McGill University
Department of Economics
Comprehensive Examination

Microeconomic Theory

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Location: Leacock 424
Date and Time: Monday, June 3rd, 2013, 10:00am-1:30pm.

Instructions:

- This exam has two parts, A and B. In each part, answer Question 1 and answer either Question 2 or Question 3.
- Calculators are allowed.
- No notes or texts are allowed.
- This exam comprises 5 pages, including this cover page.

Good luck!

Part A (50 points)

1. (30 points)

   (a) (10 points) Consider $S = \{s_1, s_2, s_3\}$ and $X = \{x_1, x_2, x_3\}$. Suppose Aniko has a preference relation $\succcurlyeq$ on Anscombe-Aumann acts $(\Delta(X))^S$ that admits an Anscombe-Aumann expected utility representation. Denote each Anscombe-Aumann act as

$$h = \begin{bmatrix} (h_{s_1}(x_1), h_{s_1}(x_2), h_{s_1}(x_3)) \\ (h_{s_2}(x_1), h_{s_2}(x_2), h_{s_2}(x_3)) \\ (h_{s_3}(x_1), h_{s_3}(x_2), h_{s_3}(x_3)) \end{bmatrix}$$

Suppose we observe the following about Aniko's preferences:

$$\begin{bmatrix} (1,0,0) \\ (1,0,0) \\ (1,0,0) \end{bmatrix} \prec \begin{bmatrix} (0,1,0) \\ (0,1,0) \\ (0,1,0) \end{bmatrix} \prec \begin{bmatrix} (0,0,1) \\ (0,0,1) \\ (0,0,1) \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} (2/3,0,1/3) \\ (2/3,0,1/3) \\ (2/3,0,1/3) \end{bmatrix} \sim \begin{bmatrix} (0,1,0) \\ (0,1,0) \\ (0,1,0) \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} (1/2,0,1/2) \\ (0,1/3,2/3) \\ (2/3,1/3,0) \end{bmatrix} \sim \begin{bmatrix} (5/12,1/6,5/12) \\ (5/12,1/6,5/12) \\ (5/12,1/6,5/12) \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} (1/2,0,1/2) \\ (0,2/3,1/3) \\ (1/3,2/3,0) \end{bmatrix} \sim \begin{bmatrix} (1/3,1/3,1/3) \\ (1/3,1/3,1/3) \\ (1/3,1/3,1/3) \end{bmatrix} \tag{4}$$

Find Aniko's subjective belief $\mu \in \Delta(S)$ and vNM utility indices (unique up to positive affine transformation)? Show your calculations.

   (b) (10 points) Suppose $S = \{s_1, s_2\}$ and X is finite. Antonio has a preference relation $\succcurlyeq$ on Anscombe-Aumann acts $(\Delta(X))^S$ that admits the following rank-dependent expected utility representation:

$$V(h) = \frac{1}{4}\max\{E_{h_{s_1}}(u), E_{h_{s_2}}(u)\} + \frac{3}{4}\min\{E_{h_{s_1}}(u), E_{h_{s_2}}(u)\}$$

where $u : X \to R$ is a vNM utility index, $h_{s_i} \in \Delta(X)$ is the corresponding roulette lottery in state s_i, and $E_{h_{s_i}}(u)$ is the expected utility of lottery h_{s_i}. Intuitively, Antonio first ranks the states by the expected utility of corresponding roulette lotteries, and then evaluates an Anscombe-Aumann act by assigning a higher weight (3/4) to the lower-ranked state.

Prove or provide counterexample to the following statements: (i) $\succcurlyeq$ is independent. (ii) $\succcurlyeq$ is convex.

(c) (10 points) Suppose $S = \{s_1, s_2\}$. Alvaro is endowed with 10 dollars to purchase two different commodities: commodity i pays a prize if state s_i realizes and pays nothing otherwise ($i \in \{1, 2\}$). Let $p = (p_1, p_2)$ be the price vector and $x = (x_1, x_2)$ be Alvaro's demand decision. Suppose we observe the following data about Alvaro's choices:

	x	p
1	(2, 3)	(2, 2)
2	(1, 3)	(1, 3)
3	(2, 2)	(3, 2)
4	(4, 3)	(1, 2)

Are Alvaro's choices consistent, i.e., can they be rationalized by some (locally non-satiated) utility function? Justify your answer.

2. (20 points) Suppose Adam and Bette live in a two-person, two-good pure exchange economy. Let (x^A, x^B) be an allocation, where $x^A = (x_1^A, x_2^A)$ and $x^B = (x_1^B, x_2^B)$. Suppose Adam has standard preferences given by

$$u^A(x_1^A, x_2^A) = \ln(x_1^A + 2) + \ln(x_2^A + 2).$$

Bette's utility function is

$$u^B(x_1^B, x_2^B) = \ln(x_1^B + 1) + \ln(x_2^B + 1) + 0.7 \ln(x_2^A + 2).$$

That is, she enjoys Adam's consumption of the second good (but not the first). Adam's endowment is $e^A = (2, 3)$; Bette's endowment is $e^B = (3, 2)$.

(a) What are the set of Pareto-efficient allocations? (Show your calculations.)

(b) In a Walrasian equilibrium, each agent chooses utility-maximizing consumption bundles subjected to his/her own budget constraint and taking what the other agent chooses as given. Find the set of competitive equilibria (prices and allocations) of this economy. (Show your calculations.) Does the first welfare theorem hold in this economy? Explain why.

3. (20 points) Consider a dynamic economy with uncertainties. Assume that there are two periods, date 0 and date 1. S possible states of nature might occur at date 1. There is one commodity, to be consumed in each state at date 1. There are I agents, each with vNM expected utility $U(c^i) = \sum_s \pi_s u(c_s^i)$, where the vNM utility index $u : R_+ \to R$ is twice continuously differentiable, strictly increasing, strictly concave, and $\lim_{c \to 0} u'(c) = +\infty$; π_s is the (common) subjective probability of state s happening at date 1 (and vector $\pi = (\pi_1, \ldots, \pi_S) \in \Delta(S)$); c_s^i is agent i's date-1 consumption contingent on state s. Assume the full set of arrow securities are traded at date 0. Finally, agent i is endowed with $\omega^i = (\omega_1^i, \ldots, \omega_S^i) \in R_+^S$. Aggregate endowment is $\bar{\omega} = \sum_i \omega^i = (\bar{\omega}_1, \ldots, \bar{\omega}_S) \in R_{++}^S$.

Prove that every competitive equilibrium consumption allocation $(c^{1*}, c^{2*}, \ldots, c^{I*}) \in R_+^{SI}$ satisfies the following two properties:

(i) Measurability: for all s, s', i, if $\bar{\omega}_s = \bar{\omega}_{s'}$, then $c_s^{i*} = c_{s'}^{i*}$.

(ii) Strong monotonicity: for all s, s', i, if $\bar{\omega}_s > \bar{\omega}_{s'}$, then $c_s^{i*} > c_{s'}^{i*}$.

Part B (50 points)

1. (25 points) Consider a simple model of bilateral trade between two agents: a buyer and a seller who owns a single indivisible object. The buyer's valuation of the object is v_b while the seller's valuation is v_s. Each agent's valuation is private information and is drawn independently from the uniform distribution on $[0, 1]$.

   The agents have decided to use the following double-auction mechanism: The seller submits a sealed bid or asking price p_s and simultaneously, the buyer submits a sealed bid or offer price p_b. If $p_b \geq p_s$, the agents trade at price $p = kp_b + (1-k)p_s$ where $k \in [0, 1]$ is a fixed parameter both agents know; otherwise, there is no trade. Both agents have quasi-linear utility functions. For example, if there is trade at price p, the buyer's utility is $v_b - p$.

   (a) Find the Bayesian Nash equilibrium of the above mechanism where each agent's bid is a linear function of his/her valuation.

   (b) What social choice function or allocation rule does the above equilibrium of the mechanism implement? Is it ex post efficient? What is the probability of trade in the equilibrium?

   (c) By varying parameter k, the probability of trade can be changed. What is the optimal k that maximizes the probability of trade?

2. (25 points) Consider a market in which a monopolist produces and sells a single product. The monopolist can be one of the two types: a high-quality type who produces high-quality product at a unit cost of 6 or a low-quality type who produces low-quality product at a unit cost of 4. (For simplicity, assume that there is no fixed cost.) Each consumer's valuation of a unit of the product is 10 if it is of high quality and 0 if it is of low quality. If a consumer buys the product, her utility is her valuation minus the price she pays. The monopolist knows his type, whereas a consumer learns the monopolist's type only if she trys the product. The prior probability that the monopolist is high-quality type is 0.5.

   The market exists for two periods. In each period, the monopolist chooses whether or not to produce and what price to charge if he produces; subsequently, a consumer decides whether or not to buy 1 unit of the product. For simplicity, assume that only the consumers who bought the product in the first period can buy in the second period; moreover, if a consumer buys the product in the first period, she learns the quality of the product and hence the monopolist's type. Thus, in the second period, she does not repeat her purchase if the quality is low and the price is strictly positive; she repeats her purchase if the quality is high and the second-period price does not exceed her valuation. Both the consumers and the monopolist have a common discount factor $\delta < 1$. All the above information is common knowledge.

The monopolist can charge different prices in the two periods. For example, the high-quality monopolist can signal his type through an "introductory price": He charges a lower price (possibly below his unit cost) in the first period than in the second period to induce the consumers to buy his product and learn his type.

(a) Assume $\delta = 0.8$. Does there exist a separating (perfect Bayesian) equilibrium in which different types of the monopolist charge different prices in the first period? If so, fully specify such an equilibrium; if not, provide a proof. Does there exist a pooling (perfect Bayesian) equilibrium in which different types charge the same price in the first period? If so, fully specify such an equilibrium; if not, provide a proof.

(b) Assume $\delta = 0.4$. Does there exist a separating equilibrium in which different types charge different prices in the first period? If so, fully specify such an equilibrium; if not, provide a proof.

3. (25 points) Dr. Watson decides to go to an insurance company to insure his car against accidents. From the insurance company's perspective, Dr. Watson could be either a safe driver or a reckless driver. The probability that Dr. Watson is a safe driver is $t \in (0, 1)$. The probability that a safe driver will suffer an accident is $p_s = 1/3$, while the probability that a reckless driver will suffer an accident is $p_r = 1/2$. Assume that there are many insurance companies in the market (so the market is competitive) and that they are all risk-neutral. Dr. Watson' utility function is $u(x) = \ln x$, where x represents his net wealth. Dr. Watson's car is equivalent to a wealth of 64 and an accident results in a loss of 63. Suppose that Dr. Watson does not have any other wealth. All insurance companies offer contracts that specify a premium ρ and a coverage amount q if an accident occurs. Dr. Watson will obviously choose the contract (ρ, q) that he most prefers.

(a) What contracts will the insurance companies offer if they can identify whether Dr. Watson is safe or reckless (i.e., information is symmetric)? Will Dr. Watson be fully insured? Explain your answer.

(b) Will the insurance companies offer the contracts obtained in (a) if they cannot identify whether Dr. Watson is safe or reckless (i.e., information is *asymmetric*)? Explain.

(c) Can there exist a pooling equilibrium (in which Dr. Watson buys the same contract regardless of his risk type) under asymmetric information? Explain your answer.

(d) Identify the contracts that could comprise a separating equilibrium (in which Dr. Watson buys different contract depending on his risk type) under asymmetric information. Will Dr. Watson be fully insured?

(e) Let $t = 2/3$. Does the contract obtained in (d) constitute an equilibrium? Explain.