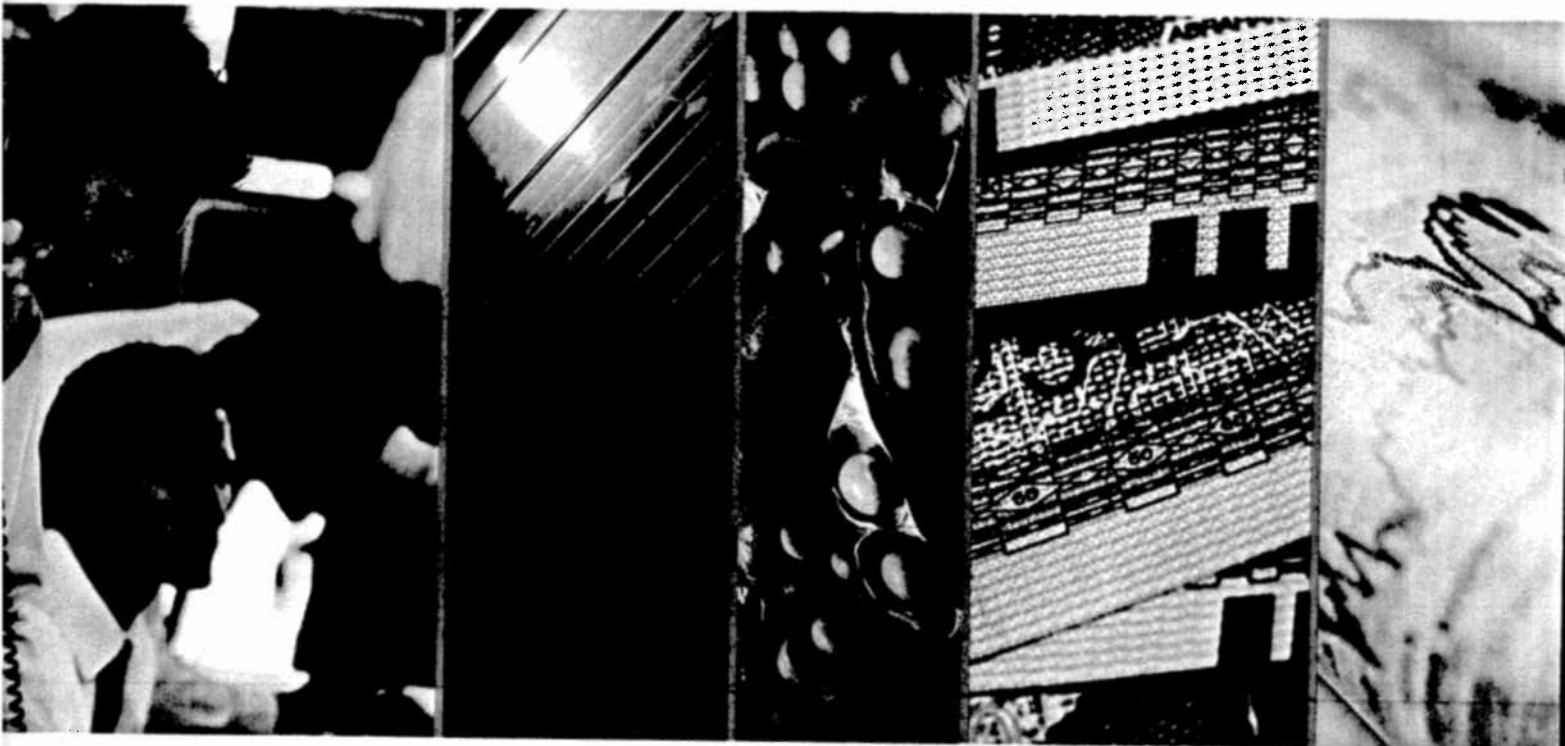


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Option Bid-Ask Spread and Liquidity

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The liquidity of securities markets has important bearings on the trading decisions and outcomes of market participants. Security prices could also be affected by liquidity if market participants demand a premium for bearing the risk of illiquidity. In this article, our focus is on the measurement of liquidity for options using a single quantitative measure. More specifically, we propose two alternative measures, both based on the end-of-day quoted dollar bid-ask spread of options. We then illustrate the usefulness of these measures using a sample of more than 2 million end-of-day option quotes for 31 large U.S. stocks (Dow Jones components plus Goldman Sachs).

The normally considered characteristics of a liquid market include high intensity of market activity (volume, number of traders and dealers/market makers, frequency of or time between orders and trades), good quality of order book (firm quotes, good depth of quotes, large order sizes at various limit prices), good quality execution (low market impact, speedy execution), and easy and quick clearing and settlement of trades. Such characteristics are not only likely related to each other, in some cases (e.g., effective firmness and depth of quotes, market impact), they are also difficult to measure. It is thus challenging to come up with a single quantitative measure that sensibly and summarily captures

the various characteristics of liquidity and yet is easy to observe, estimate and compare in a cross-section or over time. Such a measure of liquidity would arguably be more useful in the case of options given the intricate structure and arbitrage links of option prices, their diverse role as a trading instrument and the unique aspects of market making in options. Furthermore, from an empirical research perspective, detailed intraday micro-structure data on options are rather voluminous and not easily available. Thus, a meaningful quantitative measure based on the end-of-day dollar bid-ask spread could be quite useful to practitioners as well as researchers.¹

One candidate measure of option spreads is the dollar bid-ask spread relative to the mid-quote price, known as the relative spread. The argument for this measure rests on the premise that it measures the bid-ask component of the transaction cost per dollar of option investment. However, there are several reasons why such an argument lacks merit. First, although low transaction cost per dollar of investment could enhance market liquidity, for example, attracting more traders or frequency of trade, it is just one of the numerous factors that contribute to liquidity rather than a summary measure of overall market liquidity. Second, transaction cost per dollar of investment is clearly lower for a leveraged trader compared with an all-equity trader. This, however, has little to do with

market liquidity. Third, there are many exchange-traded futures contracts (e.g., futures on liquid stocks) with very low transaction cost per dollar of investment (margin), but these contracts have very shallow trading activity and cannot possibly be considered more liquid than the underlying stocks. Fourth and most importantly, in the case of option contracts, the ones with low transaction cost per dollar of investment (e.g., deep in the money options) see relatively modest trading activity in reality and are also lacking in other aspects of liquidity, such as firm quotes, good depth of quotes, speedy execution and so on.² If anything, this shows that for options, the importance of transaction cost per dollar of investment as a liquidity matter is only marginal in nature.

The relative spread measure is inherently biased toward finding lower-priced options as relatively illiquid when in reality such options are often the most liquid in terms of prompt and easy execution of trades.³ Accordingly, using the relative spread as a summary measure of option liquidity can lead to erroneous conclusions in empirical research. For example, if at- or out-of-the-money options are observed to have higher average returns on investment compared with in-the-money options, the latter having a lower relative spread and hence better liquidity according to this measure, one might inappropriately be tempted to conclude that option prices contain an illiquidity premium.

We propose two alternative summary measures of option liquidity, one using the implied volatility of the mid-quote option price to scale the dollar bid-ask spread of options, and the other expressing the bid, ask and mid-quote option prices in terms of respective implied volatilities. These measures have little imparted bias and can be used to compare the liquidity of options on the same asset at a given point in time, of the same option over time, or of options on different assets. Using more than 2 million end-of-day option quotes on 31 leading U.S. stocks (Dow Jones components plus Goldman Sachs) over the January 1996 to October 2010 period, we find that the relative liquidity ranking based on our measures is opposite to that based on using the mid-price as a deflator. More importantly, our measures produce liquidity rankings of options that are largely consistent with the well-known and widely perceived view of option liquidity.⁴ In addition, of course, the proposed measures are intuitively meaningful and easy to implement as they are based on widely reported observables. Accordingly, the proposed measures should be useful to

practitioners as well as researchers wanting a representative quantitative measure of option liquidity.

In what follows, after a summary review of the literature, the dollar and relative option spreads are discussed in detail. The two new measures of option liquidity are then proposed and some empirical results are presented. A summary and conclusions are provided at the end.

LITERATURE

Chordia et al. [2001] considered various measures of liquidity and trading activity for stocks based on quoted dollar bid-ask spread, transaction price, depths at the quoted bid and ask prices, daily volume and number of transactions. For equity options, multiple maturities and many strike prices within each maturity are available at a given point in time. Furthermore, if the underlying stock price moves significantly over time, new strikes are introduced along with the strikes that were available previously. As a result, a complete transaction database for equity options is rather voluminous and is not easily available either. But the best bid and ask prices at the end of trading day are now readily available for most equity options. As such, they offer an easy, fast, cost-effective and yet intuitively meaningful way of measuring option liquidity. One such measure that is also popular in equity and other asset markets is the relative spread. The relative spread for options can be calculated as the end-of-day quoted (best) bid-ask spread relative to the mid-quote option price. The mid-quote option price can be taken to be the arbitrage-free option price that would have prevailed in the absence of trading frictions. It is well known from the extensive option valuation literature that the cross-sectional structure of the arbitrage-free option prices and their time dynamics depend on the underlying asset price dynamics. To gain insights about the determinants of dollar bid-ask spreads of options, we start with a brief look at that literature.

Theoretical models of bid-ask spread focus on either the inventory costs or the asymmetric information costs of market making.⁵ In the inventory cost models, the bid-ask spread increases with the price level and the asset price volatility and decreases with trading volume. In the asymmetric information models, the bid-ask spread is related to the degree of the adverse selection problem. Back [1993], Biais and Hillion [1994], John et al. [1993] and Easley et al. [1998] extended

the asymmetric information approach to option bid-ask spreads. According to Cho and Engle [1999] and Kaul et al. [2001], the market makers hedge inventory and information risks in the underlying asset market, and thus, the extent of hedge and the costs of hedging determine the option bid-ask spread.

In their empirical studies, Neal [1987] and George and Longstaff [1993] considered variables believed to be related to the competitive market-making costs of the option market makers. Neal [1987] found that equity option dollar bid-ask spread is positively related to the premium level and negatively related to the contract volume; the evidence on the effect of implied volatility as a proxy for price volatility is mixed, however, as in the case of equity spreads.⁶ The explanatory power of the regressions in Neal [1987] ranged between 15% and 22%. The variables used by George and Longstaff [1993] in studying the index option spreads are the premium level, the time to maturity, the average time between trades during the day (402 daily trading minutes/number of transactions), the squared Black-Scholes-Merton delta of the option and a dummy variable representing the increase in tick size when the option premium exceeds \$3.00.⁷ They found higher premium level, shorter maturity, higher tick size for more expensive options, and lower liquidity or demand, as measured by the time between trades, to increase the bid-ask spread. However, they found it puzzling that a higher absolute delta reduces the option spread. In their sample year (1989), George and Longstaff [1993] found that their regression model explains 68.8% and 67.5% of variations in the dollar bid-ask spread of S&P 100 Index call and put options, respectively.

To summarize, there is a direct price level effect in the dollar bid-ask spread of options, and thus, if the spread is to be used as a measure of option liquidity, the spread relative to the mid-price is a candidate measure of such liquidity. It is, however, very important to note that the option price level is but one of a host of factors influencing the dollar bid-ask spread. To the extent the non-price factors do not share the same pattern as the option price, scaling the dollar spread by the mid-price may not reveal the true pattern of relative liquidity. For example, higher contract volume and less time between trades improve option liquidity (negative marginal influence on dollar spread), and it is empirically well known that the lower-priced options (at-the-money and out-of-the-money, shorter maturity) are

much more liquid according to these metrics than the higher-priced options (in-the-money, longer maturity). The price effect may still dominate the determination of dollar bid-ask spread and as such remains a viable scaling factor. Unfortunately, however, it will yield a liquidity pattern exactly opposite to the one based on volume and time between trades.

Mayhew [2002] argued that options with low volume may not lack liquidity because the market makers can hedge their risks using other options. This suggests a cross-effect in dollar spread and volume in so far as the market maker may lower the spreads of some options (with more volume) and attract trades in them to manage the risk of other options (with low volume). In addition to this indirect hedging cost of low-volume contracts, if they are also longer maturity and the hedging position is in shorter-maturity options, the latter need to be rolled over, imposing further hedging costs for the low-volume contracts. Furthermore, as the low-volume contracts often involve higher-priced options with higher absolute delta, hedging their delta risk in the underlying asset market would mean a larger asset hedge and hence cost more. In other words, the need for hedging and the costs of doing so are expected to be greater for the low-volume contracts. This is supported by the empirically observed negative effect of contract volume on dollar bid-ask spread. It is, therefore, not clear why the low-volume contracts should not be considered less liquid than the high-volume contracts.

OPTION SPREAD

In this section, we take a detailed look at the dollar and relative spread of options. We consider the inventory/hedging approach to spreads and follow with a brief discussion on the asymmetric information approach.

For simplicity, assume that there are just three call options on a stock, out of the money (K_1), at the money (K_2) and in the money (K_3), with $K_1 > K_2 > K_3$, all with the same time to maturity. A single market maker faces the problem of setting dollar bid ($C_{1B} = C_1 - 0.5X_1$, $C_{2B} = C_2 - 0.5X_2$, $C_{3B} = C_3 - 0.5X_3$) and ask prices ($C_{1A} = C_1 + 0.5X_1$, $C_{2A} = C_2 + 0.5X_2$, $C_{3A} = C_3 + 0.5X_3$) for these options around their no-arbitrage values (C_1 , C_2 , C_3) in the absence of market frictions. The market maker has a fixed cost A of running the business that is allocated to each option based on expected total volume of N ($=N_1 + N_2 + N_3$) options. There is also an

order handling cost of f per option leading to a fixed cost allocation of $F = f + (A/N)$ to each option. With probabilities p_1 , p_2 and p_3 , the market maker receives a matching trade for an option without much delay and hence generates the spread revenues X_1 , X_2 and X_3 per option; otherwise, the market maker hedges the pertinent risks at the hedging costs of H_1 , H_2 and H_3 , respectively. For simplicity, we assume that the volume and the probability of receiving a matching trade are exogenous to the spread.

We then assume that the competitive market maker sets the dollar spread to recover the average fixed cost and the expected hedging costs:

$$X_i = F + (1 - p_i) H_i, \quad i = 1, 2, 3$$

An unhedged option position would entail risk exposures to small and large changes in the underlying stock price and to a change in the implied (Black-Scholes) volatility of the option. These risks are commonly known as the delta, gamma and vega risks, respectively; the gamma and vega risks are different in magnitude but similar in nature, and these risks cannot be hedged using the underlying stock or its futures. Although implied volatilities do change and may in fact jump, we assume that the vega risk is negligible on an intraday basis. The exposures to changes in the risk-free rate and dividend yield, when they are random, are usually negligible for equity options. Accordingly, the option market maker needs to worry about managing only the delta and gamma risks of any open or unmatched option position. The hedging cost thus principally involves hedging the delta risk in the underlying asset market and the gamma risk using other options. As mentioned earlier in this article, using other options involves an opportunity cost of reduced spread there; we assume, equivalently, the hedging cost for an option is higher for hedging its gamma risk on a per unit basis.

Let us first consider the out of the money (K_1) option. Its delta and gamma risks are both small. Hence, the hedging cost H_1 is the smallest of all; in addition, if the probability p_1 of receiving a matched trade is high, the expected hedging cost for the out-of-the-money call option gets even smaller. In such a case, the average fixed cost will loom large in the dollar spread for the out-of-the-money option. Having said that, order imbalance may exist even for high-volume contracts making the probability of a matched trade smaller and the expected

hedging costs higher. For example, traders more often than not take long positions in out-of-the-money put options, making it harder (lower p) for the market maker to obtain a matched order. As the option becomes deeper out of the money, there may not be any material difference in the dollar bid-ask spreads as the size of the delta hedge for the unmatched positions becomes smaller but the probability of unmatched trades grows larger and the spread stabilizes to the average fixed cost plus a small expected hedging cost level.

The gamma risk of an at-the-money option is the highest and its delta risk is at the medium level. Additionally, given that the gamma risk of out- and in-the-money options are small, the indirect cost of hedging the gamma risk using other options is rather large for at-the-money options. Additionally, the probability p_2 of a matched trade is usually higher than for other options. The volume for these options is consistently high, and order imbalances are not that common.

The in-the-money option has little gamma risk but has the highest delta risk, requiring a large hedging position in the underlying asset. Except in instances of informed trading, to be discussed later, the volume of these options is generally low and does not have any known or persistent imbalance. But the likelihood p_3 of a quick matched trade is usually small because of infrequent orders for these options, thus the indirect cost of hedging might not be quite ignored. As the option becomes deeper in the money, the size of the delta hedge increases and the probability of matched trade drops, thus leading to an increasing expected hedging cost and, hence, dollar spread. The change in the delta gradually becomes negligible, however, and the already small likelihood of a matched trade may not change much, either. Thus, the dollar spread function may ultimately become flat. The wild card may, however, be the impact of informed trading on the dollar bid-ask spread. Because any such impact should be small for the index and exchange-traded fund (ETF) options, the bid-ask spread function will likely be flatter for the in-the-money options on these underlying assets than for the in-the-money equity options.

Let us now consider the impact of informed trading on option spread. It has been noted (Black [1975]; Easley et al. [1998]; Pan and Poteshman [2006]) that informed traders would prefer to trade in the option market. The market makers are of course not engaged in information acquisition. Hence, they would attempt to inter-

nalize the potential adverse selection cost by widening the dollar spread if they observe any abnormal increase in the trading volume (Easley and O'Hara [1992]; Kim and Verrecchia [1994]). The important issue, however, for the option bid-ask spread and liquidity is the relative importance of informed trading for various option buckets.

The production of material information about a stock is intrinsically sporadic in nature. And then, only some market participants would receive such material information at a given point in time, and even fewer would decide to trade on such information using options. Consequently, information-based trading in the options of a stock is occasional in nature at best and is unlikely to affect the spread setting of an option market maker on a day-to-day basis. Furthermore, large investors, because of their greater resources and connectivity to sources of material information, are more likely than the small/retail investors to obtain the occasional material information about a stock and then engage in information-based trading.

If the material information is directional in nature, as is often the case, for legal/governance reasons and also to remain under the radar while initiating and closing the trading positions, the large, informed traders might prefer to trade in options rather than in stocks. For this purpose, certain option strategies are likely to be more suitable for these large, informed traders. These are usually positions with high absolute delta and low absolute theta, such as positions in longer maturity in-the-money call or put options; sometimes money spreads involving in-the-money options and at-the-money options are preferred to economize on initial outlay (if net long) as well as to gain from volatility compression following the release of pending information. Such directional strategies with a minimal time value component might also be deemed suitable when the informed trader is unsure about the timing of wider dissemination of the material information. The degree of certainty about the direction of the material information, for its part, influences whether the large, informed trader would pursue a net debit (long) or credit (short) strategy.

The implication for this pattern of informed trading in options for an option market maker is that the occasional spikes in directional information-driven orders are more likely for the longer-maturity, in-the-money options. Therefore, any asymmetric information/adverse selection component, if at all, in the

market maker's bid-ask spread is likely to be higher for longer maturity, in-the-money options than for any other option bucket. By the very nature of these trades, the probability of obtaining matching orders on the other side of the spread is quite low, thus enhancing the hedging need of the market maker. Although these trades expose the market maker largely to the delta risk alone and, as such, a hedging position in the stock will be preferred, the stock hedge is large and needs to be maintained and rebalanced over an extended period of time, resulting in a relatively larger carrying cost. The large hedging cost combined with the low probability of a matched trade makes the market maker's expected cost of hedging larger for in-the-money options, more so for the longer maturity in-the-money options. Therefore, the dollar spread of these options is likely to be wider, on this account, than the other option buckets. Having said that, this differential would manifest sizably only when the market makers observe unusual ordering/trading activity; otherwise the differential is expected to be small on a day-to-day basis.

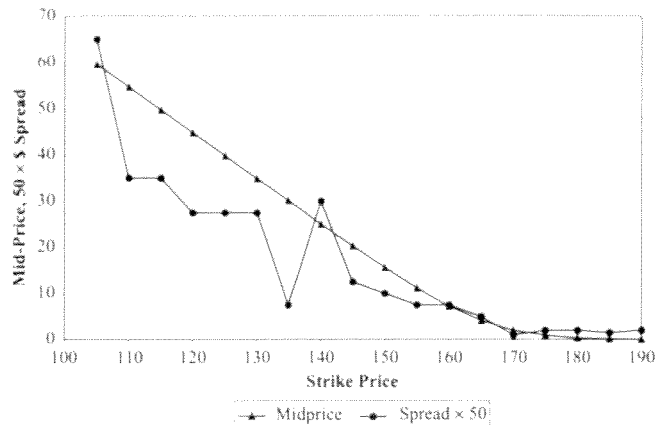
Overall, under usual circumstances, we could expect the expected hedging costs to be the smallest for out-of-the-money options. Although their delta risk cannot be ignored and their gamma risk is the highest, at-the-money options have the best chance of a matched trade, thus moderating their expected hedging costs. In contrast, for in-the-money options, the size of delta hedging is the largest, the probability of a matching trade is the lowest and the likelihood of informed trading is the highest; however, the need for gamma hedge, which is the costliest on a per unit basis, is minimal. Given that the average fixed cost allocation is the same, normally we would expect the dollar spread to be the widest for in-the-money options, followed by at-the-money and out-of-the-money options. However, the spread differential over at-the-money options may be small when information based trading is less likely.⁸ Empirically, this pattern of dollar spread prevails in most circumstances.

For the relative spread pattern, the critical issue is the rates at which the dollar spreads and the option prices (mid-prices) change across the option buckets. Because it is largely an empirical matter, we present in Exhibits 1 and 2 the February 18, 2011, closing mid-quote option price, the dollar bid-ask spread and the relative bid-ask spread percentage (of mid-quote option price) for April 15, 2011, maturity call options on IBM and the exchange-traded fund SPY, which tracks the

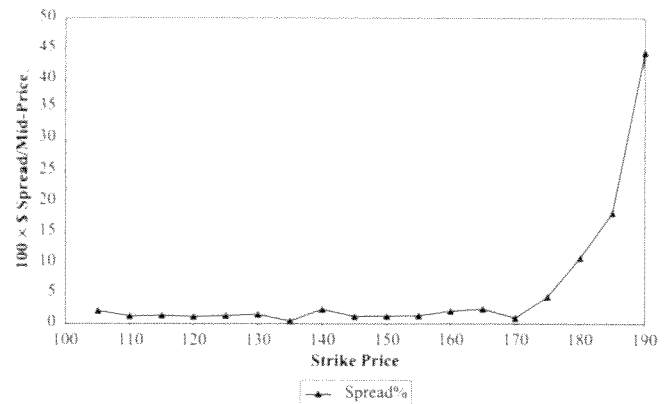
EXHIBIT 1

Closing Mid-Quote Option Price, Dollar Bid-Ask Spread and Relative Bid-Ask Spread% (of Mid-Quote Option Price) for April 15, 2011, IBM Call Options, as of February 18, 2011

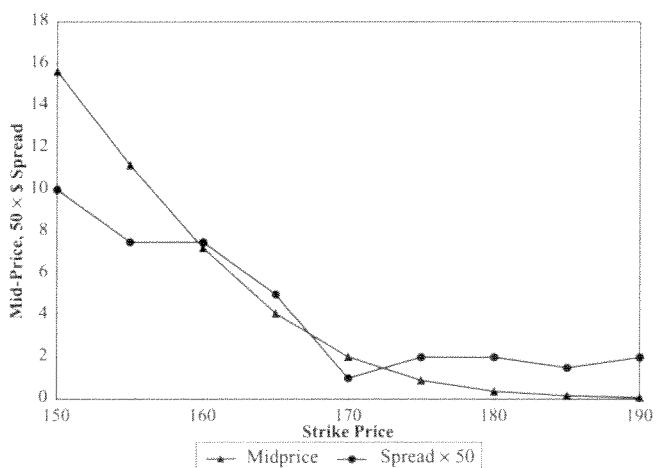
A. Mid-Quote Price and 50 × Dollar Bid-Ask Spread



C. Relative Bid-Ask Spread% (of Mid-Price)



B. Mid-Quote Price and 50 × Dollar Bid-Ask Spread, Strike > 150



D. Relative Bid-Ask Spread% (of Mid-Price), Strike < 170



Note: IBM Price: \$164.84 (February 18, 2011).

S&P 500 Index. These data are obtained from the Yahoo Finance website.

Although the dollar bid-ask spread generally increases (decreases) with the option (strike) price, this is not always the case. The SPY options show a much more variable pattern for in-the-money call options. For these options, over the entire range of strikes, the dollar bid-ask spread function looks almost like a step function, with the higher (lower) level associated with the in-the-money (out-of-the-money) options. The rather flat in-the-money segment for the SPY options and the much steeper in-the-money segment for the

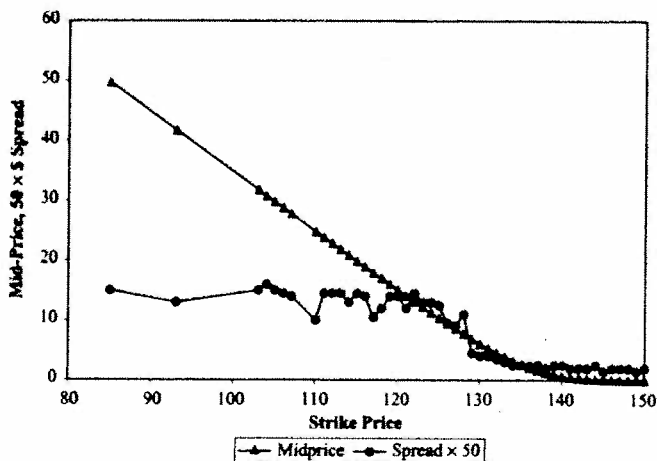
IBM options support the view that the adverse selection component of spread is more important for equity options than for ETF options.

For both series of call options, the dollar spread function is rather flat once the option becomes out of the money. This is consistent with the dollar bid-ask spread of out-of-the-money options representing largely the average fixed cost with possibly a minor expected hedging cost component. The mid-price function also becomes flatter in the out-of-the-money range, but as one would expect theoretically, the option price keeps falling. This is more clearly seen in Panel B for both

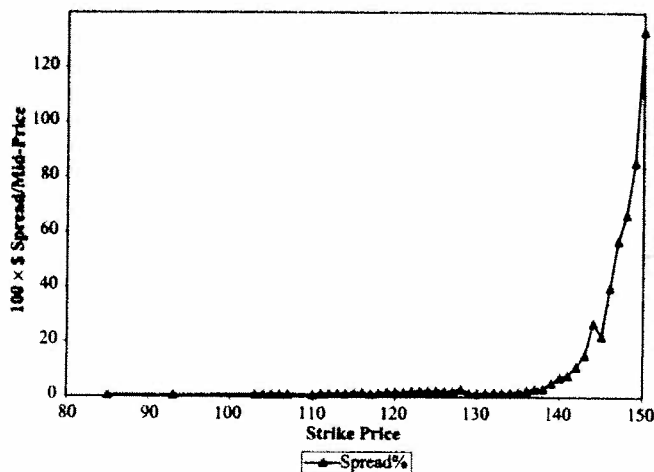
EXHIBIT 2

Closing Mid-Quote Option Price, Dollar Bid-Ask Spread and Relative Bid-Ask Spread% (of Mid-Quote Option Price) for April 15, 2011, Call Options on SPY, as of February 18, 2011

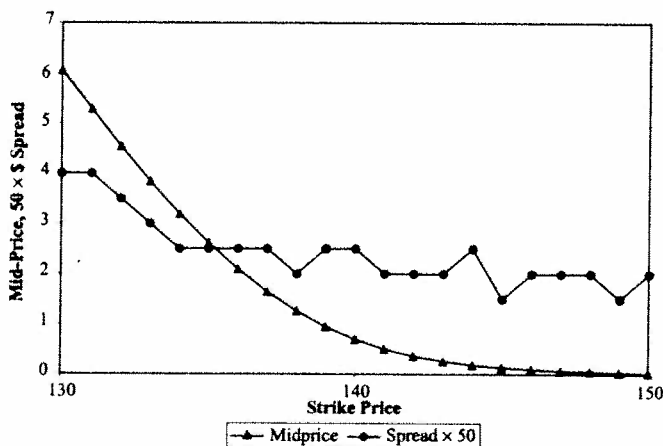
A. Mid-Quote Price and 50 × Dollar Bid-Ask Spread



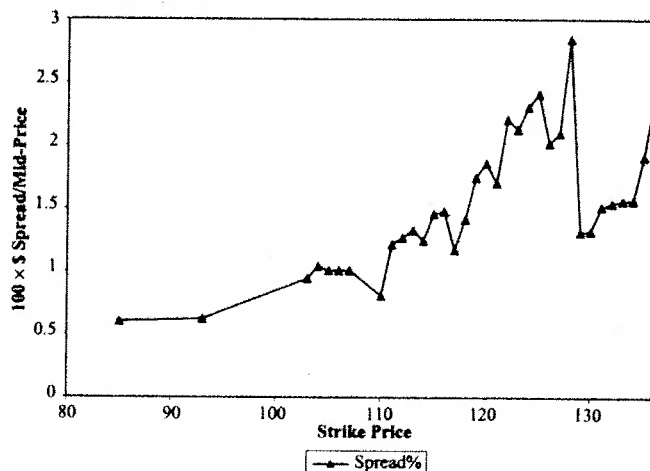
C. Relative Bid-Ask Spread% (of Mid-Price)



B. Mid-Quote Price and 50 × Dollar Bid-Ask, Strike > 130



D. Relative Bid-Ask Spread% (of Mid-Price), Strike < 136



Note: SPY Price: \$134.53 (February 18, 2011).

Exhibits 1 and 2, where we reproduce Panel A starting with the at-the-money range. As seen in Panel C for both Exhibits 1 and 2, a key implication of the flat dollar spread function combined with still falling mid-price of out-of-the-money options is that the relative (to mid-price) spread shoots up rather rapidly as it becomes deeper out of the money.

The slope of the mid-price and the bid-ask spread functions are roughly the same for strikes closer to the asset price, that is, in the traditional at-the-money range. Thus, as indicated by Panel C of both Exhibits 1 and 2,

there is no clear rising or falling pattern in the relative spread in this range. As indicated by Panel C and more clearly by Panel D of Exhibit 1, for IBM options, the same observation applies to the in-the-money range as well; the spread and the option price increases with moneyness roughly at the same rate, although the spread increase is somewhat irregular. It is, however, a different story for the in-the-money SPY options. We observe in Panel D of Exhibit 2 a general falling trend for the in-the-money SPY options as the strike price decreases and the option becomes deeper in the money. This is

due to a rather flat spread function for the SPY options with the option price function of course approaching an absolute slope of 1.0.

To summarize, given the structure of arbitrage-free option prices, it is the market maker's need for a hedge (inverse of the likelihood of a matched trade) and the relative importance of the various components of hedging cost that determine the behavior of the dollar spread relative to the option price. Important determinants of the dollar spread, such as the likelihood of a matched trade, the gamma risk and the likelihood of information-based trading, either have no monotonic relationship to the option price or are largely empirically determined. Thus, the pattern of spread relative to the mid-price says little about the components of dollar spread and, for that matter, about the relative liquidity of options. If at all, the spread relative to the mid-price is clearly biased toward a higher relative spread for options that have a relatively lower price, such as out-of-the-money (at-the-money) compared with at-the-money (in-the-money) options, and shorter-maturity compared with longer-maturity options.

In fact, comparison of liquidity based on the spread relative to the mid-price becomes perhaps even more challenging when comparing options on different assets. As an example, Exhibit 3 shows some statistics for the IBM and SPY call options we discussed previously.

EXHIBIT 3

Summary Statistics for the IBM and SPY Call Options

	Mid-Price	Spread	Spread%	Volume	Open Interest
IBM					
Total				2,057	34,589
Average	\$22.27	\$0.34	5.51	114.28	1,922
10% OTM	\$0.37	\$0.04	10.81		
ATM	\$4.10	\$0.10	2.44		
10% ITM	\$15.65	\$0.20	1.28		
SPY					
Total				62,586	338,851
Average	\$12.22	\$0.17	11.20	1331.62	7,210
10% OTM	\$0.04	\$0.03	85.71		
ATM	\$2.62	\$0.05	1.91		
10% ITM	\$14.09	\$0.24	1.70		

Note: OTM = out of the money; ATM = at the money; ITM = in the money.

Using spread% as a measure of liquidity, the SPY options would be considered overall less liquid than IBM options and especially so in the case of (10%) out-of-the-money options. However, as is clearly shown by volume and open interest, SPY options are decidedly much more liquid. Also note that the out-of-the-money and at-the-money options on IBM are more expensive, but the option price for the in-the-money IBM option is only slightly higher than the price of the in-the-money SPY option. In fact, the dollar spread of the in-the-money SPY option is greater than that of the IBM in-the-money option. We suspect this might be due to the differences in the implied volatility surface of the two option series. These observations, albeit based on very limited data, along with our earlier exposition indicate that it might be worthwhile to explore, as a measure of liquidity, the dollar spread scaled by the implied volatility.

NEW MEASURES OF LIQUIDITY

We propose two new measures of liquidity in this section and report some comparative empirical results for the new measures and the spread relative to the mid-price. The proposed first new measure is simply

$$\text{Spread relative to dollar volatility (SRDV)} \\ = 100 \times \text{Dollar spread} / \text{Daily dollar volatility of asset}$$

where

$$\text{Dollar volatility of asset} = \text{Underlying asset price} \\ \times \text{Implied volatility of option} \times \sqrt{(1/252)}$$

In this definition, the underlying asset price is in dollars and the option's implied volatility is in decimals. The new measure SRDV scales the dollar spread by the dollar volatility of the underlying asset price implied by the option. Suppose the dollar spread for an option is \$0.10, the option's implied volatility is 20% and the underlying asset price is \$50. Using 252 trading days, one standard deviation daily move in the stock is \$0.63 ($= \$50 \times 0.20 \times \sqrt{(1/252)}$). This results in a SRDV of 15.9%, meaning that the option's dollar bid-ask spread is 15.9% ($= \$0.10 / \0.63) of one standard deviation daily move in the stock. Accordingly, if the implied volatility is higher, the spread equals an even smaller (standard deviation) move in the stock, and the option should be viewed as more liquid by traders. In a similar fashion, if the implied volatility is the same for two options with

the same dollar spread, then the option on the higher-priced stock would have a lower SRDV, meaning a higher level of liquidity.

Notably, unlike the spread relative to mid-price, the new measures proposed here do not impart any bias in the liquidity measure based on the option price level. But there could be some effect of the volatility smile, although equity options are known to exhibit a rather flat volatility surface. In any case, one could adjust the proposed measures for any volatility smile bias by a multiplicative factor of average implied volatility of option bucket/average implied volatility of at the money options.

The second measure that we propose is

$$\begin{aligned} \text{Implied volatility relative spread (IVRS)} \\ = 100 \times (\text{Implied volatility of ask price} \\ - \text{Implied volatility of bid price}) \\ / \text{Implied volatility of mid-price} \end{aligned}$$

This measure is similar to the conventional dollar spread relative to the mid-price with the all-important difference that the quotes and mid-price are expressed in terms of the implied volatility units. In fact, in the over-the-counter foreign exchange options market, it is a standard practice to quote the option and standard option strategy (such as straddle, money spread) prices in terms of implied volatility units. Standardizing the prices in terms of respective implied volatilities, we avoid imparting any systematic bias into the liquidity measure. The SRDV measure may reproduce the same ranking as the dollar spread if the mid-price implied volatilities are not pronouncedly different across various option buckets. In these circumstances, IVRS could be helpful because it converts the dollar spreads into volatility spreads.

In our empirical estimation, we approximate the IVRS in the following manner:

$$\begin{aligned} \text{Implied volatility relative spread (IVRS)} \\ \text{approximation} = 100 \times [\text{Dollar spread} / (\text{Vega of} \\ \text{mid-price} \times \text{Implied volatility} \\ \text{of mid-price})] \end{aligned}$$

In order to compare the relative measures empirically, we calculate the liquidity measures for a sample of options. The sample comprises closing options and stock data, from the Wharton Research Data Services database, on 30 Dow Jones stocks plus Goldman Sachs

over the January 1996 to October 2010 period. The specific stock ticker symbols are as follows: AA, AXP, BA, BAC, C, CAT, CSCO, CVX, DD, DIS, GE, GM, GS, HD, HPQ, IBM, INTC, JNJ, JPM, KFT, KO, MCD, MMM, MRK, MSFT, PFE, PG, T, UTX, VZ and WMT. We filtered out any option data point for which no implied volatility figure was provided in the database or if the dollar spread was less than the tick size (\$0.05 for bid price < \$3.00 and \$0.10 for bid price > \$3.00); additionally, closing stock price had to be available and the call (put) option bid price had to be less than the stock (strike) price. These filters essentially screen out all questionable data points. We form six option buckets for each type (call, put) of option based on two maturity categories (short: 20 ≤ days to maturity ≤ 70 days; long: 71 ≤ days to maturity ≤ 180 days), and three moneyness categories (out of the money or OTM: 0.125 < Δ ≤ 0.375 for calls and -0.375 < Δ ≤ -0.125 for puts; at the money or ATM: 0.375 < Δ ≤ 0.625 for calls and -0.625 < Δ ≤ -0.375 for puts; in the money or ITM: 0.625 < Δ ≤ 0.875 for calls and -0.875 < Δ ≤ -0.625 for puts).

Average liquidity measures and options characteristics for the option buckets are presented in Exhibit 4 and auxiliary summary statistics are provided in Exhibit 5. As indicated by Exhibit 4, the new measure SRDV produces a liquidity ranking of option buckets that is exactly opposite to the ranking according to the spread relative to the mid-price. The SRDV measure ranks the shorter-maturity options as more liquid than longer-maturity options and out-of-the-money options as the most liquid followed by at-the-money and in-the-money options. The SRDV measure finds the shorter-maturity out-of-the-money put options to be the most liquid; this bucket is well known in practice for its strong insurance-related demand. Shorter-maturity out-of-the-money call options are also popular for covered writing purposes, and these options have the second-highest liquidity ranking according to the new measure. The volatility smile adjustment does not alter the liquidity ranking in any material way.

Equity options, however, do not normally exhibit sharp differences in implied volatilities of various option buckets. This is also visible in the average implied volatilities for various option buckets reported in Exhibit 4. As such, the liquidity ranking according to SRDV follows the dollar spread ranking in Exhibit 4. The liquidity ranking according to the IVRS measure shows that for call options, at-the-money (instead of out-of-the-money

EXHIBIT 4

Mean Dollar and Relative Spreads

A. Call														
Option Bucket	Option Bucket		Delta	Maturity (Days)	Option Midprice	Implied Volatility	Dollar Spread	Spread Relative to Price	Spread Relative to Dollar Implied Vol	Mean Spread Relative to \$Implied Vol Adjusted for Vol Smile	Implied Vol Relative Spread	Mean Implied Vol Relative Spread	Gamma	Vega
Maturity	Money	N	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Mean
All	All	1,095,327	0.537	96.60	\$5.185	34.13	\$0.227	7.92	19.24		8.03		0.0472	10.7256
Short	OTM	125,575	0.243	44.26	\$1.069	32.95	\$0.136	18.72	12.85	13.14	7.60	7.77	0.0579	7.1796
Short	ATM	118,027	0.502	44.74	\$2.956	34.07	\$0.184	7.44	15.48	15.31	7.00	6.93	0.0746	9.1828
Short	ITM	173,607	0.765	44.30	\$6.561	37.21	\$0.265	4.74	20.55	18.61	13.05	11.82	0.0558	6.8257
Long	OTM	191,127	0.250	127.57	\$1.593	31.38	\$0.147	12.63	14.86	15.96	5.11	5.49	0.0383	11.4071
Long	ATM	203,790	0.502	129.63	\$4.563	32.82	\$0.226	5.72	19.65	20.18	5.19	5.33	0.0451	15.3088
Long	ITM	283,201	0.762	128.81	\$9.967	35.59	\$0.318	3.70	25.48	24.12	9.58	9.07	0.0331	11.5737
B. Put														
Option Bucket	Option Bucket		Delta	Maturity (Days)	Option Midprice	Implied Volatility	Dollar Spread	Spread Relative to Price	Spread Relative to Dollar Implied Vol	Mean Spread Relative to \$Implied Vol Adjusted for Vol Smile	Implied Vol Relative Spread	Mean Implied Vol Relative Spread	Gamma	Vega
Maturity	Money	N	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Mean
All	All	1,105,028	-0.482	96.07	\$5.093	34.54	\$0.229	7.80	20.22		8.45		0.0486	10.5048
Short	OTM	158,486	-0.238	44.55	\$1.339	37.57	\$0.148	15.98	11.97	10.73	7.16	6.42	0.0506	7.1687
Short	ATM	120,257	-0.500	44.63	\$3.443	34.54	\$0.204	7.12	16.87	16.45	7.65	7.46	0.0753	9.0747
Short	ITM	146,979	-0.760	43.86	\$6.858	33.99	\$0.279	4.81	24.55	24.33	15.66	15.52	0.0662	6.5384
Long	OTM	267,847	-0.240	128.80	\$2.238	35.76	\$0.172	10.26	14.04	13.23	4.87	4.59	0.0314	11.9274
Long	ATM	201,100	-0.498	129.33	\$5.769	33.34	\$0.255	5.18	21.95	22.18	5.84	5.90	0.0462	15.0803
Long	ITM	210,359	-0.755	127.34	\$10.621	32.22	\$0.317	3.50	31.54	32.98	11.88	12.42	0.0438	10.4216

Notes: Closing daily option and stock data for the 30 Dow stocks plus Goldman Sachs over the January 1996 to October 2010 period. Any option data point without an implied volatility figure in the OptionMetrics database or with a dollar spread less than the tick size (\$0.05 for bid price < \$3.00 and \$0.10 for bid price > \$3.00) was screened out; additionally, closing stock price had to be available and call (put) option bid price had to be less than the stock (strike) price. Spread Relative to Price = $100 \times \text{Dollar bid-ask spread} / \text{Mid-quote option price}$. Spread Relative to Dollar Implied Vol = $100 \times \text{Dollar bid-ask spread} / [\text{Stock price} \times \text{Implied volatility in decimal} \times \sqrt{(1/252)}]$. Implied Vol Relative Spread = $100 \times \text{Dollar bid-ask spread} / (\text{Vega} \times \text{Implied volatility in decimal})$. All relative spreads are in percentages.

Sources: Wharton Research Data Services, OptionMetrics and CRSP.

according to SRDV) options are the most liquid, followed by out-of-the-money and in-the-money options, although the liquidity measure of out-of-the-money options is very close to that of at-the-money options. For put options, the IVRS and SRDV rankings are the same, with out-of-the-money put options being the most liquid. Using IVRS, however, the liquidity measure of at-the-money put options is much closer to that of out-of-the-money options.

It is also interesting to note that according to the IVRS measure, longer-maturity options are more liquid than shorter-maturity options, at least over the sample range considered here (i.e., 20 to 180 days). This could be due to the substantially higher vega of longer-maturity options, although their implied volatility is lower. In other words, when the sensitivity to volatility is high, a given dollar spread implies a low spread in terms of implied volatility, and scaling by the lower mid-quote implied volatility of longer-maturity options

does not quite offset the spread effect, thus resulting in a lower IVRS for longer-maturity options.⁹

SUMMARY AND CONCLUSIONS

In this article, we examined the merits of using the dollar bid-ask spread relative to the option mid-price as a measure of option liquidity. Because the option price is lower for shorter-maturity and out-of-the-money options, we found that scaling the dollar spread by the mid-price of the option injects a bias into the liquidity measure and ranks these options as the least liquid (most illiquid). Similarly, at-the-money options are ranked less liquid than in-the-money options according to this measure. These liquidity rankings are, of course, very hard to reconcile with the stylized fact that the shorter-maturity at-the-money and out-of-the-money options are consistently the favorites of equity option traders. By the same argument, using the spread rela-

EXHIBIT 5

Auxiliary Statistics (standard deviation, minimum, maximum) on Dollar and Relative Spreads

A. Call														
Option Bucket	Option Bucket		Dollar Spread	Dollar Spread	Dollar Spread	Spread Relative to Price	Spread Relative to Price	Spread Relative to Price	Spread Relative to Dollar	Spread Relative to Dollar	Spread Relative to Dollar	Impl Vol Relative Spread	Impl Vol Relative Spread	Impl Vol Relative Spread
Maturity	Money	N	StDev	Min	Max	StDev	Min	Max	StDev	Min	Max	StDev	Min	Max
All	All	1,095,327	\$0.181	\$0.050	\$9.800	8.38	0.19	200	18.01	1.02	851	6.0892	0.5293	548
Short	OTM	125,575	\$0.081	\$0.050	\$4.750	14.57	1.50	200	15.28	1.02	402	4.9813	0.6553	100
Short	ATM	118,027	\$0.125	\$0.050	\$5.000	4.46	0.79	200	15.86	1.23	477	4.0794	0.7070	152
Short	ITM	173,607	\$0.193	\$0.050	\$9.800	2.66	0.32	171	18.71	1.36	851	8.3947	0.7472	548
Long	OTM	191,127	\$0.096	\$0.050	\$5.000	9.28	1.14	200	15.20	1.10	502	3.1103	0.5906	94
Long	ATM	203,790	\$0.161	\$0.050	\$9.800	3.31	0.49	179	19.65	1.41	782	2.9418	0.5541	143
Long	ITM	283,201	\$0.228	\$0.050	\$9.800	2.09	0.19	128	17.92	1.43	795	6.2888	0.5293	349
B. Put														
Option Bucket	Option Bucket		Dollar Spread	Dollar Spread	Dollar Spread	Spread Relative to Price	Spread Relative to Price	Spread Relative to Price	Spread Relative to Dollar	Spread Relative to Dollar	Spread Relative to Dollar	Impl Vol Relative Spread	Impl Vol Relative Spread	Impl Vol Relative Spread
Maturity	Money	N	StDev	Min	Max	StDev	Min	Max	StDev	Min	Max	StDev	Min	Max
All	All	1,105,028	\$0.186	\$0.050	\$9.800	7.54	0.24	200	22.05	0.69	1191	7.7615	0.414121	819
Short	OTM	158,486	\$0.095	\$0.050	\$3.950	12.37	0.98	200	13.18	0.69	354	4.5990	0.662942	89
Short	ATM	120,257	\$0.145	\$0.050	\$9.800	4.16	0.57	169	17.42	1.23	687	4.3043	0.715405	182
Short	ITM	146,979	\$0.219	\$0.050	\$5.000	2.84	0.35	176	24.44	2.01	1191	11.0405	1.001139	819
Long	OTM	267,847	\$0.119	\$0.050	\$9.800	7.19	0.80	187	14.06	0.99	468	2.8149	0.558841	113
Long	ATM	201,100	\$0.181	\$0.050	\$9.800	2.91	0.37	160	21.64	1.59	829	3.2553	0.414121	278
Long	ITM	210,359	\$0.245	\$0.050	\$5.000	2.04	0.24	123	29.70	2.57	1131	8.5195	0.712488	567

Note: See Notes to Exhibit 4.

Sources: Wharton Research Data Services, OptionMetrics and CRSP.

tive to mid-price measure, it is also challenging to make meaningful comparison of the liquidity of options on different underlying assets.

The structure of option prices is well known. To better understand what drives the dollar bid-ask spread of options, we used a very simple model of inventory risk management by a competitive market maker, in line with the literature in this regard. According to this model, the odds of a matched trade and the relative importance of the costs of hedging the delta, gamma and adverse selection risks determine the behavior of the dollar spread relative to the option price. But these determinants of the dollar spread are determined empirically and do not appear monotonically related to the option price. Thus, scaling the dollar spread by the mid-price may not indicate much about the relative liquidity of options.

We introduce in this article two alternative measures of option liquidity. One of these two measures is the dollar spread relative to the dollar volatility of the stock, where the latter is estimated as the implied volatility of the option multiplied by the underlying asset price. This very simple measure scales the dollar spread

by two variables—the implied volatility specific to the option and the asset price common to all options on the same asset. These two variables are the most important determinants of the option price level, given the strike price and maturity. By not using the option price directly to scale the dollar spread, we avoid biasing the liquidity measure against lower-priced options. This new measure is also very intuitive because it measures the dollar spread in terms of a typical daily move in the underlying asset price. We also propose a second measure by expressing the bid, ask and mid-quote prices in terms of their respective implied volatilities and then expressing the quoted volatility spread as a proportion of the mid-quote implied volatility. This measure is in line with long-time industry price quotation practice in the over-the-counter foreign exchange options market. The implied volatility relative spread measure is also free from any imparted bias like the proposed measure based on dollar volatility. In addition, the implied volatility relative spread measure may be helpful in better discerning relative liquidity differences when the options do not exhibit a pronounced volatility smile.

Using end-of-day options data for 30 Dow Jones stocks and Goldman Sachs over the January 1996 to October 2010 period, we find that the new measures generally rank those options as more liquid than they are in reality known to be, according to activity-based measures, such as volume and open interest.

ENDNOTES

¹In a recent study, Grover and Thomas [2012] found that, for the Indian Nifty Fifty Index, a liquidity-weighted implied volatility index yields better forecast of realized volatility than the conventional vega or elasticity-weighted implied volatility indexes. They also provided a short review of the growing body of research concerning the effect of option market microstructure, including liquidity on market option prices or equivalently on the pattern of implied volatilities.

²According to practitioners (see www.optiontradingpedia.com/volume_and_open_interest.htm), "What liquidity of stock options contracts mean is how readily they can be bought and sold at the market price. Stock options contracts that are highly liquid, or heavily traded, can be instantly bought or sold at the prevailing market price easily and instantaneously while stock options contracts that are less liquid or 'thinly traded' tend to take a long time to fill and at very disadvantageous prices. ... In fact, if you hold very low liquidity stock options, it may even be impossible to find a buyer for those options!"

³This problem has been previously reported by Deuskar et al. [2011]. For OTC interest rate caps and floors, they noted that deep in-the-money options have lower relative bid-ask spreads (3%–4%), while some deep out-of-the-money options have bid-ask spreads almost as large as the price itself. Very importantly, as a reason, the authors recognized that some of the market maker's costs (transaction costs on hedges, administrative costs of trading and so on) are fixed and must be incurred regardless of the option premium level.

⁴According to practitioners (e.g., http://www.optiontradingpedia.com/volume_and_open_interest.htm), "The volume and open interest of each stock options contract provide an indication on their liquidity ... This suggests that at the money options are generally more liquid than in the money or out of the money options of the same stock and expiration. This phenomena is due to the fact that most speculative and hedging strategies uses at the money options more than in the money or out of the money options."

⁵Madhavan [2000] provided a review of these models.

⁶The use of implied volatility is preferred because volatility measured from transaction data is spuriously positively correlated with the bid-ask spread.

⁷The time between trades is a summary measure of liquidity or demand for options that may vary across strike price and maturity classes. Lower demand increases the average time between trades and hence the risk of uncovered option positions. The squared delta is an approximation for the variance in option price change that influences the magnitude of variation in the value of uncovered inventory positions. The premium level proxies for the incremental costs of changing inventory, and the time to maturity represents the greater risk of early exercise assignment for near maturity options as the option writers know about the assignment the next day.

⁸A testable implication is that the spread differential of in-the-money options over at-the-money options would widen prior to major information events, such as earnings releases.

⁹It is worth mentioning in this context that while gamma and vega share a similar pattern across different money buckets within a maturity (high for at-the-money and low for out-of-the-money and in-the-money), they actually move in the opposite direction across maturities within a money bucket. That is, although vega is higher for longer-maturity options, their gamma is lower compared with shorter-maturity options.

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