The Market Value and Dynamic Interest Rate Risk of Swaps

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Abstract

At the time of initiation, interest rate swaps are of zero market value to the counterparties involved. However, as time passes, the market value of the swap position of counterparty may become positive or negative. In this paper, we examine the market values and dynamic interest rate risks of existing swap positions using the one-factor general equilibrium term structure model of Cox, Ingersoll, and Ross (1985). The valuation and risk measurement framework of this paper should be useful in developing a value cum risk accounting method advocated by Merton and Bodie (1995) for better internal management and reporting purposes and for more effective regulation.

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Since their introduction in the early 1980's, the market for interest rate swaps has grown very rapidly in the past sixteen years. As of the end of 1994 the notional amount of outstanding interest rate swaps was more than $8.8 trillion[1]. It is notable that banks are now the major players in the market for interest rate swaps. For instance, as of the end of 1992 the notional amount of outstanding interest rate swaps was $6.0 trillion, and U.S. commercial banks alone held $2.1 trillion of interest rate swaps[2]. The growing popularity of interest rate swaps is due in part to the fact that interest rate swaps are simple and easy to execute and they are the relatively inexpensive instruments for hedging or for altering the interest rate risk of a firm's portfolio[3].

The U.S. commercial banks' dominance in the swap markets has recently raised many concerns about their swap transactions. These include the possibility of the failure of some large banks in the swap market leading to the collapse of the payments and credit systems, known as the *systemic risk*. Besides such dire consequences at the banking system level[4], the swap transactions of an individual bank or a business corporation also have important implications for its shareholders, creditors and other stakeholders. Efforts are being made by various regulatory and accounting overseeing agencies to better disclose and monitor the swap and other derivatives related positions of their users. Many users themselves are also instituting internal policies and mechanisms to closely track and manage their positions in the swap and other derivative markets.

An important element for the success of any external or internal effort to better regulate, disclose, or manage the derivatives positions is the measurement of the fair market value (simply the market

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[1]Source: ISDA, see *Risk*, Vol. 8, No. 7, July 1995.

[2] See Gorton and Rosen (1995).

[3] See Loeys (1985), Bicksler and Chen (1986), Smith, Smithson, and Wakeman (1986), Turnbull (1987), Arak, Estrella, Goodman, and Silver (1988), Wall (1989), Litzenberger (1992), Titman (1992) and Rendleman (1993) for discussions of motivation for interest rate swaps and their applications in hedging interest rate risk and in asset/liability management. See Sun ,Sundaresan and Wang (1993), Brown, Harlow and Smith (1994), Chen and Selender (1995), and Minton (1997) for the empirical determinants and behavior of swap spreads.

[4] If the market value of the banks' swap positions were, say, $200 billion (10% of $2 trillion notional), it will mean a liability of $200 billion for the banks if the market situation has moved in an adverse fashion for the banks. A meager 5% fall in the value of these swaps will drain the banks' market value equity by $10 billion.

value hereafter) as well as the interest rate risk of these positions. To underscore the importance of and the need for risk measurement, Merton and Bodie (1995, p.8-10) write, "To facilitate measurement, financial accounting must undergo fundamental revisions in the long run. ..central to those revisions is the creation of a specialized new branch dealing with *risk accounting*. Until a system of *risk accounting* is in place, truly effective regulation will be difficult to implement." (italics added)

Of course, it will be desirable to maintain consistency between measurements of value and risk which calls for a unified treatment of market value and risk of derivatives. For derivatives such as futures and options, there are well-developed valuation models in the finance literature for this purpose. For swaps, much has been done about their valuation when they originate. This includes the literature that deals with the pricing of the credit risks of the counterparties of a swap arrangement[5]. In comparison, unified theoretical expositions of the market value and the interest rate or market risk of *previously established* swap positions are lacking in the literature[6].

This paper offers a simple approach to determine the market value and the interest rate risk of previously established *plain vanilla* swap positions. This approach can be implemented easily and provides important insights into the valuation of existing swap positions. For example, the market value of an existing swap position is shown to be related to the value of a reference coupon bond with a fixed coupon rate and unit face value. The coupon rate of this reference coupon bond, however, varies depending on whether the swap position is that of a fixed ratepayer or receiver.

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[5] Duffie and Huang (1996), Li (1996), Sun and Wang (1996), Cooper and Mello (1991), and Hull (1989), among others, analyze the effect of credit risk on swap pricing.

[6]There is a growing practice in the industry of marking to market the existing swap positions. Other than Litzenberger (1992), we are not aware of any rigorous discussion about the theoretical basis of the industry practice. Litzenberger, however, emphasizes the role of the unique treatment of swaps under default events in explaining why the industry practice is not sensitive to credit risk and why swap spreads show relatively low cyclical variability. Duffie and Huang (1996, p.934) shows that the netting of fixed against floating payments significantly reduces the impact of credit risk on swap rates relative to bond yields.

Since interest rate swaps are interest rate contingent contracts, their market values and risks are intimately related to the term structure of interest rates. We explore these links using the one-factor general equilibrium term structure model of Cox, Ingersoll, and Ross (1985), commonly known as the CIR model[7]. Use of the CIR model allows us to derive a dynamic measure of interest rate risk for the existing swap positions that is similar to the stochastic duration measure of Cox, Ingersoll, and Ross (1979) for coupon bonds and the quasi stochastic duration measure of Chen, Park, and Wei (1986) for bond futures. As noted by Sundaresan (1991), "..precise quantification of the ..interest rate risk (exposure) is an important outcome of the model." This important benefit may be lost if a more flexible term structure model (e.g., Heath-Jarrow-Morton or Ho and Lee) or a simulation-based value-at-risk type model is used instead.

The swap valuation and risk measurement framework of this paper extends the existing literature in the following ways. *First*, while the focus in the literature is predominantly on the pricing of at-the-money swaps[8], we take the at-the-money swap rates from the market as given and examine instead the market value of previously established swap positions.

Second, we incorporate at-the-money bid and ask prices from the secondary market in valuing the long and short positions[9]. This results in differential market values for the two counterparties of the same swap contract. Our framework can also be used for investigating how the term structure of swap dealers' bid and ask price differentials impact on the market value of long and short positions.

Third, we adapt the stochastic duration measure of Cox, Ingersoll, and Ross (1979) to the case of previously established swap positions and examine the behavior of this risk measure in details. It is shown that the interest rate risk of a swap position differs in essential ways from that of a bond[10].

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[7] The CIR model has been used in several papers on swap pricing, e.g., Sundaresan (1991), Grinblatt (1995), Duffie and Huang (1996), and Li (1996).

[8] Also see Jarrow and Turnbull (1996), pp. 434-439.

[9] We ignore the lag between the payment dates and the reset dates as an issue. Sundaresan (1991) does not find the lag length as an important determinant of swap prices.

[10] Sundaresan (1991) suggests the CIR stochastic duration measure for the floating rate bond used in pricing an at-the-money swap. Li (1996) provides an excellent discussion of how swaps differ in essential ways from a portfolio

For example, it is not even possible to exactly calculate the relative variation of an at-the-money swap position.

Lastly, we broaden the simulation results to include swap maturities up to ten years. In an earlier study, Sundaresan (1991) considered maturities up to five years only. As we shall see, especially when considering the interest rate risk, swap positions of maturities longer than five years can behave quite differently from those with shorter maturities. The need for examining longer maturity swaps has also increased in recent years with the advent of longer maturity Eurodollar futures contracts.

The value and risk measurement framework of this paper can be useful from a practical point of view in several regards. *First*, our framework can be applied to evaluate the adequacy of current disclosure requirements with respect to swap positions of firms in general and the financial institutions in particular. *Second*, early warning signals to detect severe erosion in equity and excessive risk exposure can be put in place using our framework. The dynamic risk measure derived in this paper should be particularly useful in tracking the risk exposure in a changing market. *Third*, the constructs of our paper can be applied to establish proper hedge by firms wanting to hedge their swap positions or other term structure-sensitive assets or liabilities. In the same vain, the establishment and management of an internal policy of specific risk exposure targeting, such as a target stochastic duration, can be facilitated using our results. *Fourth*, since we use a general (equilibrium) framework for interest rate contingent claims, the market value and risk of economic transactions that are either equivalent or close in nature to swap positions, e.g., parallel loans, can be measured and analyzed using a common framework. Such attempts will be in the spirit of *functional regulation* or similar regulatory treatment of economically equivalent transactions advocated by Merton and Bodie (1995).

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of long position in a fixed rate bond and short position in a floating rate bond or vice versa. However, his analysis emphasizes the role of differential credit risk and does not entertain the market or interest rate risk. While in the absence of credit risk, swap cashflows are analogous to the said portfolios, the market risk of existing swap positions cannot be calculated as the difference between the market risks of the two components of the portfolios.

The rest of this paper is organized as follows. In Section 1, we describe models for determining the market values of previously established swap positions to the counterparties. Equilibrium valuation of existing swap positions using the CIR model is then discussed in Section 2. In Section 3, we address the interest rate risk of existing swap positions and derive a dynamic measure of this risk. The effects of introducing credit risk are briefly explored in Section 4. We conclude the paper in Section 5.

1. The Market Values of Swap Positions

At the date of contract initiation of a fixed/floating interest rate swap, the swap contract is usually executed *at-the-money* and the counterparties are said to have positions in a *par value* (or *at-the-money* or *at-market*) swap because there is no initial cash exchange between the two counterparties. Thus, at the date of contract initiation, an interest rate swap contract is neither an asset nor a liability to either counterparty. However, subsequent to its initial date of agreement, any changes in market interest rate can cause the value of a swap contract to become positive (an asset) to one counterparty and negative (a liability) to the other counterparty. In the following, we shall develop and discuss models for determining the market values of existing *long* (fixed ratepayer) and *short* (fixed rate receiver) swap positions.

1.1 Notation and Assumptions

To determine the market values of an existing fixed/floating interest rate swap to its counterparties, let us introduce the following notation:

W the notional principal of the swap;

m $= T - t$, the remaining time to maturity of the swap, also known as the *tenor* of the swap when the swap is originated, where T is the original maturity date of the swap and t is the swap evaluation date;

$P(j,t)$ the price at time t of a default-free unit discount (zero coupon) bond that matures at time $t+j$;

$r(m,t)$ $= 2\,[1-P(m,t)]/\,[\sum_{j=1,m} P(j,t)]$, the bond-equivalent annual yield to maturity (BEY) on the m-maturity *par value* Treasury bond at time t implied by the zero coupon yield curve[11];

i_0 the original fixed rate of interest of the swap;

$i_b(m,t)$ the dealer's bid price (also known as *pay rate*) of the *m-maturity* swap at time t; it is the fixed-rate of an at-the money swap when dealer *pays* the fixed rate;

$i_a(m,t)$ the dealer's ask price (also known as *receive rate*) of the *m-maturity* swap at time t; it is the fixed-rate of an at-the money swap when dealer *receives* the fixed rate[12];

$LS(m,t)$ the market value of an existing m-maturity swap to the *long*-swap-position-holder (the buyer) at time t;

$SS(m,t)$ the market value of an existing m-maturity swap to the *short*-swap-position-holder (the seller) at time t.

We make the following assumptions:

A1. Markets are frictionless except for the swap dealers' spreads.

A2. Any effect of credit risk is already incorporated in the dealer's spreads.

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[11] The $r(m,t)$ calculated in this manner implies absence of coupon stripping or synthetic coupon arbitrage opportunity. However, it is some times argued that the zero coupon bonds are less liquid than the underlying Treasury coupon bond from which the zeros are stripped off . Thus, according to this argument, the yield on the *par value* coupon bond implied by the zeros is an inaccurate (over) estimate of the coupon bond if it were to directly trade at par.

[12] In reality, there may be many dealers making market in a given type of swap and their bid and ask quotations may vary albeit by small amounts. For our analysis, *the dealer* is taken to be an average dealer.

A3. Swap payments are made in arrears.

A4. Reset and payment frequencies are equal to the maturity of the floating index[13], the 6-month LIBOR. The floating rate in this paper is the 6-month LIBOR flat without any spread[14].

Ignoring credit risk and swap spreads, the *new* at-the-money swap rate on a reset date is simply the BEY of a par value fixed rate bond, namely $r(m,t)$ in this paper. On an intermediate date between two reset dates, the at-the-money swap rate for a swap that is identical to the existing swap except for the fixed rate can be calculated as:

$$2\,[(1+0.5LIBORL)P(1,t)-P(m,t)]/\,[\textstyle\sum_{j=1,m} P(j,t)]$$

where *LIBORL* is the annualized 6-month LIBOR rate from the last reset date. This at-the-money rate of a new swap may be different from $r(m,t)$ as the floating rate bond may not sell at par between reset dates. For the sake of simplicity, however, we assume from hereon that the valuation date t is a reset date. As such we refer to an m-maturity swap alternatively as an m-period swap, i.e., a swap with m semiannual periods remaining to maturity.

The secondary market swap prices will reflect swap dealer's spread in addition to the theoretical at-the-money rate mentioned above. The actual process of determination of the swap dealer's bid and ask prices is outlined in Appendix A. Without loss of generality, we can express the bid and ask prices in the following manner[15]:

$$i_b(m,t) = r(m,t) + d_b(m,t), \text{ and}$$
$$i_a(m,t) = r(m,t) + d_a(m,t),$$

[13] See Sundaresan (1991) for the general case where the frequencies are different.

[14] We ignore the difference in day conventions used for calculating interest payments for the two sides of the swap.

[15] Grinblatt (1995) finds liquidity advantage of treasury bonds as a more plausible explanation of swap spreads than credit risk. Duffie and Singleton (1997) use default- and liquidity-adjusted zero coupon bond prices in determining the at-the-money fixed rate using a formula (their equation 2.4, p.5) similar to our definition of $r(m,t)$. Sundaresan's (1991, equation 3, p.418) formula for the at-par rate is also similar when the valuation date is a reset date for the next floating payment. Of course, we entertain the bid and ask spreads but ignore the modeling of credit risk and liquidity.

where $d_b(m,t)$ and $d_a(m,t)$ are dealer's bid and ask swap spreads. In general, the swap spreads can be functions of the term structure and thus can be an additional source of variation in the bid and ask prices as the interest rate situation changes in the market[16]. This indirect effect of changes in Treasury market yields is however quite small compared to the direct effect through $r(m,t)$ since the spreads themselves are quite small relative to $r(m,t)$. Hence we assume in what follows that the spreads do not depend on the Treasury yields. We, however, allow the spreads to vary with the time to maturity of the swap, m, as is the case in reality.

1.2 The Market Value of An Existing Swap Position

Most of the existing fixed/floating interest rate swaps, especially those with more active floating indices such as LIBOR and T-bill rates, can be readily traded in the secondary markets. The market value of an existing swap position can be defined as the lump sum dollar amount the dealer must receive or pay to be indifferent between stepping into the existing swap position or taking the same side in a new at-the-money swap[17]. If the dealer takes over the swap buyer's existing long position instead of taking the long side in a new at-the-money swap, she will experience an incremental cashflow of $W[(i_b(m,t)-i_0)/2]$ for the next m semiannual periods. Ignoring credit risk, the incremental

[16] For example, Duffie and Huang (1996) models the spread between the short-term default premium (over the short term LIBOR) of the two counterparties as a linear function of the short term LIBOR rate.

[17] This is essentially the same as the ISDA Code's "agreement value". Somewhat different definitions of market value can be found in the literature. For example, Duffie and Huang (1995, p.8) define market value as the price to *any* investor of a claim that pays off the same cashflow as the swap position of a counterparty. Our definition limits the investor to be a swap dealer as we are interested in marking to market the swap positions. In the presence of swap spreads, the value of a swap position to investors other than the dealer may be different. Further, we ignore credit risk which seems like a reasonable assumption for a dealer investor.

cashflow stream to the dealer is *certain* since it does not depend on the floating rate. Thus, the market value of the swap buyer's existing *long* position is[18]:

$$LS(m,t) = W[(i_b(m,t)-i_0)/2]\ [\textstyle\sum_{j=1,m} P(j,t)] \qquad (1)$$

Note that Sundaresan (1991) derives the market value as the differential between the value of the remaining floating payments and the value of the remaining fixed payments (at the original fixed rate), both valued according to the current default-free term structure. Since the value of the remaining floating payments is equal to the value of the fixed payments at the new at-the-money rate, the valuation formula in (1) is similar to Sundaresan's formula (p.418) except that we incorporate the swap dealers bid-ask spreads. Additionally, the formula in (1) highlights the fact that once the new at-the-money rate is known, the market value of an existing swap position is simply the difference in values of two fixed rate bonds (or the value of a constant annuity).

Using equation (1), a fixed rate payer can easily calculate the market value of her position by simply observing the dealer's bid price and the current term structure or the market prices of the zeros[19]. At any given point in time, an existing *long* swap position is of *positive* (an *asset*) or *negative* (a *liability*) market value to the swap buyer depending on whether $i_b(m,t)$ is *greater* than or *less* than i_0. If an existing *long* position is of positive value, $LS(m,t) > 0$, the swap is said to be *in-the-money* to the swap buyer, and the buyer has gained from holding a *long* position in the swap transaction. If $LS(m,t) < 0$, the swap is said to be *out-of-the-money* (i.e., the *"underwater"* swap) to the swap buyer, and the buyer has lost from the swap transaction.

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[18] That the market value of a swap position is the value of an annuity, namely the differential fixed coupon payments (new at-the-money fixed rate vs. the swap's original fixed rate), has been noted previously. For example, see Fabozzi (1991, pp. 1169-1170). However, there the annuity is discounted at the original fixed rate which is questionable.

[19] Industry practice (Marshall and Bansal (1992), Marshal and Kapner (1993)) is to mark swap positions to market using zero coupon swaps curve implied by *par value* or *at-market* swaps curve. These are not, however, investible rates for the incremental cashflow stream. The implied rates may also vary across dealers depending on how they calculate the swap midrate. Further, the implied rate framework is not easily amenable to the calculation of dynamic market risk of swap positions.

Similarly, the market value of the swap seller's existing *short* position can be derived as:

$$SS(m,t) = W[(i_0 - i_a(m,t))/2]\,[\textstyle\sum_{j=1,m} P(j,t)] \qquad (2)$$

Note that the magnitudes of the market values of an existing *long* position and an existing *short* position are different even if their terms (W,m,i_0) are identical. This differential value is what the swap dealer hopes to capture by making the market in swaps:

$|LS(m,t)-SS(m,t)| = [d_a(m,t)-d_b(m,t)][\sum_{j=1,m} P(j,t$**1.3 The Determinants of the Market Value of An Existing Swap Position**

The following two propositions yield useful insights into the determinants of the market value of swap positions.

Proposition 1

The market value of an existing long swap position with m periods to maturity, LS(m,t), is proportional to the discount or premium of an m-maturity unit coupon bond with an annual coupon rate of [i_0 -$d_b(m,t)$].

This can be seen by substituting for $i_b(m,t)$ in equation (1):

$$LS(m,t) = W\,[1 - B_L(m,t)\,] \qquad (1a),$$

where $B_L(m,t) = P(m,t) + [(\,i_0 - d_b(m,t))/2][\sum_{j=1,m} P(j,t)]$ (1b).

$B_L(m,t)$ is the price of an *m-maturity* coupon bond with unit face value and semiannual coupon of $C_L(m,t) = (\,i_0 - d_b(m,t))/2$. This *m-maturity* coupon bond would sell at par ($1.00) if it had a BEY of $r_{BL}(m,t) = i_0 - d_b(m,t)$.

Proposition 2

The market value of an existing short swap position with m periods to maturity, SS(m,t), is proportional to the discount or premium of an m-maturity unit coupon bond with an annual coupon rate of [i_0 -$d_a(m,t)$].

Substituting for $i_a(m,t)$ in equation (2) leads to the above proposition:

$$SS(m,t) = W\,[B_S(m,t) - 1\,] \qquad (2a),$$

$$\text{where } B_S(m,t) = P(m,t) + [(\,i_0 - d_a(m,t))/2][\textstyle\sum_{j=1,m} P(j,t)] \qquad (2b).$$

$B_S(m,t)$ is the price of an *m-maturity* coupon bond with unit face value and semiannual coupon of $C_S(m,t) = (\,i_0 - d_a(m,t))/2$. This *m-maturity* coupon bond would sell at par ($1.00) if it had a BEY of $r_{BS}(m,t) = i_0 - d_a(m,t)$. Hereafter, we shall refer to the bonds in (1b) and (2b) as the reference coupon bonds.

Equations (1a), (1b), (2a), and (2b) confirm the common knowledge that a *long (short)* swap position behaves like a *short (long)* position in a coupon bond. It is, however, important to keep in mind that while the price and the discount or premium of a bond changes by the same dollar amount, the percentage changes do not necessarily match. Since the swap values are essentially discounts or premiums, their risk characteristics may not mirror those of the reference bonds, as we shall see later in this paper.

As expected from (3), the reference coupon bonds for the *long* and *short* swap positions differ in their coupon rates in the presence of dealer's bid-ask spreads, and so do the degree of their moneyness. In general, when the *long* swap position is *in-the-money*, the *short* swap position will be *out-of-the-money*. However, exceptions may arise.

Remark 1: In the presence of swap dealer's bid-ask spreads, both counterparties of a swap may be out-of-the-money or in-the-money at the same time.

This is now shown through an example. Suppose $r(T,0)$ was 0.07, $i_b(T,0)$ was 0.07+0.0041 and $i_a(T,0)$ was 0.07+0.0045. However, the two counterparties struck the swap transaction without any intermediation at the midpoint, i_0 = 0.0743. At t, $r(m,t)$ is a bit higher at 0.0701. With the swap spreads remaining at the same levels, $i_b(m,t)$ = 0.0701+0.0041 =0.0741, and $i_a(m,t)$ = 0.0701+0.0045=0.0746. Hence, in this example, $i_b(m,t) < i_0 < i_a(m,t)$. Thus both the *long* position and the *short* position are now *out-of-the-money*. With the spreads remaining constant, this possibility will not arise if the original transaction took place through a dealer and/or if market interest rates moved enough.

A more probable case is where the swap dealer's bid and ask spreads have moved in the opposite directions. Suppose, at origination, $r(T,0)$ was 0.07, $i_b(T,0)$ was 0.0741 and $i_a(T,0)$ was 0.0745. The counterparties took positions at these bid and ask prices. At t, $r(m,t)$ is unchanged at 0.07. But due to a change in the market conditions or perhaps due to changes in the credit qualities of the two counterparties, the $d_b(m,t)$ has dropped to 0.0040 and $d_a(m,t)$ has increased to 0.0046. Thus, $i_b(m,t)=0.0740 < i_b(T,0)= 0.0741$ and $i_a(m,t)=0.0746 > i_a(T,0)=0.0745$, and both the *long* position and the *short* position are now *out-of-the-money*.

Modeling such possibilities as demonstrated above may be pertinent in the context of works such as that of Duffie and Huang (1996). There the instantaneous default risk of the counterparty who is currently out-of-the-money is used in recursively solving the market value of the swap. In light of our *Remark 1*, instantaneous default risk of both counterparties may be relevant[20].

In our default-free framework, as shown by equations (1a), (1b), (2a), and (2b), the broad determinants of the prices of the reference bonds and hence the market values of the existing swap positions per dollar of notional principal are the following: (i) the original fixed-rate of interest, i_0; (ii) the dealer's current bid or ask swap spread, $d_b(m,t)$ or $d_a(m,t)$; (iii) the remaining time to maturity of the swap, m; and (iv) the term structure of interest rates or discount bond prices, $P(j,t)$'s, $j=1,2,..,m$.

A *higher* i_0 increases the value of both reference coupon bonds, $B_L(m,t)$ and $B_S(m,t)$, and hence leads to a *lower (higher)* market value of an existing *long (short)* swap position. Increased dealer's

[20] For example, one may use the default risk of the counterparty who is relatively more (less) out-of-the-money (in-the-money). If V_A and V_B represents the values for the two counterparties A and B, the default risk of A is more relevant if $\{V_A - \max [V_A, V_B]\} <0$. Otherwise, the default risk of B is more relevant. Duffie and Huang's (1996) valuation gets quite complicated even with this simple modification.

bid-ask swap spreads, $d_b(m,t)$ *and* $d_a(m,t)$, reduce $B_L(m,t)$ and $B_S(m,t)$, and hence lead to a *higher (lower)* market value of an existing *long (short)* swap position.

Other than the minor effect of time to maturity, *m*, via the spreads, its primary impact on the market value of the swap positions is intertwined with the effect of the term structure of interest rates. Specific comments about these effects can only be made in the context of a given term structure model.

A term structure model can also be useful in linking the market value and risk of swap positions to the parameters of a stochastically evolving economy, in deriving a stochastic interest rate risk measure, and in predicting the effect of swap's time to maturity. Further, a swap portfolio may comprise of swap positions of varying maturities and there is no natural choice for a single yield measure as a determinant of the market value and risk of the swap portfolio. A (one factor) term structure model can provide such an yield measure.

2. Equilibrium Term Structure Theory and The Effects on the Market Value of Swap Positions

The finance literature is rich with term structure models[21]. Unfortunately, it is not clear which term structure model best captures the features of term structure movements in reality[22]. In this paper, we use the one-factor general equilibrium term structure model of Cox, Ingersoll, and Ross (1985),

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[21]See Rogers (1995) for an interesting recent review of the well known term structure models.

[22]Practitioners and regulators have been toiling with a similar dilemma in using *value at risk (VAR)* as a measure of market risk of involvement in derivatives (Reed, 1995). Apparently, the *VAR* of an institution depends on the specific models that are used for valuing the derivatives. There is no uniform industry standards for such models and the regulators are equally reluctant to impose such standards.

widely known as the CIR model[23]. This section starts with a brief presentation of the CIR model. This is followed by a discussion of the effects of the spot interest rate, the time to maturity of the swap position, and the equilibrium valuation parameters on the value of *long* and *short* swap positions.

2.1 The CIR Model

In the one-factor CIR model, the instantaneous default-free rate, $r(t)$, alternatively referred to as the spot rate or the short rate, is the instrumental variable for the underlying single state variable that captures the fundamental stochastic characteristics of an economy. The dynamics of the spot rate is given by:

$$dr(t) = \kappa(\theta - r(t))\,dt + \sigma\sqrt{r(t)}\;dz \qquad (4)$$

where dz is one-dimensional Wiener process, $\kappa>0$, and $\theta>0$. Known as the mean-reverting square root model of interest rate, the process in equation (4) is a continuous time first-order autoregressive process where the randomly moving spot rate is pulled toward its stationary mean, θ, at an expected rate (speed of adjustment) of κ. The instantaneous drift and variance of the spot rate are respectively $\kappa(\theta - r(t))$ and $\sigma^2 r(t)$.

The time t equilibrium price of a j-period (matures at $t+j$) default-free unit discount bond in the CIR model is given by:

$$P(j,t) = G(j,t)\exp[-r(t)H(j,t)] \qquad (5)$$

where $G(j,t) = [\,2\gamma\exp\{(\gamma+\kappa+\lambda)j/2\} / \{(\gamma+\kappa+\lambda)(\exp(\gamma j)-1)+2\gamma\}\,]^{2\kappa\theta/\sigma 2}$, $H(j,t) = 2(\exp(\gamma j)-1) / \{(\gamma+\kappa+\lambda)(\exp(\gamma j)-1)+2\gamma\}$, $\gamma = \sqrt{\{(\kappa+\lambda)^2 + 2\sigma^2\}}$, and $\lambda r(t)$ is the covariance of spot rate changes with percentage changes in optimally invested wealth (by a representative agent with

[23] The basic one-factor CIR model can be easily extended to the case of two (Cox, Ingersoll, and Ross (1985), Longstaff and Schwartz (1992)) or more factors (Chen and Scott (1993), Chen and Scott (1995)) and thus can be adapted to fit multiple points on the initial term structure using the approach of Hull and White (1990). See Longstaff (1993, footnote 3, p. 29) for the limitations of exactly fitting the whole initial term structure.

constant absolute risk aversion). The parameter λ is considered as a preference or risk premium parameter.

The equilibrium term structure in terms of the yields to maturity on the unit discount bonds of various maturities, or the equilibrium yield curve, is given by:

$$R(j,t) = [r(t)\, H(j,t) - \ln\,(G(j,t))] \,/\, j \tag{6}$$

The (equilibrium) market value of a swap position is a function of the equilibrium term structure or the set of equilibrium discount bond prices, *P(j,t)*'s[24]. Hence the spot rate, its stationary mean and its instantaneous variance, the risk premium parameter, and the speed of adjustment parameter are the economy-wide or fundamental determinants of the equilibrium market value of swap positions. For brevity, we omit the adjective *equilibrium* hereafter.

[24]The equilibrium term structure is completely specified by the equilibrium discount bond prices. Hence, we use the terms *yield curve* or *discount bond prices* interchangeably in referring to the term structure.

2.2 Effect of the Spot Rate

Proposition 3

In the one-factor CIR model, the market value of a long swap position, LS(m,t), is an increasing concave function of the spot rate, r(t).

Proof. In the one-factor CIR model, discount bond price of any maturity in equation (5) is a decreasing convex function ($P_r(j,t)<0$, $P_{rr}(j,t)>0$) of the spot rate. Thus, the price of the reference coupon bond, $B_L(m,t)$, in equation (1b), is also a decreasing convex function of the spot rate. Therefore, according to equation (1a), the market value of an existing *long* swap position, *LS(m,t)*, is an *increasing concave* ($LS_r(m,t)>0$, $LS_{rr}(m,t)<0$) function of the spot rate, *r(t)*.

It can be shown that the par value coupon bond's yield to maturity is increasing in the spot rate ($r_r(m,t)>0$), and therefore, *LS(m,t)* is increasing in *r(m,t)*. These results will hold for any one-factor term structure model where the discount bond price is a decreasing convex function of the spot rate for all maturities[25].

The direct relationship between the spot rate and the market value of an existing *long* swap position is in fact the net result of two influences.

Remark 2: A term structure change has two effects on a swap position's market value: (1) coupon effect: the incremental cashflow stream of a swap position changes, and (2) discounting effect:

[25] Two well-known examples of such one-factor term structure models are Vasicek (1977) and Dothan (1978). This result, however, may not hold universally in a two-factor model. As shown by Longstaff and Schwartz (1992), in a two-factor CIR model where the instantaneous variance of the spot rate is the second factor, the price of longer maturity discount bonds may be increasing in the spot rate.

the value of the incremental cashflow stream changes. These two changes are not necessarily in the same direction.

The coupon effect is due to change in the *par value* coupon bond's yield to maturity, $r(m,t)$, which in turn changes swap dealer's bid and ask prices, $i_b(m,t)$ and $i_a(m,t)$. The discounting effect is of course due to change in the compound valuation factor or the annuity factor, $\sum_{j=1,m} P(j,t)$. Whether these two effects are in the same direction or not depends on the nature of the term structure movement and the swap position.

Remark 3: The coupon effect and the discounting effect of a term structure shift on the market value of a long swap position are of opposing nature.

When the term structure shifts, the yield to maturity and thus $P(j,t)$ changes in the same direction for all j. If the yields rise, the par value bond's yield to maturity, $r(m,t)$, goes up leading to a higher $i_b(m,t)$ and therefore a *positive coupon effect* for a *long* swap position. However, given the inverse relationship between discount bond yields and prices, $P(j,t)$'s fall for all j leading to a decrease in the compound valuation factor, $\sum_{j=1,m} P(j,t)$, and hence the *discounting effect* of an across the board yield increase on a *long* position is *negative*. While it may appear from equation (1) that the net effect is equivocal, equations (1a) and (1b) make it clear[26].

Remark 4: For shifts in the term structure, the coupon effect dominates and an upward (downward) shift in the term structure increases (decreases) the market value of a long swap position.

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[26]In equation (1a), the netting out of the two effects lead to a short position in the hypothetical coupon bond with a coupon rate that does not depend on the spot rate. Hence the well known decreasing convexity of a coupon bond's price lead to increasing concavity of the market value of an existing *long* position.

Intuitively, this is because the yield to maturity on a par value coupon bond changes by more than the yield to maturity on just the coupon stream, given a shift in the term structure. This can be seen in the definition of $r(m,t)=2[1-P(m,t)]/[\sum_{j=1,m} P(j,t)]$. As discount bond prices fall due to an upward shift in the term structure, the numerator contributes additionally to the increase of $r(m,t)$ and hence to a stronger coupon effect.

Remark 5: The coupon effect and the discounting effect of a term structure shift on the market value of a short swap position are in the same direction.

Equations (2), (2a), and (2b) help explain the above remark.

While Remarks 2, 3, 4, and 5 above prevail in the context of the one-factor CIR model, they are in fact model independent. It is a characteristic of the one-factor CIR model that shifts in the term structure are synonymous with spot rate changes[27].

If we plot the market value of an existing *long* swap position, $LS(m,t)$, as a function of the spot rate, $r(t)$, the steepness or slope of the function will indicate the magnitude of the effect of a spot rate change. A key determinant of the magnitude of the spot rate effect is the swap position's time to maturity, m. To illustrate this matter, we plot in Figure 1, the value of an existing *long* swap position as a function of the spot rate for three different swap maturities, m = 1 (0.5 years), 8 (4.0 years), and 14 (7.0 years). The assumed values of the other parameters are: $\kappa=0.1$, $\theta=0.04$, $\lambda=0.0$, σ^2 =0.0025, i_0 =0.05, and $d_b(m,t)$ =0.0012 for m=1,8, and 14.

[27] If term structure twists are allowed over the relevant segment of swap maturity, the direction of neither the coupon effect nor the discounting effect is clear. The direction of the net effect from equations (1a) and (1b) or from (2a) and (2b) is not clear either. For a broader understanding of the effect of term structure twists, a two-factor term structure model (e.g., Longstaff and Schwartz (1992)) is needed.

Figure 1 About Here

Figure 1 shows that the function gets steeper as the swap maturity gets longer, that is the spot rate effect is *stronger* for *longer* maturity swaps[28]. This is because the *positive coupon effect* of a spot rate increase is much more dominant relative to the *negative discounting effect* for *longer* maturity *long* swap positions (with the same original fixed rate). The net effect can be seen clearly in equations (1a) and (1b). The price of the reference coupon bond with a constant (unrelated to the spot rate) coupon is more responsive to the spot rate (or *par value* coupon bond's yield) for a longer maturity. This results in a greater responsiveness for longer maturity existing *long* positions.

We now state the spot rate effect for *short* swap positions without a proof.

Proposition 4

4A. The market value of an existing short position, SS(m,t), is a decreasing convex function of the spot rate, r(t).

4B. The steepness effect of swap maturity is similar to the case of a long position. In other words, longer maturity short swap positions are more responsive to spot rate changes than shorter maturity short swap positions.

2.3 Effect of Swap Maturity

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[28]As expected, the functions are concave, although the degree of concavity is negligible given the assumed parameter combination.

While we have mentioned the influence of the swap's time to maturity on the spot rate effect, we should now look into the maturity effect by itself. To do so, we shall first describe the stochastic processes for the reference coupon bonds.

Following Cox, Ingersoll, and Ross (1979, 1985), it can be shown that the price, $B(r,\tau)$, of a default-free bond with a coupon stream $C(\tau)$, continuous or discrete, with time to maturity τ, follows the dynamics:

$$dB(r,\tau) = [\alpha(r,\tau)\, B(r,\tau) - C(\tau)]dt + \delta(r,\tau)\, B(r,\tau)\, dz \tag{7}$$

where $\alpha(r,\tau)B(r,\tau) - C(\tau)$ is the drift or expected instantaneous change in the bond price, $\delta(r,\tau)$ is the instantaneous standard deviation of return. The fundamental PDE that the equilibrium bond price must follow is:

$$B_\tau = 0.5\, \sigma^2\, r\, B_{rr} + \kappa(\theta - r)\, B_r - (rB(r,\tau) + \lambda\, r\, B_r - C(\tau)) \tag{8}$$

In the one-factor CIR model, the maturity derivative for a discount bond ($C(\tau)=0.0$), $P_\tau(j,t)$, is *negative*. For a coupon bond, however, the sign of the maturity derivative depends on the parameter combination, especially on r, λ, and $C(\tau)$.

For a given coupon rate, in general we would expect the (signed) maturity derivative of a coupon bond to decrease with the spot rate and to be positive (negative) for small (large) values of the spot rate. Thus, as the swap maturity, m, gets longer, we would expect the reference coupon bond values, $B_L(m,t)$ and $B_S(m,t)$, to become larger (smaller) at low (high) levels of the spot rate, $r(t)$[29]. Hence, according to equation (1a), we would expect the market value of existing *long* swap position, $LS(m,t)$, to be higher (lower) for longer maturity swaps at high (low) levels of the spot rate. The market value of existing *short* swap position, $SS(m,t)$, would tend to be lower (higher) for longer maturity swaps at high (low) levels of the spot rate.

The above pattern for *long* swap positions can be observed in Figure 1. When λ=0.0, the benchmark for high or low levels of $r(t)$ is roughly the annual coupon rate on the reference coupon bond, $i_0 - d_b(m,t)$. Thus, roughly at spot rates below (above) 5%, we see a longer maturity *long* swap position to have a relatively lower (higher) value. This maturity effect can be seen more clearly in Figure 2 where we plot the market values of *long* swap positions against swap maturity (ranging from 6 months to 10 years) for three alternative levels (1%, 4%, 10.5%) of the spot rate.

Figure 2 About Here

The above swap maturity effect implies, in *general*, that *in-the-money (out-of-the-money) long (short)* swap positions will tend to *decline (increase)* in market value as they approach maturity. The opposite is true for *out-of-the-money (in-the-money) long (short)* swap positions. This general type of swap maturity effect is similar to that *usually* found for the coupon bonds that sell at discount or premium from their *par value*. An *in-the-money (out-of-the-money)* swap position is like a *premium (discount)* coupon bond, as is evident from equations (1a) and (2a). The intuitive reason behind this usual maturity effect is simple, it is namely the gravitational pull toward the terminal boundary condition: bonds or swaps selling below or above *par* before maturity will have to sell at *par* at maturity.

It should be mentioned, however, that while the above pattern of maturity effect is usually the case, it is by no means universal. It is possible to have a non-monotonic relationship between swap maturity and the market value of an existing swap position. See, for example, the 1% spot rate curve in Figure 3 and the 10.5% spot rate curve in Figure 4. The only assumption that is different between these two figures and Figure 2 is that relating to the value of the risk premium parameter, λ. Similar examples can be constructed by varying the other general equilibrium pricing parameters, θ and κ.

Figure 3 About Here

Figure 4 About Here

[29]What is a low or a high level for the spot rate in this context depends on the parameter combination.

It is interesting to compare the term structure of the at-the-money swap rates and the market values of previously established swap positions. For this purpose, let us assume that the CIR parameters are such that monotonic term structures prevail. As reported by Sundaresan (1991, p.424), when the spot rate is low (high) and the term structure of zero coupon bond rates is upward (downward) sloping, the term-structure of at-the-money swap rates is also upward (downward) sloping. This might suggest that when the spot rate is low, the term structure of market values of previously established *long* swap positions (with a common original fixed rate) is upward sloping as well. [This is because the dealer has to pay a higher fixed rate for a new at-the-money swap as the maturity gets longer].

As we have discussed above (Figures 1 and 2), the term structure of the market values of *long* swap positions is in fact downward sloping when the spot rate is low. While it is true that the new at-the-money fixed rates increase with swap maturity, all these rates are still lower than the original fixed rate of the swaps. Thus previously established *long* swaps are still at a disadvantage to the new ones and this advantage is protracted over a longer time span for a longer swap maturity.

Sundaresan's (1991) findings and our findings regarding the effects of the term structure of interest rates on the term structures of new at-the-money rates and the market values of existing swap positions are summarized below in the following remark.

Remark 6

Assume that the existing swap positions have a common original fixed rate, i_0, and that the swap dealer's spreads, $d_b(m,t)$ and $d_a(m,t)$, are constant.

6A. In the one-factor CIR model, when the spot rate, r, is low and thus the term structure of interest rates of zero coupon bonds, R(j,t), is upward sloping, in general:

(i) the term structures of new at-the-money swap rates, $i_b(m,t)$ and $i_a(m,t)$, are upward sloping (Sundaresan)[30];

[30] Sundaresan (1991) has a single at-the-money rate as he does not include dealer's bid-ask spreads.

(ii) the term structure of the market values of existing long swap, LS(m,t), positions is downward sloping;

(iii) the term structure of the market values of existing short swap, SS(m,t), positions is upward sloping.

6B. In the one-factor CIR model, when the spot rate, r, is high and thus the term structure of interest rates of zero coupon bonds, R(j,t), is downward sloping, in general:

(i) the term structures of new at-the-money swap rates, $i_b(m,t)$ ***and*** $i_a(m,t)$***, are downward sloping (Sundaresan);***

(ii) the term structure of the market values of existing long swap, LS(m,t), positions is upward sloping;

(iii) the term structure of the market values of existing short swap, SS(m,t), positions is downward sloping.

6C. In the one-factor CIR model, depending on the model parameters, it is possible to have non-monotonic term structures of the market values, LS(m,t) and SS(m,t), of existing swap positions even when the term structure of interest rates on zero coupon bonds, R(j,t), and thus the term structure of new at-the-money rates, $i_b(m,t)$ ***and*** $i_a(m,t)$***, are monotonic in maturity.***

2.4 Effects of Other CIR Model Parameters

The effects of the parameters, λ, κ, and θ, on the current market value of an existing swap position can be derived using their effects on the discount bond prices. Table 1 provides a summary of these effects along with the effects of the spot rate and the swap maturity.

Table 1 About Here

3. Dynamic Interest Rate Risk of Swap Positions

Credit risk and interest rate are the two major types of risk inherent in an interest rate swap position. This section provides a detailed examination of the interest rate risk of existing swap positions.

The unanticipated or stochastic variations in the market interest rates leading to variations in the value of a swap position are at the source of the interest rate risk of a swap position. In the following, we derive a dynamic measure of interest rate risk for existing swap positions using the framework of Cox, Ingersoll, and Ross (1979). The behavior of this risk measure as it relates to the spot rate and swap maturity is then examined.

3.1 Dynamic Measure of Interest Rate Risk

The square root of the diffusion coefficient of a bond (with or without coupon) is proportional to the square root of the diffusion coefficient of the spot rate, $\sigma\sqrt{r}$:

$$\delta(r,\tau) = |B_r/B|\ \sigma\sqrt{r} \tag{9}$$

The proportionality factor, $|B_r/B|$, is called the *relative variation* of the bond. As pointed out by Cox, Ingersoll, and Ross (1979), this is the correct metric of risk in a stochastic interest rate environment. The *stochastic duration* of a coupon bond is defined as the time to maturity of a zero coupon bond that has the same *relative variation* (and thus the same diffusion coefficient).

In the one-factor CIR model, the *relative variation* of a zero coupon bond is given as a function of its time to maturity, j:

$$|P_r(j,t)/P(j,t)| = H(j,t) = 2(exp(\gamma j)-1)\ /\{(\gamma + \kappa + \lambda)(exp(\gamma j)-1)+2\gamma\} \tag{10}$$

where $\gamma = \sqrt{\{(\kappa + \lambda)^2 + 2\sigma^2\}}$. From equation (1b) or (2b), if we calculate the *relative variation* of a coupon bond to be X, then the *stochastic duration* at time t, $SD(t)$, of the coupon bond can be found by inverting equation (10):

$$SD(t) = H^{-1}(X) = ln\ [1+\{2\gamma X/(2-pX)\}]\ /\ \gamma \tag{11}$$

where $p=\gamma+\kappa+\lambda$. In other words, $SD(t)$ is that value of j for which $H(j,t)$ =X. The *stochastic duration* of a zero coupon bond is thus its maturity by definition.

Proposition 5

In the one-factor CIR model, the stochastic duration at time t of the long and the short swap positions are respectively:

$$\boldsymbol{SD_{LS}(t) = H^{-1}(RV_{LS}) = \ln [1+\{2\gamma RV_{LS} /(2-\ p\ RV_{LS})\}] / \gamma} \tag{12}$$

$$\boldsymbol{SD_{SS}(t) = H^{-1}(RV_{SS}) = \ln [1+\{2\gamma RV_{SS} /(2-\ p\ RV_{SS})\}] / \gamma} \tag{13}$$

where $RV_{(.)}$ represents the corresponding relative variation:

$$\boldsymbol{RV_{LS} =|LS_r /LS(r,\tau)| = |B_{L,r} /(1-B_L(r,\tau)|} \tag{14}$$

$$\boldsymbol{RV_{SS} = |SS_r /SS(r,\tau)| = |B_{S,r} /(B_S(r,\tau)-1)|} \tag{15}$$

Proof. For notational convenience, let us normalize the notional principal of a swap position to one dollar. The stochastic differential of the market value of a *long* swap position is as follows (dropping the swap maturity and time subscripts):

$$d(LS) = [-\alpha_L(r,\tau)LS + C_L\ (\tau)]dt + \delta_{LS}(r,\tau)\ LS\ dz \tag{16}$$

where $\alpha_L(r,\tau) = r\ \ + (\lambda r B_{L,r} /B_L(r,\tau))$, and the square root of the diffusion coefficient of the swap value is proportional to the square root of the spot rate's diffusion coefficient, $\sigma\sqrt{r}$:

$$\delta_{LS}(r,\tau) =|LS_r /LS(r,\tau)|\sigma\sqrt{r} = |B_{L,r} /(1-B_L(r,\tau)|\sigma\sqrt{r} \tag{17}$$

The proportionality factor in (17) is the *relative variation* of the *long* swap position in (14). Using the corresponding relative variation as *X* in (11) results in (12). A similar proof applies to the *short* position's stochastic duration in (13).

Several points should be noted about the *relative variation* and the *stochastic duration* measures of interest rate risk for swap positions. *First*, the *relative variation* measures are not defined if the swap positions are exactly *at-the-money* (reference coupon bonds are *at par*). This, however, does not mean that the risk of a swap position that is close to being *at-the-money* is negligible. As we shall discuss shortly, our simulations show that the *relative variation* of swaps that are close to being *at-the-money* are distinctively and substantially larger. Thus in a cross-sectional comparison and for hedging or other risk management purposes, the exactly *at-the-money* swap positions can be classified in the riskiest category[31].

Second, for a given combination of λ, κ, θ, and σ, there is a maximum value, $X_{max}(\lambda,\kappa,\theta,\sigma)$ of *relative variation* for which the *stochastic duration* measure is meaningful[32]. There is no guarantee that the *relative variation* of swap positions will be bounded by this ceiling. There are two ways this problem can be handled. One way is to calculate the *stochastic duration* measures using the following slightly modified versions of equations (12) and (13):

$$SD_{LS}(t) = H^{-1}(rv_{LS}) = A \ln [1+\{2\gamma\ rv_{LS}(t) /(2- p\ rv_{LS})\}] / \gamma \qquad (18)$$

$$SD_{SS}(t) = H^{-1}(rv_{SS}) = A \ln [1+\{2\gamma\ rv_{SS}(t) /(2- p\ rv_{SS})\}] / \gamma \qquad (19)$$

where $rv_{LS} = X_{max}(\lambda,\kappa,\theta,\sigma)\ (RV_{LS}/U)$, $rv_{SS} = X_{max}(\lambda,\kappa,\theta,\sigma)\ (RV_{SS}/U)$, U is an arbitrary large number, and A is a scaling factor for suitable presentation of the *stochastic duration* measures[33].

The use of equations (18) and (19) leaves the cross-sectional ranking of the dynamic risk of swap positions in tact. Hence equations (18) and (19) can be used without any qualification for comparisons dealing with swaps alone. The *stochastic duration* measures from these equations cannot, however, be used meaningfully for a comparison of swap positions to other interest rate contingent claims including the zero coupon bonds. This is because the *stochastic duration* values from equations (18) and (19) do not any more mean that the swap positions have the same degree of interest rate risk as the zero coupon bonds of maturity equal to the calculated stochastic duration values.

In a situation where the interest rate risks of swap positions are to be compared against other interest rate contingent claims, an alternative will be to simply compare the unadjusted *relative*

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[31] In reality, existing (previously established) swap positions will rarely be exactly *at-the-money*. So the aforementioned problem of dynamic risk measurement may not arise at all. In case it does, one way to handle this measurement problem will be to assign a small nonzero value for the swap position in calculating the *relative variation* measure.

[32] This point was previously noted by Chen, Park, and Wei (1986).

[33]In simulations over many parameter combinations, we find reasonable values for U and A to be 100,000 and 1,000 respectively.

variation measures from equations (14) and (15) for swaps and the *relative variation* measures of other claims. After all, *relative variation* is the proper metric of relative risk, its monotonic transformation, the *stochastic duration* measure, is merely intended to represent the riskiness in time units.

Third, since the *stochastic duration* measure is a positive monotonic transformation of the *relative variation* measure, the direction of effect of the various parameters or variables of interest on the *stochastic duration* or interest rate risk can be observed from the *relative variation* measure.

3.2 The Behavior of Relative Variation and Stochastic Duration

In this subsection, we first compare the dynamic interest rate risk of an existing swap position to that of the reference coupon bond. The effects of the spot rate and the swap maturity on the dynamic interest rate risk of an existing swap position are then discussed. For the sake of brevity, we limit our discussion to the case of an existing *long* swap position.

3.2.1 Long swap position vs. reference coupon bond

To recollect, the reference coupon bond's maturity is the same as the swap's remaining time to maturity and its coupon rate (paid semiannually) is equal to the swap's original fixed rate less the swap bid spread.

Remark 7

The interest rate risk of a swap position is substantially greater than that of the corresponding reference coupon bond.

Table 2 About Here

This can be seen from a comparison of the respective relative variations. Table 2 presents the percentage changes in the market values of the *long* swap and the corresponding reference coupon bond for a change of 10 basis points in the spot rate for swap maturities ranging from 6 months to 10 years and for initial spot rate varying from 1% to 10%. To highlight the effect of interest rate uncertainty, $\lambda=0.0$ is assumed in Table 2.

Table 2 results clearly indicate how volatile the plain vanilla swap positions are compared to the coupon bonds. The relative variation of a swap position is at least a few times greater than that of the reference coupon bond across all initial spot rate situations and all maturities. Even a very short maturity, say 1 year, swap position is much more volatile than a 10 year coupon bond.

Remark 8

Variations in the interest rate risk across different interest rate environments and maturities are greater for a swap than the reference coupon bond.

Table 2 results substantiate this remark. A distinct risk characteristic of swap positions compared to their coupon bond counterparts is that a spot rate change leads to a change in the annuity level (incremental cashflow). As we noted earlier in this paper, this cashflow effect is quite dominant.

3.2.2 Spot rate and long swap's relative variation

Let us now examine the nature of relationship between the spot rate, r, and the *relative variation*, RV_{LS}, of an existing *long* swap position. Differentiating RV_{LS} with respect to r and rearranging, we find that the direction of the relationship depends on:

$$DIFF_r = (|LS_r|_r / |LS_r|) - (|LS|_r / |LS|)$$

This is the difference between the proportionate or relative change in the *absolute* first partial of the *long* swap value and the relative change in the *absolute* value of the *long* swap. The relative changes in question depend on the level of the spot rate and the swap maturity in a complicated fashion.

Figure 5 About Here

Figure 5 presents the $DIFF_r$ for a 50 basis point change in the spot rate and the adjusted *stochastic duration*, $H^{-1}(rv_{LS})$ [34], of a *long* swap position as a function of the spot rate for three swap maturities (1 year, 5 year, and 10 year). The *stochastic duration* increases (declines) as the spot rate goes up when $DIFF_r$ is positive (negative). Roughly speaking, $DIFF_r$ is positive (negative) when the *long* swap is *out-of-the-money (in-the-money).* This leads us to the following remark.

Remark 9

9A. The interest rate risk of an existing long swap position is an increasing function of the spot rate for out-of-the-money swaps and is a decreasing function of the spot rate for in-the-money swaps.

9B. Swaps that are close to being at-the-money carry the greatest interest rate risk.

The above pattern of a swap position's interest rate risk is primarily driven by the relative change in the *absolute* value of the *long* swap. While the value of the long swap is monotonic increasing in the spot rate, its *absolute* value is not. Also, the absolute value of the swap has a kink at the exactly *at-the-money* point. However, the behavior of risk in Remark 9 cannot attributed to this kink as the said pattern emerges long before the exactly *at-the-money* point is reached.

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[34] *U* and *A* were set to 100,000 and 1,000 respectively.

An important implication of the risk pattern in Figure 5 is offered in the following remark.

Remark 10

Swaps that are out-of-the-money or in-the-money by the same amount have similar interest rate risks.

This contrasts with the commonly held view among regulators and practitioners that the holder of an *in-the-money* swap has more to lose. This view is usually based on the notion that the interest rate movements that have proved to be favorable to one side of the swap mean potential distress for the other side of the swap and hence an increased possibility of default by the latter party. Our analysis shows that too much emphasis on the default risk of existing swaps may seriously distort the relative market or interest rate risk of swaps.

3.2.3 Swap maturity and long swap's relative variation

Differentiating RV_{LS} with respect to m (or τ) and rearranging, we find that the direction of the relationship between swap maturity and its market risk depends on:

$$DIFF_m = (|LS_r|_m / |Ls_r|) - (|LS|_m / |LS|)$$

This is the difference between the relative change in the *absolute* first partial (with respect to the spot rate) of the *long* swap value and the relative change in the *absolute* value of the *long* swap, both relative changes occurring as the swap maturity increases.

Figure 6 presents the $DIFF_m$ for a 0.5 year or 6 month change in the swap maturity and the adjusted *stochastic duration*, $H^{-1}(rv_{LS})$ [35], of a *long* swap position as a function of the swap maturity for three spot rate levels (1%, 4%, and 10.5%). The risk structure is markedly different across the three spot rate levels.

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[35] *U* and *A* were set to 100,000 and 1,000 respectively.

Figure 6 About Here

At the 1% level of the spot rate, $DIFF_m$ is positive for swaps up to 3.5 year maturity and the interest rate risk increases with maturity over this range. Four year and longer maturity swaps have a negative $DIFF_m$ and the interest rate risk declines with maturity over this range. The minimum risk swap's maturity is somewhere between 3.5 and 4 years[36]. At 4% level of the spot rate, the interest rate risk is monotonically decreasing in swap maturity. When the spot rate is quite high at 10.5%, a situation opposite to that of the low spot rate obtains. Now, $DIFF_m$ is negative for swaps up to 6.0 year maturity and the interest rate risk decreases with maturity over this range. Six and a half year and longer maturity swaps have a positive $DIFF_m$ and the interest rate risk increases with maturity over this range. The minimum risk swap's maturity is somewhere between 6.0 and 6.5 years.

A key factor behind the reciprocal interest rate risk structures in low and high interest rate environments is that the first partial with respect to the spot rate is relatively high when the spot rate is low and then gets smaller with higher spot rates.

The next remark points out some key insights about the market risk structure of swap positions.

Remark 11

A. Longer maturity swaps are not necessarily riskier than shorter maturity swaps with identical fixed rates. The market risk structure of swaps depends critically on the current level of the spot rate.

B. As the spot rate (and hence the term structure) evolves stochastically, the interest rate or market risk structure of different maturity swaps may completely reverse itself.

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[36]By setting $DIFF_m = 0.0$, the minimum risk swap maturity can be found numerically.

Remark 11 underscores the distinction between swaps and bonds as well as the need for dynamic measurement of the market risk of swaps. Sundaresan suggests the relative variation (equivalently, stochastic duration) measure of Cox, Ingersoll and Ross (1979) to determine the dynamic interest rate risk of the floating rate bond used in pricing an at-the-money swap. While this may be a relevant magnitude, as we have seen the interest rate risk of a swap position differs in essential ways from that of a floating rate bond.

4. The Effect of Credit Risk

We have not dealt with credit risk of swaps so far in this paper. Recent studies (e.g., Duffie and Huang (1996), Li (1996)) suggest that the credit risk sensitivity of the at-the-money swap prices is quite modest. This is in conformity with the secondary market swap practices where most counterparties are of low credit risk and swap quotations are not conditioned on the credit risk of the counterparties. One may, however, wonder what bearings if any credit risk may have on the market risk of existing swap positions. We are unable to find discussions on this matter in the literature. In what follows, we briefly explore how credit risk might affect market risk and vice versa. A more detailed investigation of this matter is deferred for future research.

In section I, the market value of an existing swap position was determined assuming that the incremental cashflows to the dealer are certain and accordingly used the default-free discount factors, *P(j,t)'s*, to value these cashflows. We also assumed (Assumption A2) that any effect of credit risk is already embodied in the dealer's spread quotation which we take as given. These assumptions preclude any bearing that credit risk may have market risk and vice versa.

Fortunately, under certain assumptions about credit risk pricing, the default-free framework of our paper can be largely retained to accommodate credit risk of swaps. These assumptions are basically those made by Duffie and Singleton (1997)[37]: (a) default risk is exogenous, i.e., does not

[37] Sundaresan's (1991) treatment of credit risk uses a similar set of assumptions as well.

depend on the swap value; (b) default risk leads to a stochastic credit spread, s_L or s_S in the short rate for the counterparties over the risk-free short rate, r; this leads to default-adjusted short rate processes, $R_L = r + s_L$ or $R_S = r + s_S$ for the counterparties; (c) credit qualities of the counterparties do not change substantially during the life of the swap; (d) credit qualities of the counterparties are similar enough for both to be rated the same as that of a LIBOR borrower; thus a single set of default-adjusted discount factors, $L(j,t)$*'s*, based on the LIBOR rates can be used; and (e) the credit spread, s, follows a mean-reverting square root process:

$$ds(t) = \kappa_S (\theta_S - s(t))\, dt + \sigma_S \sqrt{s(t)}\ dz_s \qquad (20)$$

where dz_s is one-dimensional Wiener process that is independent of the process dz which drives the spot rate, r.

The default-risk adjusted term structure of discount bond prices is then given by:

$$L(j,t) = P(j,t)\, p(j,t)] \qquad (21)$$

$$\text{where } p(j,t) = a(j,t) \exp[-s(t)b(j,t)] \qquad (22)$$

and

$a(j,t) = [\, 2\, \gamma_S \exp\{(\gamma_S + \kappa_S + \lambda_s)j/2\} / \{(\gamma_S + \kappa_S + \lambda_s)(\exp(\gamma_S\, j)\text{-}1)+2\gamma_S \}\,]^{2\, \kappa_S\, \theta_S / \sigma_S\, 2}$,

$b(j,t) = 2(\exp(\gamma_S\, j)\text{-}1) / \{(\gamma_S + \kappa_S + \lambda_s)(\exp(\gamma_S\, j)\text{-}1)+2\gamma_S \}$,

$\gamma_S = \sqrt{\{(\kappa_S + \lambda_s)^2 + 2\sigma_s^2\}}$.

Notice that the default-risky discount bond prices, $L(j,t)$, result from adjusting the default-free prices, $P(j,t)$, downwards by the risk adjustment term structure, $p(j,t)$. By our assumption of independent sources of uncertainty for the spot rate and the credit spread, $p(j,t)$*'s* are not functions of the spot rate. This simplifies the analysis of market risk while effectively establishing a two-factor CIR model.

Proposition 6

In the two-factor CIR model, where the risk-free rate r(t) and the credit spread s(t) are the two independent factors, the market value of an existing long position, LS(m,t), is:

$$LS(m,t) = W\, [1 - B_L(m,t)\,] \qquad \textbf{(23a),}$$

$$\textit{where } B_L(m,t) = L(m,t) + [(i_0 + d_{b,mid} (m,t))/2][\textstyle\sum_{j=1,m} L(j,t)] \qquad \textbf{(23b).}$$

The market value of an existing short position, SS(m,t), is:

$$SS(m,t) = W\,[B_S(m,t) -1\,] \qquad \textbf{(24a),}$$

$$\textit{where } B_S(m,t) = L(m,t) + [(i_0 - d_{a,mid} (m,t))/2][\textstyle\sum_{j=1,m} L(j,t)] \qquad \textbf{(24b).}$$

Proof: The swap midrate or the at-the-money rate without dealer's market making spreads is now determined as: $r_s(m,t)= 2\ [1-L(m,t)]/\ [\sum_{j=1,m} L(j,t)]$, the bond-equivalent annual yield to maturity (BEY) on the *m*-maturity *par value* LIBOR quality coupon bond at time t. Entering the market-making spreads of the dealer, the bid and ask prices of the dealer are now expressed as: $i_b(m,t) = r_s(m,t) - d_{b,mid}(m,t)$, and $i_a(m,t) = r_s(m,t) + d_{a,mid}(m,t)$. Substituting these in (1) and (2) respectively and using *L(j,t)'s* instead of *P(j,t)'s* to discount the cashflows result in the above formulas.

Remark 12

The introduction of credit risk in a CIR framework increases (decreases) the market value of an existing long (short) position.

The default-risky discount bond prices are uniformly lower with credit risk which of course increases the at-the-money midrate offered by the dealer. For a given original fixed rate, this reduces the reference coupon bond prices which leads to our remark above. It is, however, likely that the original fixed rate itself is higher with credit risk. To what extent this will negate the value changes depends on the evolution of the two term structures, *P(j,t)* and *p(j,t)*, since the inception of the swap.

Even though the two term structures, *P(j,t)* and *p(j,t)*, are assumed to be driven by independent uncertainties, their marginal impacts on the default-risky discount factors, coupon bond values, and swap values are not independent. For example, when the spot rate *r* increases, holding the short credit spread *s* at the current level, by how much the long swap value *LS(m,t)* goes up now depends on the prevailing risk-adjustment term structure, *p(j,t)*. Along the same line, it is difficult to determine a priori how the (relative variation and) interest rate risk of swaps will be affected by the introduction of credit risk. Note also that the relative variation with respect to the credit spread will in turn be affected by the prevailing default-free term structure.

It is interesting to observe that in the presence of credit risk, there are in fact two sources of market risk for swap positions: (a) interest rate risk or market value fluctuations driven by unanticipated changes in the default-free term structure, and (b) credit spread risk or market value fluctuations induced by stochastic evolution of the term structure of default risk premium. While some researchers (e.g., Duffie and Huang (1996)) have modeled the dependence of credit risk on the market value of swap, the dependence of market risk (variance of market value) on the credit spread remains to be explored. From our brief encounter above, it seems that insightful patterns of interaction between the interest rate risk and credit spread risk may emerge from a detailed examination of these matters. Given the scope of the subject matter, it is deferred for future research.

5. Conclusions

In this paper we build upon Sundaresan's (1991) work to model the marked to market values of the two counterparties' positions in a previously established plain vanilla swap. These values are shown to be functions of swap-specific factors that include the relative sizes and the different remaining maturities of the swap contracts, the original fixed interest rates, and the current market prices for the *par value* swaps with the same maturities. Using the general equilibrium term structure model of Cox, Ingersoll, and Ross (1985) and following Cox, Ingersoll, and Ross (1979), we have also shown how to measure the interest rate or market risk of an existing swap position. This risk measure is dynamic in the sense that it changes over time as a function of the stochastic evolution of the spot rate and the term structure of interest rates. It should thus be useful in managing interest rate exposure using swaps and swap related derivatives[38].

[38] Kim and Koppenhaver (1992) find a positive relation between the long-term interest rate exposure of a bank and the likelihood and extent of swap market participation.

Our models of market value and interest rate risk are mutually consistent and thus offer a unified framework for a *value cum risk accounting method* for swaps. This framework is also general enough to be applicable to other functionally equivalent transactions of a firm. The valuation and risk measurement framework of this paper should thus help the development of a more effective reporting and regulatory system for derivatives transactions as suggested by Merton and Bodie (1995). Our framework can also be useful in secondary market swap management[39].

The market value and risk of interest rate swaps are intimately linked to those magnitudes for Treasury securities as indicated by the market convention of quoting the swap prices as spreads over the corresponding maturity Treasury yields. However, our analysis indicates important differences between the values and the interest rate risks of bonds and existing swap positions. The value of a swap position is shown to be the discount or premium, as the case may be, from the face value (equal to the notional principal of swap) of a reference coupon bond. Thus the swap positions behave like the discount or premium rather than the price itself of a coupon bond. The importance of this difference is clearly visible in the behavior of the interest rate risk.

The simulation results of this paper show that the interest rate risk of a swap position is substantially greater than that of the same maturity coupon bond. Swap positions that are closer to being at-the-money seem to carry the greatest interest rate risk. The interest rate risk diminishes as the swap position becomes either deeper out-of-the-money or deeper in-the-money. Unlike coupon bonds and contrary to our initial perceptions, we find that the shorter maturity swaps can often exhibit greater volatility to unanticipated interest rate variations than the longer maturity ones. Thus, as noted by Litzenberger (1992, p.831), *indeed* " .. there is more to these plain vanilla swaps than first meets the eye."

Much work remains to be done in this area. The sensitivity of the results that rely upon the one-factor CIR model need to be looked into under alternative term structure assumptions,[40] under credit

[39] Chen and Millon (1989) discuss various methods of swap management.

[40] See Rogers (1995) for a review of term structure models. Kalotay, Williams and Fabozzi (1993) develop duration measure for the Black-Derman-Toy binomial term structure model while Bierwag (1996) proposes a measure of duration under the Ho-Lee binomial term structure model. Recently, Chang and Ho (2000) have derived a duration

risk, or perhaps using new risk measurement procedures (e.g., Bierwag (1996), Wu (1996)). Analyses of more complex swaps, e.g., differential swaps, amortizing swaps, etc., and swap derivatives are expected to reveal more intricacies and perhaps surprises in terms of their relationship to bonds. Given data availability, this paper's predictions regarding the market value and risk of existing swap positions may also be empirically tested.

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measure similar to Bierwag (1996) for the Heath-Jarrow-Morton term structure model. The duration measure in Wu (1996), on the other hand, applies to the Vasicek and CIR term structure models.

Acknowledgements

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Appendix A: Determination of Dealer's Quotes on Swap Prices

By market convention, dealer's bid and ask prices are quoted as spreads over the bond equivalent asked yield to maturity, $r_{on}(m,t)$, of the corresponding maturity *on-the-run* (most recently issued) Treasury security:

$$i_b(m,t) = r_{on}(m,t) + d_{b,on}(m,t), \text{ and}$$
$$i_a(m,t) = r_{on}(m,t) + d_{a,on}(m,t),$$

where $d_{a,on}(m,t)$ - $d_{b,on}(m,t)$ is the spread dealer hopes to make as a market maker[41]. For example, a swap dealer's quote for 5-year (m=10) fixed-for-floating swap may be like T+ 45 - T+41, which means the swap dealer is willing to receive (pay) a fixed rate for 5 years at the current BEY on the 5-year *on-the-run* Treasury note plus 45 (41) basis points against paying (receiving) the floating 6-month LIBOR flat. If the current BEY on 5-year *on-the-run* Treasury note, $r_{on}(10,0)$, is 7%, in our notation, $d_{b,on}(10,0)=0.0041$, $d_{a,on}(10,0)=0.0045$, $i_b(10,0) = 0.0741$, and $i_a(10,0) = 0.0745$.

While the dealers quote their bid and ask prices as spreads over the BEY of *on-the-run* Treasury security, these prices are first arrived at by subtracting and adding spreads over what is called the swap *midrate*, $r_{mid}(m,t)$:

$$i_b(m,t) = r_{mid}(m,t) - d_{b,mid}(m,t), \text{ and}$$
$$i_a(m,t) = r_{mid}(m,t) + d_{a,mid}(m,t),$$

where usually $d_{a,mid}(m,t) = d_{b,mid}(m,t)$[42]. To continue our example, the swap midrate, $r_{mid}(m,t)$, was first calculated at 7.43%, and then 2 basis points ($=d_{a,mid}(m,t)=d_{b,mid}(m,t)$) were subtracted and added to arrive at the quoted bid and ask prices of $i_b = 0.0741$ and $i_a = 0.0745$ respectively.

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[41] With the massive growth in the swap market and increased competition among dealers, the bid spread and the ask spread over Treasury have declined over time. The market-making spread (ask-bid) of the dealer has also narrowed over the years and is now typically less than 10 basis points. See the recent study of Brooks and Malhotra (1994).

To calculate the swap *midrate*, $r_{mid}(m,t)$, dealers typically employ arbitrage-free valuation approach. For short-dated (maturity less than 2 years) fixed-for-floating swaps, dealers generally use the forward LIBOR rates implied by the Eurodollar strip (strip of Eurodollar futures contracts), to calculate the no-arbitrage fixed rate, i.e., the swap *midrate* [43]. The rationale is that the dealer can hedge the floating LIBOR exposure (pay or receive) by taking appropriate position in the Eurodollar strip.

Whether short-dated or long-dated, dealers can also hedge their unmatched swap positions by taking appropriate positions in the Treasury securities, cash and/or futures. For example, if the dealer is paying fixed rate (5-year Treasury + 41 bp) on $25 million in exchange for 6-month LIBOR flat, the dealer can hedge by short selling 6-month T. Bills of $25 million face value and using the proceeds to buy 5-year Treasury. Any basis risk between T.Bill and Eurodollar exposures can be hedged by taking a position in TED (T. Bill over Eurodollar) futures. Once the match is found, the dealer can lift the hedge.

It should also be mentioned that while LIBOR is the most popular floating rate index, other interest rates (e.g., T. Bill yield) are also used as the floating rate index in fixed-for-floating swaps. In any case, the yields, explicit or implied, of the hedging vehicles (cash and/or futures) relevant to dollar-denominated interest rate swaps are intimately related to the basic U.S. interest rates, namely the Treasury zero coupon yields or term structure of interest rates. As the term structure changes, dealer's bid and ask prices will change irrespective of which specific variant of arbitrage valuation is used to set the swap *midrate*. Since we intend to analyze how market conditions affect the value and risk of existing swap positions, it seems reasonable to directly (rather than indirectly) link the bid and ask prices to the term structure of interest rates.

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[42] See Marshall and Kapner (1993) for industry practices in the swap market.

[43] For some examples, see Bautista and Mahabir (1994) and Marshall and Kapner (1993, pp. 147-154).

Theoretically speaking, the swap midrate is that fixed coupon rate which equates the value of fixed coupon payments to the value of the floating rate payments. As shown by Sundaresan (1991), the latter is like the value a floating rate bond net of the present value of its principal. If the difference in default risk premium of the two counterparties is not substantial, both set of payments can be discounted using the same term structure of zero coupon rates. Otherwise, two separate term structures are needed to value the two sets of payments (Jarrow and Turnbull, 1996, p.439). However, Duffie and Huang's (1996) results indicate that (a) the swap default spread between the two counterparties is small (a swap default spread of less than 1 basis point for a bond yield spread of 100 basis points), and (b) the swap default spread is rather insensitive to yield curve movements (less than 2 basis points change in swap default spread as the difference between the short LIBOR rate and the 5-year LIBOR rate varies by 650 basis points). Like Sundaresan (1991), we do not consider default risks for the most part of our analysis. Given Duffie and Huang's (1996) results, this may not be a costly assumption.

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Figure 1

The market value, *LS(m,t)*, of an existing *long* swap position with a notional principal of $100 and maturity *m* at time *t*, as a function of the spot rate, *r(t)*. The parameter values assumed are: $\kappa=0.10$, $\theta=0.04$, $\lambda=0.0$, $\sigma=0.05$, $i_0=0.05$, and $d_b(m,t)=d_b=0.0012$.

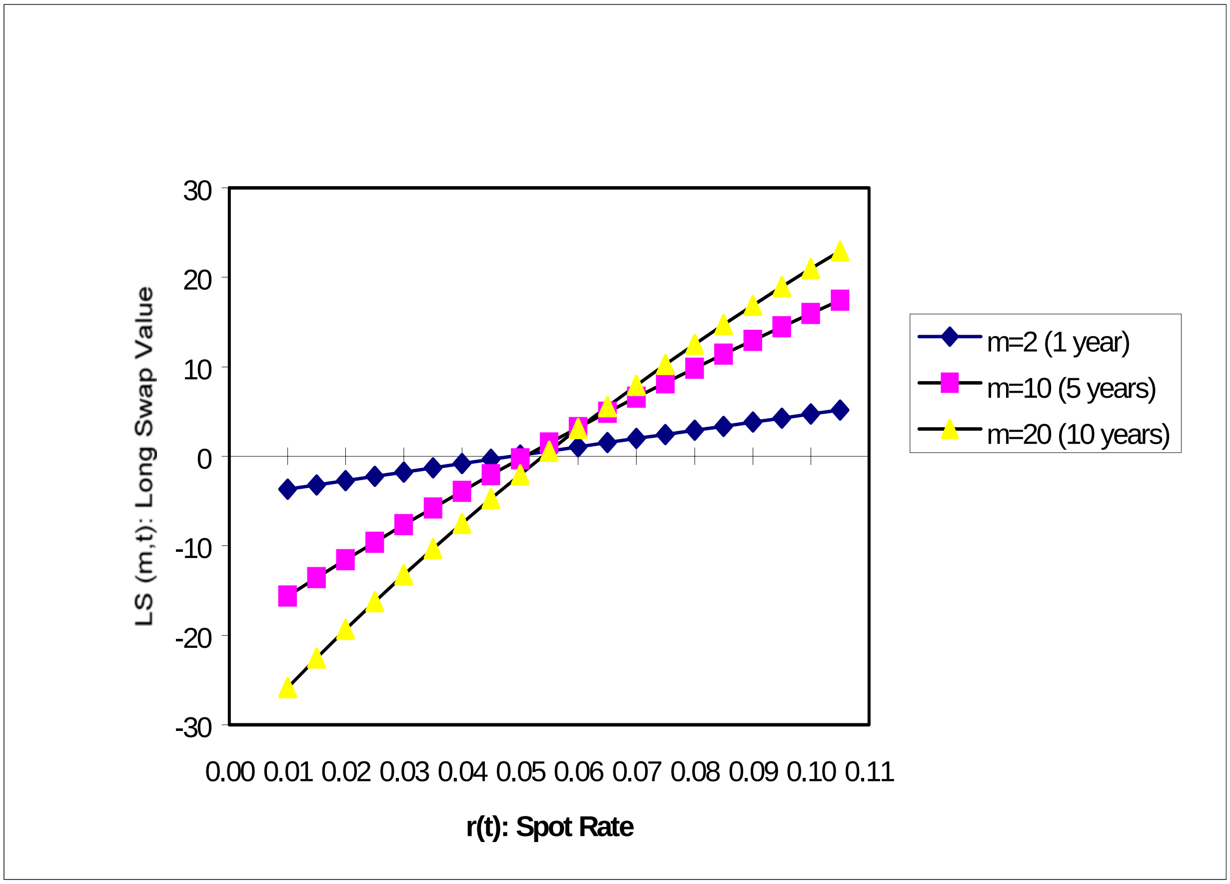


Figure 2

The market value, *LS(m,t)*, of an existing *long* swap position with a notional principal of $100, at time *t*, as a function of the swap maturity, *m*, for alternative levels of the spot rate, *r(t)*. The parameter values assumed are: $\kappa=0.10$, $\theta=0.04$, $\lambda=0.0$, $\sigma=0.05$, $i_0=0.05$, and $d_b(m,t)=d_b=0.0012$.

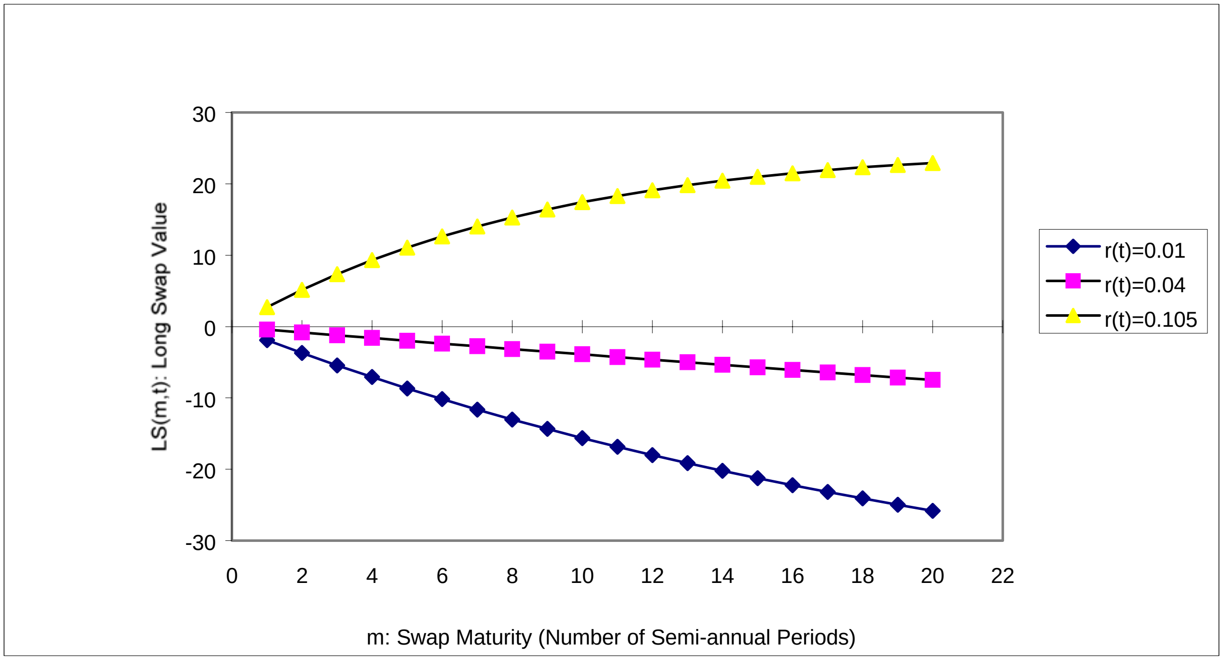


Figure 3

The market value, *LS(m,t)*, of an existing *long* swap position with a notional principal of \$100, at time *t*, as a function of the swap maturity, *m*, for alternative levels of the spot rate, *r(t)*. The parameter values assumed are: $\kappa=0.10$, $\theta=0.04$, $\lambda=-0.3$, $\sigma=0.05$, $i_0=0.05$, and $d_b(m,t)=d_b=0.0012$.

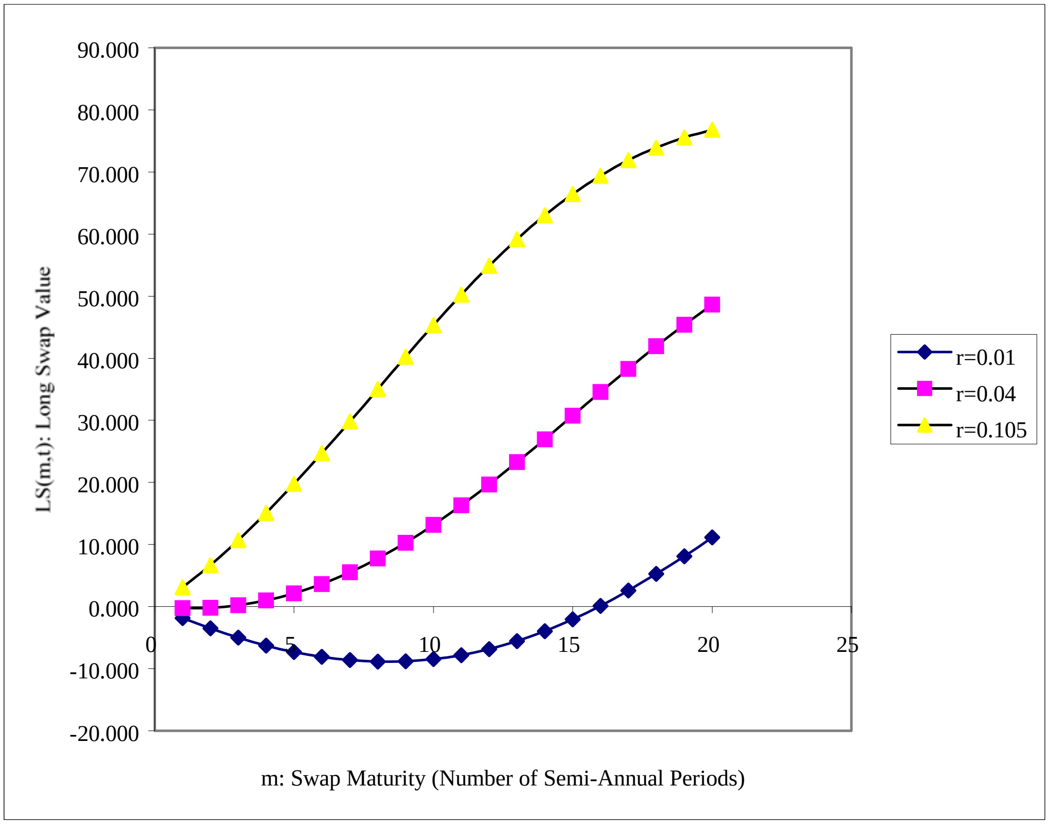


Figure 4

The market value, *LS(m,t)*, of an existing *long* swap position with a notional principal of $100, at time *t*, as a function of the swap maturity, *m*, for alternative levels of the spot rate, *r(t)*. The parameter values assumed are: $\kappa=0.10$, $\theta=0.04$, $\lambda=0.2$, $\sigma=0.05$, $i_0=0.05$, and $d_b(m,t)=d_b=0.0012$.

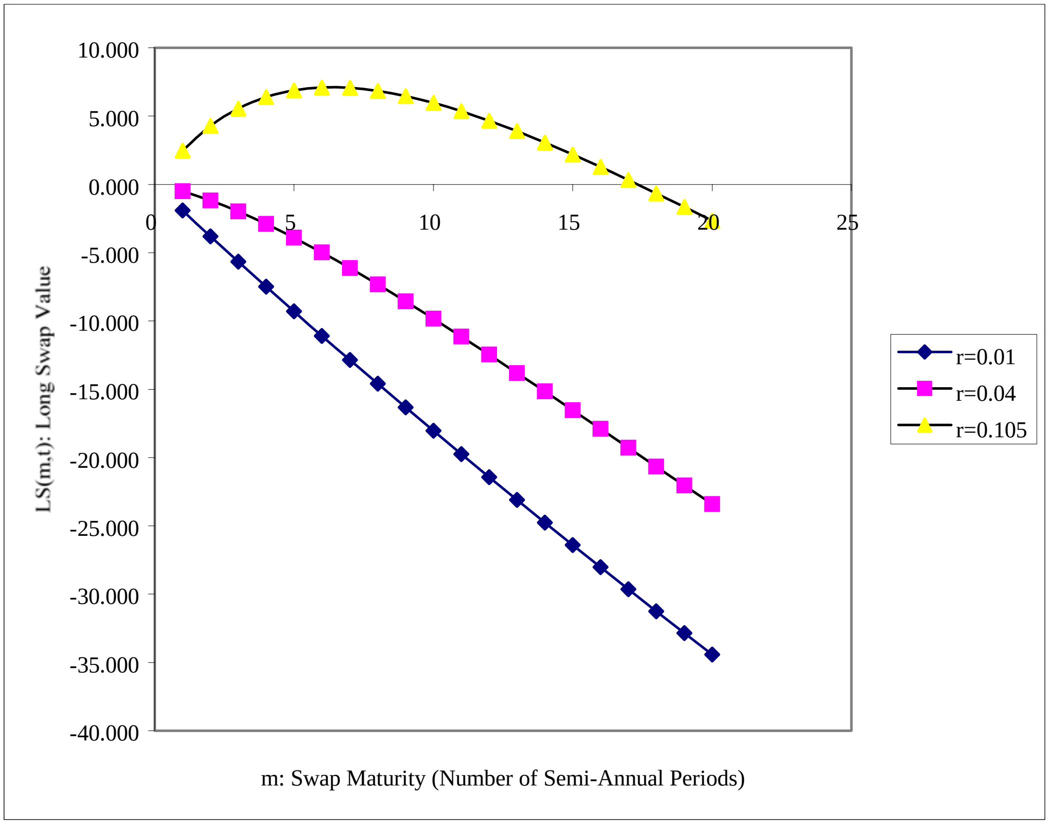
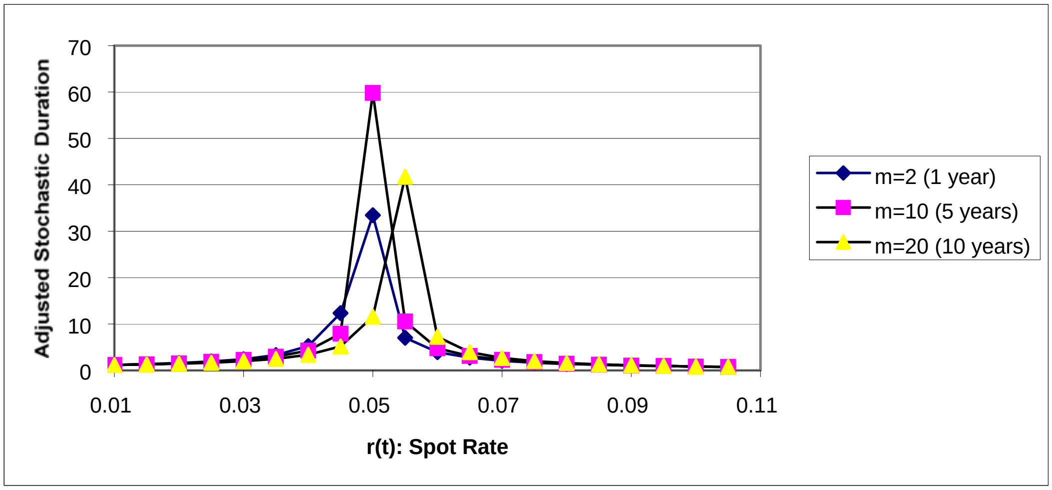


Figure 5

The adjusted stochastic duration of an existing *long* swap position with a notional principal of \$100, at time *t*, and the relative change in the absolute value of the first partial (with respect to the spot rate) of long swap value minus the relative change in the absolute value of long swap, both as a function of the spot rate, *r(t)*, for alternative levels of the swap maturity, *m*. The relative changes are for a change of 0.005 in the spot rate, *r(t)*. The parameter values assumed are: $\kappa=0.10$, $\theta=0.04$, $\lambda=0.0$, $\sigma=0.05$, $i_0=0.05$, and $d_b(m,t)=d_b=0.0012$.



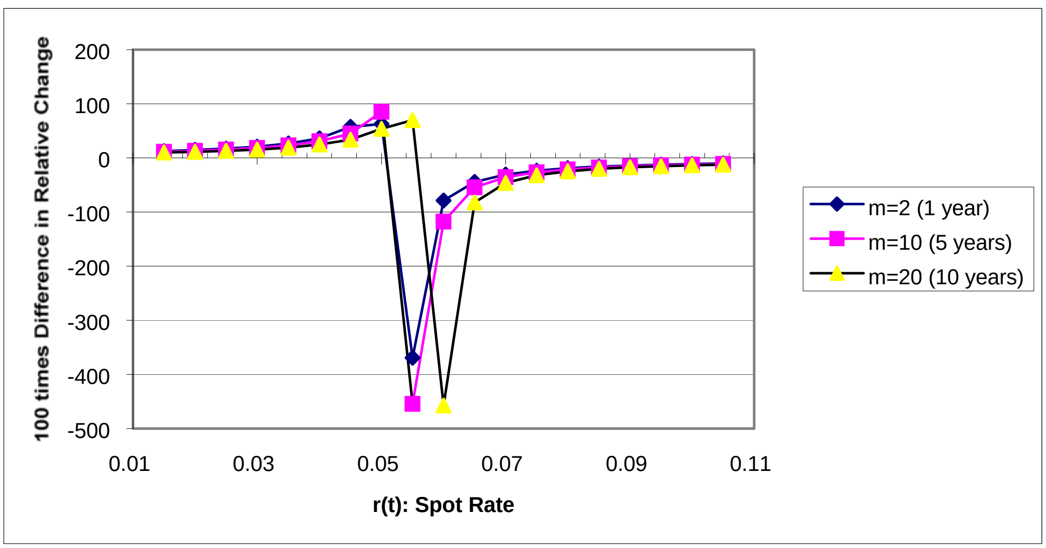


Figure 6

The adjusted stochastic duration of an existing *long* swap position with a notional principal of \$100, at time *t*, and the relative change in the absolute value of the first partial (with respect to the spot rate) of long swap value minus the relative change in the absolute value of long swap, both as a function of the swap maturity, *m*, for alternative levels of the spot rate, *r(t)*. The relative changes are for a change of 0.5 year in the swap maturity, *m*. The parameter values assumed are: $\kappa=0.10$, $\theta=0.04$, $\lambda=0.0$, $\sigma=0.05$, $i_0=0.05$, and $d_b(m,t)=d_b=0.0012$. In the top panel, the r=0.01 series is scaled to *[(10xAdjusted Stochastic Duration)-11]*50* and *the r=0.105* series is scaled to *[(10xAdjusted Stochastic Duration)-7]*50*. In the bottom panel, the *r=0.01* series is scaled to *[1000xDifference in Relative Change]*8* and the *r=0.105* series is scaled to *[1000xDifference in Relative Change]*3*.

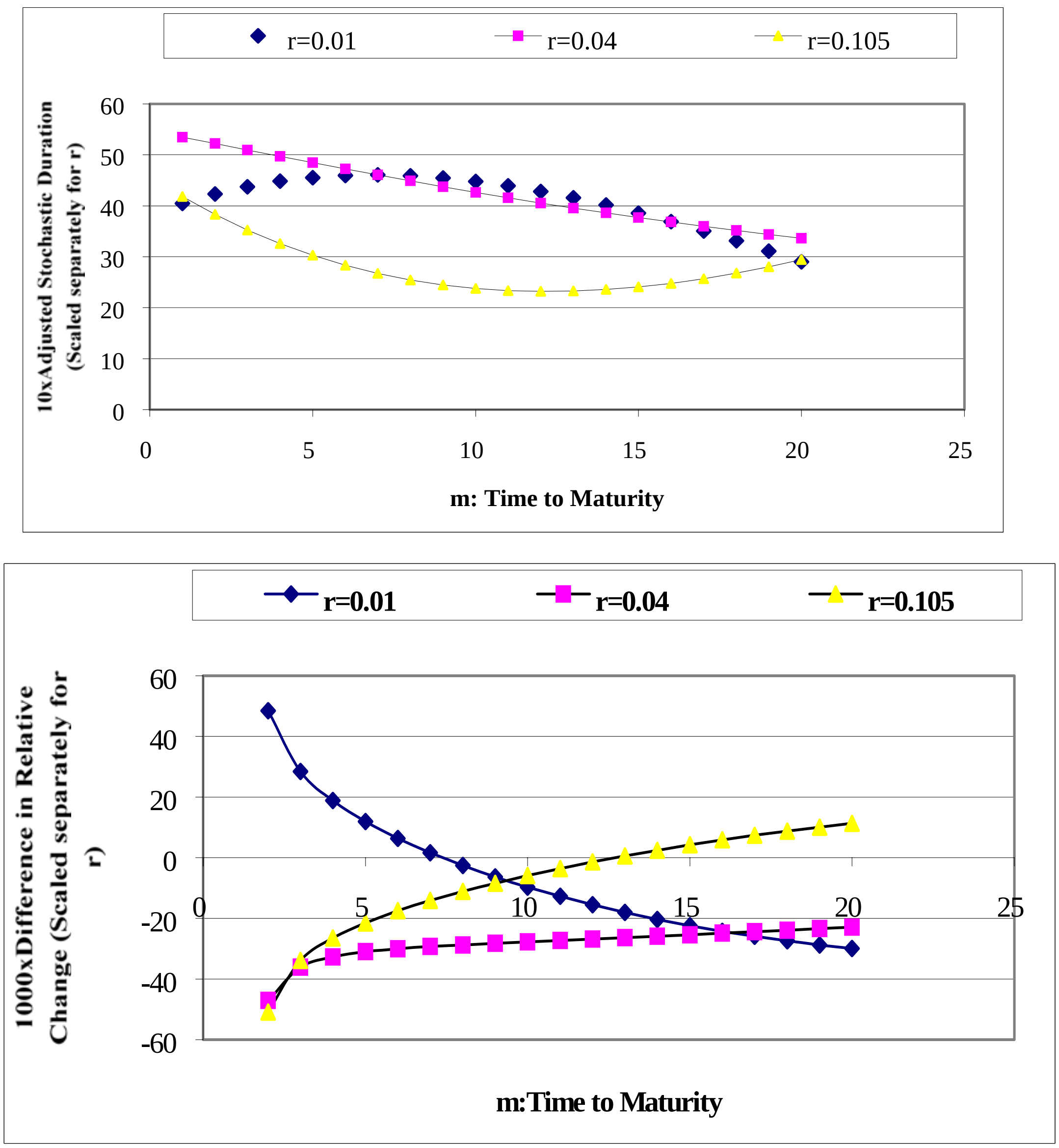


Table 1

Summary of the swap valuation effects of the spot rate, the time to maturity of the swap, and the parameters of the one-factor general equilibrium term structure model of Cox, Ingersoll, and Ross (1985)

Parameter	Effect on long swap value: First Derivative	Effect on long swap value: Second Derivative		Effect on short swap value: First Derivative	Effect on short swap value: Second Derivative
r(t)	+*	--		--*	+
m	-- (in-the-money)** +(out-of-the-money)			-- (in-the-money)** +(out-of-the-money)	
λ	--	+		+	--
κ	+	--	$r(t) < \theta$	--	+
	--	+	$r(t) > \theta$	+	--
θ	+	--		--	+

* The magnitude is higher for longer maturity.

** In general, but not always.

Table 2

Percentage changes in the value of a *long* swap position and the corresponding reference coupon bond for a change of 10 basis points in the spot rate from its initial level, $r(t)$. The notional principal of the *long* swap and the face value of the coupon bond are both set equal to \$1. The parameter values assumed are: $\kappa=0.10$, $\theta=0.04$, $\lambda=0.0$, $\sigma=0.05$, $i_0=0.05$, and $d_b(m,t)=d_b=0.0012$.

A. Reference coupon bond

Spot rate, *r*	Time to maturity, *m* (number of semiannual periods) 1	2	4	6	8	10	12	14	16	18	20
0.010	0.049	0.094	0.175	0.245	0.305	0.358	0.403	0.441	0.475	0.504	0.529
0.020	0.049	0.094	0.175	0.245	0.305	0.357	0.402	0.440	0.473	0.502	0.526
0.030	0.049	0.094	0.175	0.245	0.305	0.356	0.401	0.439	0.471	0.499	0.524
0.040	0.049	0.094	0.175	0.245	0.304	0.356	0.399	0.437	0.469	0.497	0.521
0.050	0.049	0.094	0.175	0.244	0.304	0.355	0.398	0.436	0.468	0.495	0.518
0.060	0.049	0.094	0.175	0.244	0.303	0.354	0.397	0.434	0.466	0.493	0.515
0.070	0.049	0.094	0.175	0.244	0.303	0.353	0.396	0.433	0.464	0.490	0.513
0.080	0.049	0.094	0.175	0.244	0.303	0.353	0.395	0.431	0.462	0.488	0.510
0.090	0.049	0.094	0.175	0.244	0.302	0.352	0.394	0.430	0.460	0.485	0.507
0.100	0.049	0.094	0.175	0.243	0.302	0.351	0.393	0.428	0.458	0.483	0.504

Table 2 Continued

Percentage changes in the value of a *long* swap position and the corresponding reference coupon bond for a change of 10 basis points in the spot rate from its initial level, $r(t)$. The notional principal of the *long* swap and the face value of the coupon bond are both set equal to \$1. The parameter values assumed are: $\kappa=0.10$, $\theta=0.04$, $\lambda=0.0$, $\sigma=0.05$, $i_0=0.05$, and $d_b(m,t)=d_b=0.0012$.

B. *Long* swap

Spot rate, *r*	Time to maturity, *m* (number of semiannual periods) 1	2	4	6	8	10	12	14	16	18	20
0.010	2.627	2.635	2.646	2.651	2.651	2.646	2.637	2.625	2.611	2.594	2.576
0.020	3.542	3.538	3.523	3.502	3.474	3.442	3.407	3.368	3.328	3.286	3.244
0.030	5.451	5.410	5.319	5.219	5.115	5.007	4.898	4.789	4.681	4.576	4.473
0.040	11.892	11.610	11.049	10.502	9.978	9.483	9.018	8.585	8.184	7.815	7.474
0.050	63.516	73.950	114.146	276.313	541.007	131.759	73.775	50.771	38.505	30.925	25.802
0.060	8.616	8.762	9.106	9.525	10.030	10.636	11.362	12.234	13.288	14.573	16.162
0.070	4.611	4.636	4.702	4.786	4.890	5.012	5.155	5.318	5.502	5.708	5.938
0.080	3.143	3.143	3.151	3.169	3.198	3.236	3.284	3.340	3.404	3.477	3.557
0.090	2.381	2.371	2.358	2.354	2.356	2.366	2.382	2.403	2.430	2.461	2.497