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Volatility and expected option returns: A note

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ABSTRACT

We show analytically that the relationship between asset volatility and expected option return is ambiguous. Numerical results elaborate how the direction and magnitude of the relationship depend on asset beta and volatility levels, and option moneyness and maturity.

1. Introduction

Empirically cross-section of option returns appear related to volatility of the underlying asset (Goyal and Saretto, 2009; Vasquez, 2012; Cao and Han, 2013; Karakaya, 2014). Prior theoretical research (Galai and Masulis, 1976; Johnson, 2004; Broadie et al., 2009) suggests an inverse (direct) relationship between expected call (put) option return and asset volatility, that is, a negative (positive) return vega, for call (put) options.[1]

In this paper, we argue that the expected option return (*EOR*)-volatility relationship may depend on the source of volatility differential, namely idiosyncratic or systematic, the latter with an additional opposite effect via the physical expected return on the asset, popularly known as the *DRIFT*. Hu and Jacobs (2015), hereafter HJ, considered these effects separately using the Black–Scholes setup, holding either the *DRIFT* or the volatility constant.[2] In this paper, we extend HJ's important work by considering the two effects simultaneously.

Our numerical results suggest that the strength and the direction of the *EOR* and volatility relationship depends critically on the level of asset beta and volatility and maturity and moneyness of the option.

2. Pricing framework

The payoffs at maturity T of the European call and put options with a strike price of K and asset price of S_T at maturity are:

$$C_T = Max[0, S_T - K] \quad (2.1)$$

$$P_T = Max[0, K - S_T]. \quad (2.2)$$

In the Black–Scholes option pricing model, the physical asset price dynamics is described by the Geometric Brownian Motion:

$$dS/S = \mu\, dt + \sigma\, dz \quad (2.3)$$

where μ is the asset's *DRIFT*, σ is the volatility of continuously compounded asset return and dZ is a Standard Normal variable.

Under the above dynamics and given risk-free rate of r and dividend yield of q on the asset the expected option payoffs under the risk-neutral (Q) and physical ($*$) probability distributions are:

$$E^Q(C_T) = S\, exp\{(r-q)T\}N(d_1) - KN(d_2) \quad (2.4)$$

$$E^Q(P_T) = K[1 - N(d_2)] - S\, exp\{(r-q)T\}[1 - N(d_1)] \quad (2.5)$$

$$E^*(C_T) = S\, exp\{(\mu-q)T\}N(d_1^*) - KN(d_2^*) \quad (2.6)$$

$$E^*(P_T) = K[1 - N(d_2^*)] - S\, exp\{(\mu-q)T\}[1 - N(d_1^*)] \quad (2.7)$$

with $d_1 = ln(g)/\sigma\sqrt{T} + 0.5\sigma\sqrt{T}$, $g = S\, exp\{(r-q)T\}/K$, and $d_2 = d_1 - \sigma\sqrt{T}$, and $d_1^* = ln(g^*)/\sigma\sqrt{T} + 0.5\sigma\sqrt{T}$, $g^* = S\, exp\{(\mu-q)T\}/K$, and $d_2^* = d_1^* - \sigma\sqrt{T}$.

The arbitrage-free option prices are:

$$C = exp(-rT)E^Q(C_T) \quad (2.8)$$

$$P = exp(-rT)E^Q(P_T). \quad (2.9)$$

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[1] Coval and Shumway (2001) considered expected option returns in relation to moneyness.

[2] Entertaining a more general stochastic volatility option pricing model is not important in this regard. See Broadie et al. (2009) and Hu and Jacobs (2015).

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Following the conventional definition of gross return, the expected gross option return is the physical expectation of option payoff at maturity relative to its current price (the discounted risk-neutral expectation, Eqs. (2.8) and (2.9)). Accordingly, the expected gross call (*ECOR*) and put (*EPOR*) option returns are given by:

$$ECOR = E^*(C_T)/C = exp(rT)E^*(C_T)/E^Q(C_T) \quad (2.10)$$

$$EPOR = E^*(P_T)/P = exp(rT)E^*(P_T)/E^Q(P_T). \quad (2.11)$$

In the Mean–Variance/CAPM context, $\mu = r + \lambda\ \sigma$, with $\lambda = \rho\ \lambda_M$, where ρ is the stock's correlation with the market portfolio and λ_M is the price of unit volatility risk. It is seen that $d_1^* = d_1 + \lambda\sqrt{T}$ *and* $d_2^* = d_2 + \lambda\sqrt{T}$.

3. Volatility and expected option return

HJ hold the asset risk premium unchanged as the asset volatility changes. The general vegas without this restriction are:

$$Call\ Return\ Vega : \partial ECOR/\partial\sigma = exp(rT)X_C/[E^Q(C_T)]^2 \quad (3.1)$$

$$Put\ Return\ Vega : \partial EPOR/\partial\sigma = exp(rT)X_P/[E^Q(P_T)]^2 \quad (3.2)$$

$$\text{where } X_C = E^Q(C_T)\partial E^*(C_T)/\partial\sigma - E^*(C_T)\partial E^Q(C_T)/\partial\sigma \quad (3.3)$$

$$\text{and } X_P = E^Q(P_T)\partial E^*(P_T)/\partial\sigma - E^*(P_T)\partial E^Q(P_T)/\partial\sigma. \quad (3.4)$$

ECOR will decrease with volatility if $X_C < 0$ and *EPOR* will increase with volatility if $X_P > 0$. The expressions X_C and X_P are similar to but more general than HJ's corresponding sign-determining expressions *EX* and *B*:

$$exp(-rT)X_C = S\ exp\{(\mu - q)T\}\sqrt{T}[EX + CN(d_1^*)\lambda\sqrt{T}] \quad (3.5)$$

$$exp(-rT)X_P = S\ exp\{(\mu - q)T\}\sqrt{T}[B - PN(-d_1^*)\lambda\sqrt{T}]. \quad (3.6)$$

The additional terms $CN(d_1^*)\lambda\sqrt{T}$ and $PN(-d_1^*)\lambda\sqrt{T}$ are due to the *DRIFT*/systematic (varying *DRIFT*) effect of volatility. For positive systematic risk assets ($\lambda > 0$), they are positive. Call (put) return vega will remain negative (positive) as in the case of idiosyncratic risk, if the magnitude of the *DRIFT* effect $CN(d_1^*)\lambda\sqrt{T}$ ($PN(-d_1^*)\lambda\sqrt{T}$) is small relative to *EX* (*B*).

4. Numerical analysis

Given the lack of analytical intractability, we numerically consider the various possibilities using the CAPM for asset prices and the Black–Scholes model for option prices. We assume $r = 5\%$, $q = 0\%$, $\sigma_M = 20\%$, $\lambda_M = 0.35\%$ (market portfolio risk premium of 7%), and $\rho = 0.50$ ($\lambda = 0.175\%$). Alternative asset betas considered are 0.10, 0.50, 1.00, 1.50 and 2.00. Asset volatility (σ) is varied from 10% to 110% in increments of 0.10%. With the initial stock price (S) set at $100, moneyness ($K/S$) is varied from 0.70 to 1.30 leading to 1001 different strike prices (K) from $70 to $130 in increments of $0.06. With $T = 0.085$ (one month), 0.25 (three months) and 1.00 (one year), we evaluated a total of 15,030,015 option return cases (parameter combinations). After excluding cases with either call or put option price below $0.05, we were left with 13,895,625 cases.

For each case, two return vega measures (for each of call and put), *Vega_I* and *Vega_S*, are calculated, along with two corresponding sets of *DRIFT*, *ECOR* and *EPOR*. *Vega_I* is calculated in the idiosyncratic vega spirit of HJ, that is, the *DRIFT* effects $CN(d_1^*)\lambda\sqrt{T}$ and $PN(-d_1^*)\lambda\sqrt{T}$ are ignored. The measure *Vega_S* is calculated in the systematic vega spirit, that is, the *DRIFT* effects $CN(d_1^*)\lambda\sqrt{T}$ and $PN(-d_1^*)\lambda\sqrt{T}$ are included.

Given the way we calculate and report the return vega, it is the change in *EOR* (in percentage) for 1% increment in volatility (in percentage). For the sake of brevity and ease of discussion, Table 1 presents summary statistics on expected asset returns, beta (varying with volatility), expected option returns and the return vega measures. For this purpose, we classified the volatility range into five buckets, *Lowest* (10%–30%), *Low* (31%–50%), *Middle* (51%–70%), High (71%–90%) and *Highest* (91%–110%). Similarly, five strike price buckets were formed based on *K* (same as percentage of asset price $S = 100$), *Lowest* (70–82), *Low* (83–94), *Middle* (95–106), *High* (107–118) and *Highest* (119–130).

4.1. Overall: panel A of Table 1

Considering the entire set of 13,895,625 cases, the average *Vega_I_C* is negative (−0.9171) and the average *Vega_I_P* is positive (+0.3214), as argued by HJ. But considering *DRFIT* effect, the average *Vega_S_C* is now positive (+0.0399) and *Vega_S_P* is positive but of a significantly lower value (+0.0609). Further, the differential between *Vega_I* and *Vega_S*, appears quite substantial in magnitude, +0.9570 (call) and −0.2605 (put), or 104.35% (call) and 81.05% (put) of *Vega_I*.

Given large standard deviations and wide ranges (Min to Max) of the EORs and return vegas, we now delve into various subsets.

4.2. Constant DRIFT (Beta) buckets: panel B of Table 1

As expected and similar to HJ's Table 4, with higher constant *DRIFT*, *ECOR_I* (*EPOR_I*) increases (decreases), and *Vega_I_C* (*Vega_I_P*) is negative (positive) in all buckets. An additional finding is that the strength of HJ-type volatility-*EOR* relationship (magnitude of return vega) is directly related to the constant *DRIFT* (asset beta) and is much stronger and steeper for call options than put options.

4.3. Volatility buckets: panel C of Table 1

Consistent with HJ the average *Vega_I_C* (*Vega_I_P*) is negative (positive) in all five volatility buckets, varying from −5.09 (+1.21) to −0.06 (+0.06). The return vega magnitudes indicate that the *EOR*-idiosyncratic volatility relationship is the strongest (weakest) in a cross-section of below (above) average volatility assets.

Vega_S_C is positive everywhere except the *Lowest Volatility* case. Thus, with the *DRIFT* related effect, a direct instead of an inverse volatility-*ECOR* relationship prevails. This is further supported by the average *ECOR* increasing with volatility, from 19.06% (*Low Volatility*) to 21.78% (*Highest Volatility*). For put options, the *DRIFT* effect does not reverse the direction, but reduces the return vega magnitude considerably.

4.4. Maturity buckets: panel D of Table 1

With the *DRIFT* effect, call (put) return vegas become less negative (positive) and reverses sign for 1.0 year maturity call options. Further, the term structure of *EOR* and return vega magnitude appear steeply upward (downward) sloping for call (put) options. Thus empirical work should consider liquid enough longer maturity options for stronger results.

4.5. Strike price buckets: panel E of Table 1

The average *Vega_I_C*s are all negative and the magnitude increases with higher strike price. The *DRIFT* effect reverses the sign of the vega from negative to positive for the *Lowest* (+0.15 instead of −0.08), *Low* (+0.13 instead of −0.17), and *Middle Strike* (+0.07 instead of −0.55) buckets. For the *High* (−0.03 instead of −1.35) and *Highest* (−0.12 instead of −2.42) *Strike* buckets, the return vega is considerably reduced but remains negative.

Table 1
Expected option return (*EOR*) and return Vega.

Constant beta	Volatility bucket	Time to maturity, year	Strike price bucket	*N*	Statistic	Constant DRIFT (%) (*DRIFT_I*)	Varying beta	Varying DRIFT (%) (*DRIFT_S*)	*ECOR* (%), Constant *DRIFT* (*ECOR_I*)	*ECOR* (%), Varying *DRIFT* (*ECOR_S*)	*ECOR Vega*, Constant *DRIFT* (*Vega_I_C*)	*ECOR Vega*, Varying *DRIFT* (*Vega_S_C*)	*EPOR* (%), Constant *DRIFT* (*EPOR_I*)	*EPOR* (%), Varying *DRIFT* (*EPOR_S*)	*EPOR Vega*, Constant *DRIFT* (*Vega_I_P*)	*EPOR Vega*, Varying *DRIFT* (*Vega_S_P*)
A. Overall:																
All	All	All	All	13,895,625	Average	12.14	1.58	16.06	19.57	20.36	−0.92	0.04	−8.13	−9.56	0.32	0.06
					Std Dev	4.76	0.69	4.82	38.36	13.27	8.89	0.23	13.20	5.01	0.62	0.16
					Min	5.70	0.25	6.77	0.59	3.04	−1,108.82	−4.93	−98.99	−37.02	0.00	−0.68
					Max	19.00	2.75	24.25	2340.06	69.88	0.00	0.94	4.68	−1.52	6.49	2.06
B. By constant beta:																
0.10	All	All	All	2,779,125		5.70	1.58	16.06	3.82	20.36	−0.05	0.04	1.20	−9.56	0.05	0.06
0.50	All	All	All	2,779,125		8.50	1.58	16.06	9.87	20.36	−0.28	0.04	−3.27	−9.56	0.22	0.06
1.00	All	All	All	2,779,125	Average	12.00	1.58	16.06	18.26	20.36	−0.71	0.04	−8.37	−9.56	0.36	0.06
1.50	All	All	All	2,779,125		15.50	1.58	16.06	27.68	20.36	−1.33	0.04	−13.00	−9.56	0.46	0.06
2.00	All	All	All	2,779,125		19.00	1.58	16.06	38.25	20.36	−2.21	0.04	−17.24	−9.56	0.52	0.06
C. By volatility buckets:																
All	Lowest	All	All	2,047,710		12.14	0.53	8.69	52.81	21.37	−5.09	−0.06	−24.37	−12.53	1.21	0.14
All	Low	All	All	2,851,470		12.14	1.00	12.03	20.35	19.06	−0.47	0.04	−10.60	−10.47	0.37	0.07
All	Middle	All	All	2,996,445	Average	12.14	1.50	15.51	13.96	19.34	−0.17	0.06	−5.74	−9.39	0.16	0.05
All	High	All	All	3,000,000		12.14	2.00	19.01	11.40	20.51	−0.09	0.06	−3.33	−8.55	0.09	0.04
All	Highest	All	All	3,000,000		12.14	2.50	22.51	9.93	21.78	−0.06	0.06	−1.90	−7.82	0.06	0.03
D. By time to maturity:																
All	All	0.085	All	4,232,185		12.14	1.67	16.70	6.40	7.92	−0.21	−0.01	−4.32	−5.81	0.13	0.02
All	All	0.250	All	4,708,020	Average	12.14	1.57	15.98	13.61	14.59	−0.58	0.00	−7.39	−8.84	0.26	0.04
All	All	1.000	All	4,955,420		12.14	1.51	15.58	36.49	36.47	−1.85	0.13	−12.09	−13.44	0.54	0.12
E. By strike price buckets:																
All	All	Lowest	All	2,469,410		12.14	1.70	16.89	12.32	16.56	−0.08	0.15	−10.71	−14.24	0.40	0.17
All	All	Low	All	2,835,395		12.14	1.56	15.95	14.22	17.15	−0.17	0.13	−10.65	−11.89	0.46	0.14
All	All	Middle	All	2,998,425	Average	12.14	1.50	15.51	18.18	19.15	−0.55	0.07	−8.74	−9.14	0.38	0.05
All	All	High	All	2,901,885		12.14	1.54	15.78	23.84	22.64	−1.35	−0.03	−6.16	−7.10	0.22	−0.01
All	All	Highest	All	2,690,510		12.14	1.62	16.32	28.82	26.13	−2.42	−0.12	−4.57	−5.90	0.14	−0.03

For put options, *Vega_S_P* diminishes significantly but remains positive for the *Lowest*, *Low* and *Middle Strike* buckets while reversing to negative values for the *High* and *Highest Strike* buckets.

HJ's empirical exercise concerns idiosyncratic volatility and close to at-the-money options. Further research may determine if their findings would hold with systematic volatility and various other option buckets.

5. Summary and conclusions

We provided an analytical expression for the volatility-expected option relationship (return vega) that includes both the idiosyncratic and the systematic risk (*DRFIT*) effects. Our numerical analysis shows that the relationship weakens considerably with the *DRIFT* effect and reverses the direction in some cases. Without the *DRIFT* effect, the relationship is strong among higher beta and lower volatility assets, and amidst longer maturity and out-of-the-money call and put options.

Our findings indicate that not just the volatility level of the underlying assets, but also the nature of volatility, idiosyncratic/unpriced or systematic/priced, is important for the cross-section of option returns.

Herskovic et al. (2016) find that a common idiosyncratic volatility (*CIV*) factor bears a negative risk premium. In that case, the *DRIFT* effect of volatility on expected option return should be weaker. More empirical research is thus called for the *EOR* effect of the source of volatility differential in the cross-section of underlying assets.

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