McGill University
Department of Economics
Comprehensive Examination

Microeconomic Theory

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Location: Leacock 429
Date and Time: Tuesday, June 3rd, 2014, 10:00am – 1:30pm

Instructions:

- This exam has two parts, Part A and Part B. In Part A, answer both questions, but between Questions 2(a)(iii) and 2(a)(iv), you only need to answer one. In Part B, answer any 2 questions out of the 3.
- Calculators are allowed.
- No notes or texts are allowed.
- This exam comprises 5 pages, including this cover page.

Good luck!

Part A

1.

Suppose a firm's possibility set $Y(q)$ is parametrized by a parameter $q \in \mathbb{R}$ as follows:

$$Y(q) = \{y \in \mathbb{R}^2_+ : 2\psi(q)y_1 + y_2 \leq 8\},$$

where $\psi : \mathbb{R} \mapsto \mathbb{R}_{++}$ is a continuous function such that $\psi(q) > 1$ for all q. (In particular, $\psi(q) > 1$ means $||y|| \leq 4$ for all $y \in Y(q)$ and all $q \in \mathbb{R}$.) So, any sequence of $y^j \in B(q^j)$ is bounded. Its profit $\pi^*(q)$ is defined by:

$$\pi^*(q) = \max_{y \in Y(q)} p \cdot y.$$

(a). Show that the correspondence $Y : \mathbb{R} \rightrightarrows \mathbb{R}^2_+$ is upper hemicontinuous.

(b). Show that the correspondence $Y : \mathbb{R} \rightrightarrows \mathbb{R}^2_+$ is lower hemicontinuous.

(c). Prove that π^* is continuous in q.

Suppose $X \subseteq \mathbb{R}^m$ and $Y \subseteq \mathbb{R}^n$.

A compact-valued correspondence $f : X \rightrightarrows Y$ is *upper hemicontinuous* if for any domain sequence $x_j \to x$ and any range sequence y_j such that $y_j \in f(x_j)$, there exists a convergent subsequence $\{y_{j_k}\}$ such that $\lim y_{j_k} \in f(x)$.

A compact-valued correspondence $\phi : X \rightrightarrows Y$ is *lower hemicontinuous* if for all $\{x_m\} \in X$ such that $x_m \to x \in X$ and $y \in \phi(x)$, there exist $y_m \in \phi(x_m)$ such that $y_m \to y$.

Bolzano–Weierstrauss Theorem. If $\{x_j\}_{j=1}^{\infty} \subseteq R^m$ is a bounded sequence, then there exists a convergent subsequence $\{x_{j_k}\}_{k=1}^{\infty}$.

Theorem of the Maximum, Berge. If $\phi : Y \to \mathbb{R}$ is a continuous function and $\Gamma : X \rightrightarrows Y$ is a continuous correspondence with nonempty and compact values, then $y^* : X \rightrightarrows Y$ defined by $y^*(x) = \arg\max_{y \in \Gamma(x)} \phi(y)$ is an upper hemicontinuous correspondence and $\phi^* : X \to \mathbb{R}$ defined by $\phi^*(x) = \max_{y \in \Gamma(x)} \phi(y)$ is a continuous function.

2.

Let X be a finite set.

(a). Suppose $\succcurlyeq$ is a preference relation on $\Delta(X)$, the set of lotteries on X.

(i) Provide a statement of the Independence axiom.

(ii) We say $\succcurlyeq$ is *convex* if

$$\pi \succcurlyeq \rho \text{ and } \sigma \succcurlyeq \rho \Longrightarrow \alpha\pi + (1-\alpha)\sigma \succcurlyeq \rho$$

for all $\pi, \rho, \sigma \in \Delta(X)$ and all $\alpha \in [0, 1]$. Either prove or provide a counterexample to the following TRUE OR FALSE statement: If $\succcurlyeq$ is independent, then it is convex.

Answer ONLY ONE of questions (iii) and (iv).

(iii) Suppose there exists function $u : X \mapsto \mathbb{R}_{++}$ such that

$$U(\pi) = \prod_{x \in X} u(x)^{\pi(x)},$$

is a utility representation of $\succcurlyeq$. Either prove or provide a counterexample to the following TRUE OR FALSE statements: (i) $\succcurlyeq$ is independent. (ii) $\succcurlyeq$ is convex.

(iv) Suppose $\succcurlyeq$ has the following utility representation:

$$U(\pi) = \frac{1}{2} \max_{x \in X} \pi(x) + \frac{1}{2} \min_{x \in X} \pi(x).$$

Either prove or provide a counterexample to the following TRUE OR FALSE statements: (i) $\succcurlyeq$ is independent. (ii) $\succcurlyeq$ is convex.

(b). Suppose $\succcurlyeq$ is a preference relation on $2^X \setminus \emptyset$, the family of nonempty subsets of X, satisfying the following two conditions:
Monotonicity. If $B \subseteq A$, then $A \succcurlyeq B$.
Lower union closure. If $A \succcurlyeq B$ and $A \succcurlyeq C$, then $A \succcurlyeq B \cup C$.
Prove that there exists a function $v : X \mapsto \mathbb{R}$ such that

$$U(A) = \max_{x \in A} v(x)$$

is a utility representation of $\succcurlyeq$, i.e., the decision maker evaluates a set by its best element.

Part B (50 points)

Answer any 2 qeustions.

Question 1. MATCHING: Consider a two-sided matching market where M is the set of men, W the is set of women, and each agent has strict preferences. Let μ^M be the M-optimal stable matching. Prove the following result (known as Blocking Lemma): Let μ be any individually rational matching and M' be the set of all men who strictly prefer μ to μ^M. Then if $M' \neq \varnothing$, there is a blocking pair (m, w) for μ such that $m \in M \setminus M'$ and $w \in \mu(M')$, where $\mu(M')$ is the set of women who are matched with men in M' under matching μ.

Question 2. SIGNALING: Consider two firms, an *incumbent* and a potential *entrant* in a market with a homogeneous good. The market exists only for 2 periods and in each period, the market inverse demand function is given by $P(Q) = 18 - Q$, where Q is the aggregate quantity on the market. In the first period, only the incumbent is in the market and its unit cost is either $c^h = 6$ or $c^\ell = 0$. After observing the first-period behavior (i.e., quantity choice) of the incumbent, the entrant decides whether or not to enter the market. The entrant's unit cost is $c = 3$, but it must also pay a fixed cost of $K = 20$ if it decides to enter the market. If the entrant enters, the two firms engage in a *Cournot* (i.e., quantity) competition in the second period. The discount factor is 1 so the incumbent maximizes the sum of profits from the two periods.

(1) Assume that the incumbent's unit cost is commonly known to both firms. For each of the two values of the incumbent's unit costs (i.e., $c^h = 6$ or $c^\ell = 0$), find the first-period quantity decision of the incumbent and both firms's second-period quantity decisions *if* the entrant enters the market. Will the entrant choose to enter the market?

Now assume that the incumbent's unit cost is unknown to the entrant.

(2) Can the first-period decisions made by the incumbent's two types in (1) be supported by a separating perfect Bayesian equilibrium? Explain.

(3) Show that there is a separating perfect Bayesian equilibrium in which the first-period quantity decisions are $q^h = 6$ (for high-cost type incumbent) and $q^\ell = 12$ (for low-cost type incumbent). Fully specify such an equilibrium.

(4) Assume that the probability that the incumbent has high cost ($c^h = 6$) is $\frac{1}{2}$. Is there a pooling equilibrium in which both types of incumbent produce the same quantity of 9 in the first period? What if the probability that the incumbent has high cost is $\frac{1}{10}$? Justify your answer rigorously.

Question 3. MORAL HAZARD AND INCENTIVE CONTRACTING: Consider an individual who owns a warehouse that is subject to fire damage. Suppose that the warehouse, if it burns, suffers a damage of $\ell = \$100,000$. The owner of this warehouse can take precautions against the event of fire – for simplicity, we assume that the owner can either "be careful" or "be negligent". The owner's decision whether to be careful or negligent affects the probability of a fire: if she is careful, the probability that a fire occurs is $\pi_1 = 0.1$, while if she is negligent, the probability of a fire is $\pi_2 = 0.4$.

The owner's preferences are described by a von Neumann and Morgenstern utility function $\sqrt{w - e}$ where w is her net wealth and e is her monetary disutility of taking precautions. Assume that $e = 0$ if she is negligent and $e = \$10,000$ if she is careful. The owner has an initial wealth of $w_0 = \$110,000$. (Thus, $w = w_0 - \ell$ if she has a fire and $w = w_0$ if she has no fire.)

The owner can insure herself against fire damage by purchasing insurance on a *competitive* market. The insurance companies, however, cannot monitor the owner's level of care. An insurance contract is a pair (p, s) where p is the premium to be paid in all contingencies and s is the amount reimbursed to the owner in case of a fire.

(1) Suppose that insurance companies are compelled to offer complete coverage. What insurance policy will the insurance companies offer? Will the warehouse owner buy insurance and what level of care will she take?

(2) Suppose that the insurance companies can offer insurance that insures only a half of the damage incurred. Show that the warehouse owner is better off in this case than if the insurance companies were compelled to offer complete coverage.

(3) Set up the problem of designing optimal insurance policy. (Since the market is competitive, the optimal policy has to make the warehouse owner as well off as possible.) How would you go about finding this optimal policy without using Lagrangian?

(4) Now, assume that the insurance market is monopolistic. Set up the problem of designing optimal insurance policy. How would you go about finding this optimal policy without using Lagrangian?