McGill University
Department of Economics
Comprehensive Examination

Microeconomic Theory

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Location: Leacock 424
Time: May 28, 2015, 9:00 am—12:30 pm.

Instructions:

• This exam has two parts, A and B. Answer ALL four questions. The questions have equal weight.

• Calculators are allowed.

• No notes or texts are allowed.

• This exam consists of 3 pages, including this cover page.

Good luck!

Part A

1.

Let $X = [0,1]$. Binary relation R on X captures the relation that two numbers are "approximately the same", and satisfies the following six properties:

(1) For all $x \in X$, xRx.

(2) For all $x, y \in X$, xRy implies yRx.

(3) For all $x \in X$, the set $\{y \in X : yRx\}$ is closed in X.

(4) If $w \geq y \geq x \geq z$, then $wRz \Rightarrow yRx$.

(5) For all $x \in X$, there exists an interval around x such that yRx for all y in the interval.

(6) Define $M(x) =: \max\{y|yRx\}$ and $m(x) =: \min\{y|yRx\}$. Then M and m are weakly increasing functions and are strictly increasing whenever they do not have values 0 or 1.

Answer the following questions.

(a). Briefly explain in words the intuition behind each property. (Hint: Try to relate to concepts we learnt from class.) Do the properties fit your impression of "approximately the same"?

(b). For some strictly positive number ϵ, define a relation R_ϵ by $yR_\epsilon x$ if $|x-y| \leq \epsilon$. Does R_ϵ satisfy each of the six properties? Either prove or provide a counterexample to your answer. What does a larger ϵ mean?

2.

Let $X = \{x_1, \ldots, x_n\}$. Suppose that preferences $\succcurlyeq$ over the set of lotteries $\Delta(X)$ is defined by

$$\pi \succcurlyeq \rho \quad \Longleftrightarrow \quad \sum_{i=1}^{n} \left(\pi(x_i) - \frac{1}{n}\right)^2 \leq \sum_{i=1}^{n} \left(\rho(x_i) - \frac{1}{n}\right)^2.$$

That is, a DM with $\succcurlyeq$ prefers a lottery that is less dispersed relative to the uniform lottery $(\frac{1}{n}, \ldots, \frac{1}{n})$.

Either prove directly or provide a counterexample to the following statements:

(a). $\succcurlyeq$ is independent.

(b). $\succcurlyeq$ is convex.

(c). $\succcurlyeq$ is Archimedean.

Part B

1. *Bargaining*

Consider a random proposer version of the Rubinstein bargaining game played between Ann and Bob with an exogenous risk of delay. In each period Ann is selected as the proposer with probability 1/4 and Bob with probability 1/2. With probability 1/4 neither player is selected and the game moves to the next period. The proposer (if and when there is one) makes an offer (x_A, x_B) such that $x_A + x_B \leq 1$. The responder may then either accept the offer or reject it. Accepting the offer ends the game and Ann and Bob receive x_A and x_B, respectively in that period. Rejecting the offer leads the bargaining to continue to the next period. Ann and Bob have the same discount factor, δ.

Write down a subgame perfect equilibrium of this game.

2. *Sequential Equilibrium/Signaling*

Ann has two possible types, hard-working and lazy, with equal probability. Ann may choose to either finish high school or drop out. If she finishes high school, Bob must decide whether or not to hire Ann (Ann moves first, followed by Bob). Ann knows her own type while Bob simply knows either type to be equally likely. If Ann drops out both players get 0. If Ann finishes high school but is not employed by Bob then Bob gets 0 while Ann gets $-x$ if hard-working and $-y$ if lazy, where $y > x > 0$ and $1 > x$, but no further restrictions on y. If Ann finishes high school and is employed by Bob then Bob gets a if Ann is hard-working and b if Bob is lazy, with $a > b$ and $a > 0$. No further restrictions on b. Also Ann, in this case, gets $1-x$ if hard-working and $1-y$ if lazy.

For what values of a, b, x, y is there a

a) sequential equilibrium in which both types drop out?

b) separating equilibrium?