursive form. **(9 points)**

d) Find and interpret the first-order conditions for utility maximization. **(12 points)**

e) Impose general equilibrium conditions and derive the national accounting identity in this economy. **(9 points)**

f) Is money super-neutral in the deterministic steady state of this economy? **(12 points)**