How Costly Are Limited Liability Rules?

Michel A. Robe

School of Business
University of Miami
5250 University Drive
Coral Gables, Florida 33124

mrobe@homer.bus.miami.edu

(305) 284-1884

Faculty of Management*
McGill University
1001 Sherbrooke Street W.
Montreal, Canada H3A 1G5

robe@management.mcgill.ca

(514) 398-5204

Current version: November, 1998

---

* Mailing address. I am grateful to S. Bhattacharyya, P. Fishe and H. Robe for very helpful discussions and many suggestions. Many thanks also to T. Burch, B. Demonty, P. Faynzilberg, C. Fluet, K. Jacobs, R. Marquez, D. Mauer, S. Pallage, M. Vanwijck and seminar participants at UQAM and McGill for useful comments. Initial financial support for this project came from a Dean's Summer Research Grant at the University of Miami. The usual disclaimer applies.

How Costly Are Limited Liability Rules?

ABSTRACT

We quantify the cost of limited liability rules in a traditional model of investment financing under moral hazard and risk aversion. Under limited liability, we show that external debt and the granting of absolute priority to debtholders are key (but not the unique) ingredients of the optimal financing package. Removing liability limits makes possible a superior contract that punishes the entrepreneur harshly for bad performance. We interpret these harsh penalties in terms of debtors' prisons, and quantify the deadweight losses brought about by the judicial imposition of liability limits. For reasonable parametrizations, the losses are significant. Our results indicate that the costs associated with unlimited liability and debtors' prisons must be substantial to explain the ubiquity of limited liability rules in modern economies.

JEL-Classification: G32, D82

Keywords: Financial Contracting, Security Design, Limited Liability, Debtors' Prison, Moral Hazard

I. Introduction

In early Roman times, the law gave borrowers every incentive to work hard to repay their debts because failure to pay meant losing, for years to come, both their freedom and the right to the fruits of their labor. As recently as the nineteenth century, the threat of bondage was a key ingredient of financial agency relationships in many countries. Bondage and debtors' prison, however, are seldom legal in modern economies. Instead of being menaced with seizure and bondage, borrowers and other agents now enjoy limited liability.

One might think that the standard agency model with *ex-ante* action choices (Holmström, 1979), used to analyze many moral hazard issues in financial economics, would predict the optimality of limited liability. Yet, for reasonable parametrizations of that model, we show that the threat of bondage is an integral component of the optimal or "second-best" contract. This is true even though we posit that it is the risk-averse entrepreneur (i.e., the "agent") who designs the contract and maximizes his expected utility.

This result raises several questions. First and foremost, if limited liability is not optimal, how large is the deadweight loss brought about by a ban on debtors' prisons-- i.e., by a legal mandate of liability limits? Second, do familiar mechanisms, such as imposing performance targets on agents or giving absolute priority to creditors, emerge as optimal responses to societal demands that debtors' prisons be abolished? Finally, can a case be made that legal bounds on entrepreneurs' liability may have contributed to the advent of more sophisticated financial instruments, such as warrants?

We answer those questions by solving a standard one-period model of investment financing under moral hazard. We solve the model numerically because, with or without liability limits, the equations that define the optimal contracts cannot be solved analytically. We employ commonly used parametrizations that validate the "first-order approach," and calibrate the model in line with empirical evidence showing that moral hazard itself is quite costly.

In this environment, the optimal financing contract can stipulate monetary transfers from the agent to the principal that potentially exceed the project's output and, thus, the entrepreneur's current resources. Because such excessive transfers correspond to negative consumption for the entrepreneur, they are well-defined for some typical preference choices and have a natural interpretation in terms of debtors' prison.

When the entrepreneur exhibits either constant absolute or constant relative risk aversion, our first result is that a legal requirement that his income from the project never be negative is almost always binding. That is, across all reasonable parameter combinations, limited liability rules make it optimal for the agent to receive no income unless output exceeds a given threshold. In an employment context, this finding means that compensation contracts should normally specify a performance target for managers. In our investment-financing framework, the same result implies that external debt is a key component of the optimal financing package and that creditors' claims are given absolute priority if output falls below a pre-specified level.

In the absence of limited liability rules, negative consumption (i.e., bondage) is possible when the entrepreneur has exponential preferences. Our second result is that the entrepreneur does prefer contracts that entail such harsh penalties when output is low. For example, across a wide range of parametrizations, he optimally precommits to a monetary penalty for zero output that exceeds the entire income he expects from the project.

The law in almost all countries, however, bans bondage and debtors' prisons. How large is the deadweight loss caused by legally-mandated liability limits?

Our third result is that the deadweight loss from limited liability rules, over and above the deadweight cost due to moral hazard *per se*, is substantial. At the calibrated parameter values, incentive-compatibility requirements cut the entrepreneur's certainty equivalent consumption by three fourths from its first-best level -- i.e., from what it would be if there were no incentive problems. In absolute terms, that deadweight loss amounts to 13.5% of the firm's assets. Imposing limited liability reduces the entrepreneur 's certainty-equivalent consumption by a further 4.6%, which is equivalent to about 0.6% of company assets. This extra loss is substantial, given that our calibrated rate of return on investment is 12.1% (under moral hazard but with no legal limit on liability).

Our results are robust: we find similarly large extra deadweight costs across a variety of technologies and parameter combinations. Indeed, the extra loss due to limited liability rules exceeds 4.7% of assets in some cases. Furthermore, there exist reasonable parameter choices for which investing is unprofitable when the entrepreneur's monetary payoff from the project is not allowed to become negative -- even though the same investments would be profitable in the absence of liability limits.

Given that limited liability rules are costly, a natural question is what qualitative impact they have on the optimal contract. We have already argued that contracting parties will give absolute priority to creditors' claims in an attempt to control the costs caused by limited liability. We also argue that legal bounds on liability may foster the use of sophisticated financial instruments, such as warrants. We show that the severity of the deadweight loss due to limited liability rules increases if agents can only issue debt and equity but, in contrast, does not worsen as long as warrants can be issued as well.

Except under special conditions, we know that two-piece, piece-wise linear sharing rules are not optimal in the present environment regardless of the liability regime. Still, debt (to control agency conflicts) and equity (for risk sharing) were for ages the main financing tools.[1] The erstwhile predominance of these simple contracts makes our fourth result interesting.

When agents are restricted to these simple securities, we find that the deadweight losses caused by exogenous liability limits remain large and, in many cases, increase significantly. In contrast, the magnitude of the deadweight loss caused by exogenous liability limits can be controlled almost perfectly whenever warrants can be issued alongside debt and equity. This is because a combination of equity, straight debt and a warrant issue is almost optimal in the modeled environment.[2]

One possible interpretation of these results is that primitive contracting environments, similar to those in which debt bondage and debtors' prisons once flourished, are also those in

[1] Rent and share tenancy contracts have, likewise, long been a fixture of agricultural societies.

[2] See Robe (forthcoming). Put differently, a three-piece, piece-wise linear output sharing rule is almost optimal in the standard agency model studied here.

which limited liability rules are potentially most costly and, hence, least likely to emerge. An alternative interpretation of our results is that the bounds on agents' liability, imposed by many countries during the course of the nineteenth century, may have contributed to the development of modern financial instruments such as warrants.

The remainder of the paper proceeds as follows. Section II reviews the related literature. Section III describes the setup. Section IV brings in liability limits. Section V parametrizes the model. Sections VI to VIII present the results. Section IX and X discuss their robustness to the choice of parametrization and contracting environment, respectively. Section XI concludes.

II. Related Work

It has long been recognized that harsh punishments may provide ideal, but infeasible, incentives in moral hazard problems.[3] Many papers have accordingly examined the impact of liability limits on the solution to agency problems.[4] Most of those other papers, however, seek to characterize that impact analytically under conditions, such as risk neutrality, that keep the model tractable. Innes (1990), for example, derives the optimal contract in a standard agency setting in which a risk-neutral entrepreneur makes *ex-ante* choices under limited liability. He shows that a debt contract implements the optimal non-decreasing sharing rule. Our setup is similar -- with the key distinction that the entrepreneur is risk-averse. As a result, even though debt and absolute priority rules (or a performance target) remain essential components of the optimal sharing rule under limited liability, a straight debt contract alone is almost never optimal.[5]

Our approach is different. We assess the prevalence and the magnitude of departures from limited liability in a typical contracting setting in which productive efficiency must be traded-off

[3] See Mirrlees (1976) for an early discussion.

[4] Recent reviews of this literature are presented in Sappington (1991) and Demougin and Fluet (forthcoming).

[5] Innes (1993) builds on Innes (1990) to show that, even under risk neutrality, a pure debt contract is almost never optimal if the principal's payoff function can depend on both the output and the output's price. In that setting, the optimal contract entails not only straight debt but commodity futures and commodity call-option contracts as well. In our setup, the optimal contract under limited liability can be approximated almost perfectly by a combination of equity, straight debt and a single warrant issue – see Robe (forthcoming).

against risk-sharing, and then quantify the deadweight costs of exogenous (legal) liability limits in such an environment.[6]

The present paper is therefore connected to the analyses of Stoughton (1993), Boyd and Smith (1994) and Robe (1998), in that we quantify the deadweight costs associated with some exogenous contracting restrictions under moral hazard.

Stoughton (1993) models a risk-averse investor who wishes to allocate her wealth between two assets. After eliciting research effort from an analyst, she must motivate him to truthfully report his information. When the return distribution is symmetric, a quadratic contract helps control the appeal to the analyst of undoing his effort choice *via* portfolio selection. The cost to the investor of inducing truth-telling (suboptimal risk sharing with the agent) is negligible, and her expected utility is nearly first-best, when she is almost risk-neutral and is large relative to the agent. We focus instead on the impact of limited liability rules in an environment where, even without such rules, the optimal contract is far from first-best and moral hazard is very costly.

Boyd and Smith (1994) and Robe (1998) show numerically that, given limited liability, very small welfare losses ensue if corporate financing is restricted to standard securities. Boyd and Smith show that straight debt and internal equity are almost optimal financing instruments, in a costly state verification environment in which cash-flows are not observable. When the agent's action (rather than output) is unobservable, Robe demonstrates that only negligible efficiency gains are achievable by issuing more complicated securities than equity, one layer of straight debt and one class of warrants. We use a framework similar to Robe's to show that, in contrast, limited liability requirements themselves are quite costly and that debtors' prisons can be a key component of the optimal contract.

---

[6] We abstract from situations in which output can only be verified at a cost, the agent has or chooses to acquire better ex-ante information than the principal, the agent makes unobservable choices after observing the state of nature, or the principal can monitor the agent's actions. Risk-neutral analyses showing that liability limits do have a first-order qualitative impact in such settings can be found, respectively, in: Diamond (1984); Demougin and Garvie (1991) and Lewis and Sappington (1997); Sappington (1983); and Demougin and Fluet (forthcoming).

Our result, that liability limits strongly affect effort choices in the presence of moral hazard, presents an interesting counterpoint to recent empirical evidence documented by Esty (1998) that liability rules affect bank managers' risk-taking behavior significantly.

Finally our finding that it may be optimal to expand an entrepreneur's liability beyond his firm's resources complements results derived in other financial models. Winton (1993), for example, shows that unlimited liability may (but need not) be optimal when shareholders must balance improved incentives to monitor managers against the reduced liquidity of their shares. The main difference here is that we take an environment in which no investment would take place if the entrepreneur could trade his shares, and then quantify the cost of limited liability rules. Thus, our contribution is both to interpret severe penalties in terms of the optimality of bondage or debtors' prisons and, more importantly, to show that these institutions can bring about significant efficiency gains.[7]

Our treatment of debtors' prisons is related to a recent paper by Welch (1995). He uses a costly state verification setup to show that, given limited liability and risk neutrality, meting out non-monetary penalties to defaulted borrowers helps reduce fraud and (together with bankruptcy protection laws) can overcome inefficiencies caused by the borrower's inability to precommit. Our analysis differs in two main respects. First, all the penalties imposed on our entrepreneur are monetary and, hence, directly benefit investors. That is, bondage and debtors' prisons here are viewed as an institution that makes possible the collection of monetary penalties in excess of the agent's current resources. We provide support for this modeling of debtors' prison through examples of seizures in Roman times and of debtors' prisons in England.[8] Second, we consider the financing choices of entrepreneurs whose decisions have a significant impact on company performance and who retain a significant residual claim on their firms. Because it is improbable

---

[7] Our conclusion, that liability limits may hurt welfare, is also related to the prediction of efficiency wage models that involuntary unemployment would disappear if workers could post bonds or forfeit lifetime retirement benefits – see, e.g., Carmichael (1991).

[8] Replacing monetary penalties by non-monetary ones in our setup should result in allocations that, while superior to those attainable under limited liability, are dominated by the outcomes made possible by the threat of harsh monetary penalties – see also Chu and Jiang (1993).

that such individuals hold well-diversified portfolios, we assume that they are risk averse. The difference between our results, in which debt financing is not optimal, and those of Innes (1990) or Welch (1995), who find that it is, justifies the importance of these risk-sharing considerations.

III. Model

We consider a traditional one-period model of investment financing under moral hazard, similar to Robe (1998). A cash-strapped entrepreneur (the "agent") must raise a fixed amount I to finance a new project. The project's attributes are common knowledge but its uncertain return, y, is a function of the agent 's unobservable effort level, a. The output y can be viewed as a random variable with distribution $F(y,a)$ and density $f(y,a)$ parametrized by a. The support of y, denoted $Y \subset \Re^{+}$, is independent of a.

The return y is the entrepreneur's only source of income. His total utility is a separable function of his income, c, and effort, a: $U(c,a) \equiv u(c)-v(a)$. The functions $u(\bullet): \Re \to \Re$ and $v(\bullet): \Re^{+} \to \Re^{+}$ are three times continuously differentiable, with $u(\bullet)$ strictly increasing concave and $v(\bullet)$ increasing convex. Unlike the agent, outside investors (the "principal") are posited to be risk neutral. This assumption is a good approximation for situations in which investors holds a well-diversified portfolio of projects and are indifferent to any given project's specific risk. For simplicity, the discount rates of both principal and agent are assumed to be 0.

The entrepreneur designs the financing contract to maximize his expected utility, subject to two constraints. The first constraint is individual rationality (IR). Outside investors will not provide funds unless they expect at least a competitive rate of return on I (their investment). Without loss of generality, that competitive rate is set equal to 0.

The second constraint is incentive compatibility (IC). Once they finance the project, investors face moral hazard because the agent's effort level, a, is not observable. Consequently, the output-sharing rule must be consistent with the effort level promised by the entrepreneur. We replace this requirement by the more manageable constraint that the entrepreneur select an effort level at which, given the financing terms, his utility is at a stationary point. Jewitt (1988) identifies conditions on $F(\bullet)$ and $u(\bullet)$ that validate this "first-order approach" in the Holmström (1979) agency model and, thus, in the present setting.[9,10]

There is no other market imperfection, such as taxes or bankruptcy costs. The output is verifiable at no cost. And, the statistical properties of the stochastic output *y*, the production process $F(\bullet)$, and the preferences of principal and agent, are all common knowledge.

Let *t(y)* be the entrepreneur's monetary payoff in terms of the project's output, *y*. In a "first-best" world in which *a* is costlessly verifiable, the risk-averse entrepreneur would exchange all rights to the output against a fixed sum from the risk-neutral investors. Given that *a* is not verifiable and that *t(y)* is his sole source of income, he faces the "second-best" problem:

(1)
$$\max_{a,\, t(.)} \left[\int_Y u(t(y))\, f(y,a)\, dy \;-\; v(a) \right]$$

$$\text{s.t.:} \quad \text{(IC)} \quad \int_Y u(t(y))\, f_a(y,a)\, dy \;=\; v'(a)$$

$$\text{(IR)} \quad \int_Y [\, y - t(y)\,]\, f(y,a)\, dy \;=\; I$$

IV. Liability Limits

In the above model, the optimal contract may call for monetary transfers from the agent to the principal in excess of the project's output. One might view these events simply as situations in which the entrepreneur does not enjoy limited liability. To the extent that the payment *t(y)* constitutes his only source of income, however, transfers that exceed the project's output will also exceed the entrepreneur's resources. Such negative consumption is well defined under some of

---

[9] Solutions to our problem and to the Holmström (1979) model, in which the principal optimizes subject to eliciting agent participation, define points on the same utility-possibility frontier and are therefore qualitatively similar.

[10] One restriction on $F(\bullet)$ is that it have a concave, monotone likelihood ratio. The other two restrictions on $F(\bullet)$ are weaker than assuming a random production function with decreasing marginal returns in each state of nature. Examples of distributions that meet these conditions are the Gamma, Poisson and Chi-square. Finally, restrictions on $u(\bullet)$ are satisfied by any constant absolute risk-averse utility function and by any non-decreasing relative risk averse utility function with coefficient of relative risk aversion strictly greater than 0.5.

the very specifications for the agent's preference that should validate the first-order approach. Hence, these preference specifications may also imply that the second-best sharing rule that solves program (1), say $t^*(y)$, differs from the "third-best" sharing rule, say $s^*(y)$, that solves the contracting problem when the agent has limited liability (LL):

(2)
$$\max_{a,\, s(.)} \left[\int_Y u(s(y))\, f(y,a)\, dy \ - v(a) \right]$$

s.t.: (LL) $0 = s(y) = y$

(IC) $$\int_Y u(s(y))\, f_a(y,a)\, dy \ = \ v'(a)$$

(IR) $$\int_Y [\, y - s(y)\,]\, f(y,a)\, dy \ = \ I$$

We claim that monetary penalties on the entrepreneur greater than the project's output -- $t^*(y) \leq 0$ -- are not only well defined, they also have a natural interpretation in terms of debt bondage or debtors' prisons. Two examples of how these institutions operated in practice demonstrate our point. In early Roman law, creditors could take hold of their debtors upon default and, after a sixty-day grace period, put them to death or sell them into slavery ("*legis actio per manus iniectionem*"). These outcomes might seem inconsistent with the model considered in this paper, because they seem to imply that there exist levels of the output *y* for which the allocations of the principal and of the agent would not sum up to *y*. In reality, however, the Roman debtor was often made to work for his creditor until the fruits of his labor had repaid the debt. In early English common law, likewise, a debtor had in theory no means to repay his creditor once put in prison for default. Still, his lack of even basic rights once in jail, and the legality of the mistreatments that ensued, could be interpreted as government-sanctioned incentives for relatives to pay off his debt.[11]

---

[11] The Roman law example is based on Vigneron (1998). Our discussion of English debtors' prison draws on Welch (1995), who gives an overview of their history and summarizes the relevant financial economics literature. Stein and Subrahmanyam (1996) and de Sainte Croix (1988) document instances of bondage in the last two centuries.

Although seizing debtors for non-fraudulent default on commercial debt used to be normal, the practice is now obsolete and illegal.[12] This raises several questions. For reasonable parametrizations of the model, are deviations from limited liability statutes the norm rather than the exception? What is their magnitude? If the model predicts that large, recurrent limited liability violations are optimal, do significant deadweight losses result from exogenously limiting the agent's liability?

V. Parametrization

To answer the above questions, the model must be parametrized and solved numerically. As pointed out in Robe (1998), whose parametrization we follow closely, numerical solutions are needed for two reasons. One, the first-order conditions for program (1), which yield the optimal contract in the absence of limited liability requirements, cannot be solved analytically. Two, the limited liability condition (LL) in program (2) rules out any guarantee that the first-order approach is even valid. Consequently, one must confirm numerically, for each parameter combination, that the solution to program (2) does implement the promised effort level.

A. Functional forms

The entrepreneur's utility from consumption $u(\bullet)$ and the technology $f(\bullet)$ are chosen to satisfy the Jewitt (1988) conditions. In most of the paper, the entrepreneur has exponential preferences over consumption and exhibits constant absolute risk aversion $g > 0$:

(3) $$u(c) \equiv -e^{-gc}$$

This utility specification is ubiquitous in the financial agency literature. In Section VI, we show that our result that liability limits are binding for reasonable parametrizations is robust to the alternative assumption of constant relative risk-averse preferences.

The output is assumed to follow a Gamma distribution with parameters t and $\frac{t}{aa}$:

[12] One of the last industrialized countries to abrogate the institution was Belgium in 1980 – see t'Kint (1991, p.21).

$$\textbf{(4)} \qquad f(y,a) \equiv \frac{y^{t-1} e^{\frac{-yt}{aq}}}{\left(\frac{aq}{t}\right)^{t} (t-1)!} \quad , \quad q > 0,\ t \geq 1$$

The main advantage of choosing density function (4) is computational. Preferences (3) and technology (4) ensure that all the integrals that make up the first-order conditions of programs (1) and (2) have a closed form, which is central to the precision of the numerical solutions.

These closed-form solutions become more complex as t increases. Accordingly, we set $t = 1$ in Sections VI and VII so as to improve computational tractability. With $t = 1$, (4) is an exponential. Thus, the density *f*(•) is monotone decreasing in the level of output and most of the probability mass is concentrated on very low output levels. In Section VIII, we set the shape parameter $t \geq 2$ so that *f*(*y*,*a*) is a bell-shaped, unimodal return distribution. Our main results are robust to the particular value taken by t.

Finally, the entrepreneur is assumed to suffer power disutility from effort:

$$\textbf{(5)} \qquad v(a) \equiv \frac{a^n}{A} \quad , \quad n>1$$

The scaling factor A>0 and the disutility parameter n yield two degrees of freedom and make (5) general enough for computations. In addition, given that *v*(•) is convex, the Euler conditions for programs (1) and (2) show that the functional form of the second-best and third-best contracts do not depend on the functional form of *v*(•). Therefore, choosing (5) implies no loss of generality *per se* -- see, e.g., Rogerson (1985) and Faynzilberg and Kumar (1997).

B. Parameter values

The above functional forms require that several parameters be calibrated: *I*, g *n*, *A*, q and the impact of the entrepreneur's effort on the project's performance. We choose values for these parameters to match U.S. data, under the assumption that program (1) summarizes the conditions faced by an entrepreneur seeking financing.

Agent's contribution

There is no recognized empirical estimate of the extent to which managerial effort helps increase project or firm performance. Haubrich (1994) submits that assuming a CEO's effort can increase his firm's value by a maximum of 4% is consistent with empirical evidence on CEO turnover in Weisbach (1988). While Haubrich uses that estimate to simulate a Grossman and Hart (1983) framework that is closely related to ours, his focus is on large corporations. In contrast, we focus on agency relationships in which the agent's effort should increase expected output very significantly and moral hazard is likely to be very costly. Consequently, rather than posit that the entrepreneur's actions can increase expected output by 4% at most, we assume instead that his *marginal* productivity at the second-best effort level lies in a range around 4%. Precisely, with technology (4), the percentage marginal productivity of effort is given by:

(6) $$\frac{\partial E[\, y|a,q\,]/\partial a}{E[\, y|a,q\,]} = \frac{1}{a}$$

We set that parameter equal to 2%, so that the calibrated value of a in the second-best equilibrium, say a^*, is equal to 50. Our qualitative results are robust to alternative value choices for a^* -- see Section IX. Accordingly, we only report results for $a^* = 50$. Note that a^* should be viewed as an index of the agent's effort rather than as a number of hours worked.

Project profitability

Let R denote the rate of return on the amount invested, I. We set R = 12.1%, the nominal annual rate of return on common stocks from 1926 to 1988 (Brealey and Myers, 1991). This value is consistent with the calibration of Boyd and Smith (1994) who, for various industries between 1972 and 1991, document annual gross rates of return on corporate assets ranging from 6.1% to 15.5 %. Still, setting R = 12.1% may underestimate the profitability of entrepreneurial firms, as it is well known that the latter have higher expected rates of return than larger ones. In sensitivity tests, we therefore also employ values for R between 5% and 40%. With technology (4), the expected return on the project is given by: $E[y\,|\,a,q] - I = a\,q\, - I$. At the calibrated second-best effort level, a^*, it must thus be true that: $q = \frac{I.\,R}{a^*}$. Without loss of generality, we set $I = a^*$. Then, for $a^* = 50$ and $R = 12.1\%$, we obtain $q = 1.121$.

Risk tolerance

Mehra and Prescott (1985), in their seminal paper on the equity premium puzzle, argue that (0, 10] is a sensible range for the coefficient of risk aversion if individuals have constant relative risk averse preferences. This range is now widely accepted. With exponential utility (3), though, the entrepreneur displays constant absolute risk aversion g We therefore bound g so that his relative risk aversion, measured at his expected income level, is in the interval (0, 10]. Given values of g between 0.1 and 0.6 and given our choices for q and n, the entrepreneur's relative risk aversion evaluated at his expected income, $g(a^* q - I)$, runs from 1.25 to 6.83 in the first-best case, and from almost zero to 5.48 in the second-best scenario. We do not consider values of g below 0.1 because doing so often leads to extremely high rates of return on assets (more than 50%). Values of g above 0.6 generate excessive average levels of relative risk aversion in some cases and are therefore ruled out as well.

Cost of effort

We conclude with the entrepreneur's disutility from effort, which depends on n and A. We set the scaling factor $A = I^2$ to keep the disutility from effort in the same range as the expected utility from consumption. Finally, we use the first-order conditions of program (1) to calibrate n. For the parameter values already chosen ($I = a^* = 50$, $A = 2500$, $q = 1.121$ and $g = 0.5$) we get $n = 1.574$. We therefore set $n = 1.574$ as the base-case value for our computations, and use values of n between 1.2 and 1.63 for robustness checks. Higher values of n are not employed because, in the third-best environment, the entrepreneur strictly prefers not to work if $n = 1.64$ -- for all values of g Table 1 summarizes our parameter choices in the CARA specification.

<Insert Table 1>

C. Scenarios

To keep the results' discussion manageable, we focus on three parameter combinations. (i) The first or "base case" scenario corresponds to the parameter values just calibrated: $q = 1.121$; $I = 50$; $g = 0.5$; and $n = 1.574$. (ii) The second or "best-case" scenario is identical to the first, except for the lower intensity of managerial disutility from effort n. We set $n = 1.472$ in order to obtain a rate of return on assets (ROA) of $R = 12.1\%$ in the *third-best* environment. The second-best ROA is therefore higher than it is in the base-case scenario: 14.1%, as opposed to 12.1%. Because incentive compatibility and its implementation are less costly whenever the agent is not strongly work-averse, reducing n makes it less necessary to punish him harshly for low output levels. As a result, limited liability rules should be less costly. (iii) In our third or "worst-case" scenario, we instead select the levels of absolute risk-aversion ($g = 0.115$) and disutility from effort ($n = 1.6$) such that imposing liability limits induces large deadweight costs. Because the entrepreneur's absolute risk-aversion is low, it is optimal to punish him harshly for low output levels. Consequently, the imposition of liability limits is very costly. We set $I = 50$ and $q = 1.121$ in all three scenarios, because additional computations with other values of I and q showed that neither parameter had any qualitative impact on our conclusions.

VI. Optimal Contract under Limited Liability

Using the parameter values in Table 1, we can solve for the optimal financing contracts.

When limited liability rules compound incentive-compatibility problems, Figure 1.a shows that the entrepreneur receives no income in equilibrium when output is low. The Figure depicts the entrepreneur’s monetary payoff in the most-likely scenario, but the results are readily generalized. For *all* combinations of the parameter values listed in Table 1, the entrepreneur is rewarded only to the extent that the output he helps realize exceeds a given level stipulated in the contract. Put differently, with constant absolute risk averse (CARA) preferences, liability limits are always binding for reasonable parameter choices.[13]

This result implies two intuitive, testable predictions. First, because the optimal sharing rule in our setup is qualitatively similar to that in the original Holmström (1979) model, our

[13] In the Appendix, we discuss similar results under the alternative specification that the entrepreneur exhibits constant relative risk aversion (CRRA). By considering both CARA and CRRA utility, we cover all of the standard preference choices in economics and finance. Note that, in the agency theoretic literature, these preference choices are meant to ensure the validity of the first-order approach. Yet, once liability limits are explicitly incorporated into the analysis, our results establish that the familiar Euler equation usually describes the optimal sharing rule only for output levels in excess of a given threshold.

analysis shows that compensation contracts should specify a performance target for managers whenever productive outcomes depend principally on managerial effort.[14,15]

Second, in terms of investment financing, the above result implies that external debt -- together with a promise to grant absolute priority to creditors' claim -- is a key component of the optimal financing package under limited liability.

These predictions extend an earlier finding by Innes (1990) that, if the entrepreneur were risk-neutral, then straight debt financing would be optimal in the presence of limited liability rules. In that setting, debt is the monotone contract that maximizes the entrepreneur's incentives to aim for high output by giving him nothing in low output states. A major difference here is that, because the entrepreneur is risk averse, ensuring productive efficiency has a cost in terms of risk-sharing. Consequently, straight debt alone is almost never optimal in the present model[16] -- but our results show that, for all reasonable parameter combinations, it is indispensable.

VII. Debt Bondage

Once we remove legal limits on liability, we find that the optimal sharing rule under CARA preferences calls for investors' payoff to exceed the available output and for the entrepreneur to make up the difference when output is sufficiently low. For *all* combinations of parameter values listed in Table 1, the contract that solves program (1) punishes the agent for low output levels over and above what is feasible given the available output. Put differently, with CARA preferences, optimal deviations from limited liability are the norm rather than the exception in our standard financial agency model. Figure 1.b depicts the second-best contract for the base-case scenario.

<Insert Figures 1, 2, 3>

---

[14] We are grateful to Pat Fishe for pointing out this interpretation.

[15] Notice also that minimum-wage laws in the present framework would correspond to situations in which $s(y) \geq W$, where $W > 0$ is the mandatory minimum wage. It is easy to show numerically that setting $W > 0$ raises the target and increases the pay-for-performance sensitivity of the contract in the output range directly beyond the target.

[16] To see this, note in Figure 1.a that the agent's monetary payoff is far from a linear function of output after fixed-claim holders have been paid off. Indeed, given limited liability, the optimal contract here can be approximated almost perfectly by a layer of straight debt *together with* external equity and a warrant issue – see Robe (1998).

The deviations from limited liability built into the second-best contract are, moreover, massive. Take, for example, $t^*(0)$ -- the amount that investors demand from the entrepreneur when the output is 0. This number is an upper bound for deviations from limited liability in the model, because the entrepreneur's optimal compensation $t^*(y)$ is increasing in y under the Jewitt (1988) conditions. In order to obtain a meaningful measure, we express $t^*(0)$ as a fraction of the expected output, E[y]. With exponentially distributed output, Figure 2 shows that the deviations are significant. Indeed, if we instead express the transfer in terms of the agent's expected income, E[y]-I, Figure 3 shows that the punishment for low output levels are extremely large: in the worst output state, $y=0$, the agent *always* faces a monetary punishment larger than his expected income.

When averaged over the support of output, limited liability violations are naturally smaller than $t^*(0)$. For the same range of parameter values as in Figures 2 and 3, we find that the average *level* deviations are indeed smaller. Because the entrepreneur is strictly risk-averse, however, using levels seriously underestimates the importance of these deviations. The next section addresses this concern by computing the welfare losses from exogenous liability limits.

VIII. Costs of Limited Liability Rules

In each contracting environment (first-, second- or third-best), we can compute the entrepreneur’s monetary payoffs, expected utilities and certainty-equivalent consumption levels (CEC) -- i.e., the consumption levels that would give the entrepreneur the same utility with certainty. In order to quantify the deadweight losses brought about by various contracting constraints, we contrast the CEC in each environment to the beginning-of-period assets (I). We also discuss how much further from their first-best level the entrepreneur's CEC, expected utility and the rate of return on assets (ROA), *R*, fall when the basic agency problem is compounded by restrictions on admissible sharing rules. These comparisons give us natural benchmarks to assess the importance of imposing liability limits, relative to the importance of contracting *per se*.[17] Table 2 summarizes our quantitative results.

[17] We compare (i) the losses caused jointly by the incentive-compatibility constraint and the liability limits to (ii) the losses induced by the incentive-compatibility constraint alone. By attributing the entire difference between the two

A. Deadweight Losses from Moral Hazard

Compared to the first-best environment, in which effort is verifiable, the welfare losses due only to incentive-compatibility requirements are very substantial. In the base-case scenario, the entrepreneur's CEC falls by three-fourths simply because effort is unobservable -- see Table 2, Panel B. Precisely, his CEC falls from a first-best 9.04 (18.1% of the assets *I*) to a second-best 2.25 (4.5% of *I*). The deadweight loss reaches 37.6% of assets in the worst-case scenario (his CEC drops from a first-best 24.3 to a second-best 5.5). These large welfare costs are consistent with empirical evidence that moral hazard can be very onerous (Ferrall and Shearer, 1994).

<Table 2: Expected Utility, CEC and ROA>

The deadweight losses from moral hazard in our various scenarios are similarly large according to other welfare measures. For example, given that the entrepreneur has negative exponential utility, his expected utility ranges from -1 (if he abandons the project and thus neither works nor consumes) to 0 (with no labor and infinite consumption). Thus, we can express the drop in the entrepreneur’s expected utility that results from moral hazard as a fraction of the difference between the first-best utility level and the upper bound 0. A comparison of the first-best and second-best expected utility levels reported in Table 2 (Panel A) shows that this ratio is large – more than 137% in the base case scenario.

B. Limited Liability Costs: "Base-Case" Scenario

A key conjecture of this paper is that the optimal contracts with and without limited liability have very different efficiency and welfare properties. The last column in Table 2 (Panels A and B) confirms this proposition by showing that the extra cost of ruling out limited liability rules is significant.

In the base-case scenario, judicial liability limits cause an additional expected utility loss to the entrepreneur equal to 15.5% of the loss due to moral hazard alone (Panel A). The efficiency loss is large as well: the rate of return on assets falls from 12.1% to 8.9% (Panel C).

deadweight-cost levels to liability limits, we implicitly assume that the cost of ensuring incentive compatibility is the same regardless of the contracting environment.

The reason is that the entrepreneur works much less under limited liability than he would in the second-best environment.

A comparison of the second and third columns in Table 2 (Panel B) puts the significance of these losses in perspective. Following the imposition of liability limits, the agent's CEC falls by 0.62% of the assets $I = 50$. This welfare cost is serious, given that (i) the second-best rate of return on assets R = 12.1% and (ii) in *every* output state, the entrepreneur's consumption would have to increase from its third-best level by 11.9% for him to be indifferent to the presence of exogenous liability limits.

C. Limited Liability Costs: "Worst-Case" Scenario

When the agent is very work-averse ($n = 1.6$) but is not very risk-averse ($g = 0.115$), he is optimally motivated by a sharing rule in which his monetary payoff is very negative over a wide range of low output levels. As a result, the deadweight losses of ruling out negative consumption are larger than for other parameter combinations in Table 1.

Contrasting the second and third columns in Table 2 (Panel B) shows that the agent's CEC would have to be raised by 76.5% from its third-best level for the manager to get the same utility as in the second-best environment. That is, debt bondage (or debtors' prisons) is a key component of the optimal contract. Alternatively, one can express the CEC drop in terms of the resources invested. Following the imposition of liability limits, the agent's CEC falls by more than 4.7% of the investment I. This deadweight loss is very large, amounting to almost a fifth of the second-best rate of return on investment in that environment ($R = 24.8\%$).

IX. Simple Contracts

Given that limited liability rules are very costly, a natural question is what qualitative impact they have on the optimal contract. Our results in Section VI imply that entrepreneurs will issue debt and precommit to granting absolute priority to debtholders in an attempt to control the costs caused by limited liability. In this Section, we argue that legal bounds on liability may also foster the use of sophisticated financial instruments, such as warrants. Precisely, we show that the severity of the deadweight loss due to limited liability rules increases if agents can only issue debt and equity but, in contrast, does not worsen as long as warrants can be issued as well.

Indeed, suppose that agents can only employ two-piece, piece-wise linear sharing rules. Except under special conditions (log utility), we know that such simple contracts are not optimal in the present setting. Still, debt and equity were the primary financing instruments for centuries: only more recently have securities like warrants become ordinary.[18] It is therefore natural to assess the impact of liability limits in a world dominated by these simple contracts.

When agents are forced to use piecewise linear contracts with a single kink, Table 3 shows that the deadweight losses caused by exogenous liability limits remain significant. In the base-case scenario, for example, banning debt bondage reduces the entrepreneur's CEC by 0.45% of assets (see columns 2 and 3, Panel B).

<Insert Tables 3 and 4>

The worst-case scenario illustrates that limited liability can become especially devastating in a simplified contracting environment. In Section VIII.C we found that, under optimal contracting, the exogenous imposition of liability limits brings about a CEC decrease equivalent to 4.7% of assets. This deadweight loss, already very large, nevertheless pales in comparison with the loss when the entrepreneur can only issue debt and equity. Given limited liability, the project is simply abandoned – even though its rate of return would be 21% if the entrepreneur could precommit to bondage when output is low (Panel C). Thus, very restrictive contracting environments, similar to those in which debt bondage and debtors' prisons prevailed, magnify the deadweight losses from banning debtors' prisons.[19]

In contrast, a comparison of Tables 2 and 4 shows that, if a warrant issue can be bundled with debt or equity, then the magnitude of the deadweight loss caused by exogenous liability

[18] Likewise, rent and share tenancy contracts have traditionally been a hallmark of rural societies (Basu, 1992).

[19] In the best-case scenario, admittedly, the deadweight loss caused by liability limit seems negligible: less than 0.1% of assets. The reason, however, is that the simple sharing rule is rigidly linear across most of the output range in *both* liability regimes and, hence, that liability limits matter relatively little under debt-and-equity financing.

limits can be contained almost as well as with the optimal contract that solves program (2). This is because, in both liability regimes, a three-piece piece-wise linear sharing rule (equivalent, in a capital raising context, to issuing straight debt, equity and warrants) is almost optimal in the present setup -- see Robe (1998).

One interpretation of these results is that primitive contracting environments, in which debt bondage and debtors' prisons once flourished, are also those in which limited liability rules are potentially more costly and, thus, less likely to emerge. An alternative interpretation is that limited liability statutes, enacted by many countries over the last two centuries, may have contributed to the development of some modern financial instruments such as warrants.

X. Robustness

Parameter choices

Figures 2 and 3 plot the extent of the entrepreneur's maximum potential liability in terms of the values taken by various parameters. Both figures show that the results in Section VI are not qualitatively affected by the parameter choices: limited liability violations are large and typical.

Likewise, a comparison of the various scenarios in Table 2 shows that, even in the best-case scenario, imposing legal liability restrictions leads to large welfare and efficiency losses: the CEC drop due to liability limits in that case exceeds 0.30% of assets (see Table 2).[20]

What is more important, for some parameter values, the project is not profitable under liability limits even though it would have been profitable in their absence. For example, given the other base-case parameter values (I = 50, q = 1.121 and g= 0.5), investment is unprofitable under limited liability when the cost of effort index n exceeds 1.64. In that case, the entrepreneur strictly prefers not to undertake the project unless he can precommit to bondage when output is

[20] Our results are likewise robust to the value of the marginal productivity of effort at the optimum. Changing that number from 2% to 10% (a*=10) or 1% (a*=100) and appropriately re-calibrating the other parameters (including the firm's size and the coefficient of absolute risk-aversion g), we found broadly similar results. Tables summarizing all these robustness checks are available upon request.

low. In contrast, debtors' prisons keep the project remains profitable for work aversion levels as high as $n = 1.73$. When $n = 1.65$, for instance, the second-best ROA is still 10.63%.

Technology

One possible reason for the uniformly large deviations from limited liability in Figure 2 might be that, with an exponential technology, the probability distribution of output is monotone decreasing in the level of output and much of the probability mass is located on low output levels. As a result, the contract might optimally impose large penalties on low output levels.

To disprove this conjecture, we raise the shape parameter t in technology (4). By letting $t > 1$ we can test our results' robustness to situations where $f(y,a)$ has the "hump shape" of many return distributions. We set $t = 2$ to ensure computational ease, re-calibrate the model's key parameters (the risk aversion g and the cost of effort index n), and carry out an analysis similar to that of Sections VI-VII.

Under limited liability, optimal contracts directly comparable to their counterparts with exponential technology are readily computed. It is also immediate from Figure 4 that potential deviations from limited liability are even more significant than with an exponential technology. The worst penalties are so large, in fact, that no continuous solution to program (1) can usually be found numerically once g is higher than 0.05.[21]

<Insert Figure 4>

We conclude by measuring the welfare costs of liability limits when entrepreneurs can issue at most three traditional securities: straight debt, equity and warrants. We find that the

---

[21] The resulting expected rates of return on assets *I* generally exceed 35%. This number may well be acceptable for the small firms modeled in this paper, as companies whose managers hold a significant equity stake and exert a crucial yet unobservable effort level are likely to be small and to have higher-than-average profitability. It is, nevertheless, far above 12.1% -- the nominal annual rate of return on common stocks during the 1926-1988 period. Furthermore, to obtain lower expected rates of return, the levels of relative risk aversion at the entrepreneur's mean consumption would have to be less than 1. In contrast, typical values in the financial economics literature start at 1.5 (Telmer, 1993; Lucas and Heaton, 1996). In the absence of *any* bound on liability, inadmissibly high rates of return on investment would be required for solutions to program (1) to yield higher levels of relative risk aversion.

results of Section IX are robust to the technology. Comparing the last two columns in Table 6 (Panel B), for example, shows that the CEC drop due to liability limits amounts to 1.38% of assets in the base-case -- a greater loss than with the exponential technology. Furthermore, a comparison of the worst-case scenarios in Tables 5 and 6 confirms that the deadweight loss due to limited liability rules can become much more severe if firms are only allowed to sell debt and equity -- but can be much better contained if warrants can be issued as well.

XI. Conclusion

In this paper, we characterize both quantitatively and qualitatively the impact of limited liability on the optimal financing contract, in a standard one-period model of investment financing under moral hazard and risk aversion.

For reasonable parametrizations of the model, we show that the threat of bondage is an integral component of the optimal contract in the absence of limited liability rules. This is true even though we posit that it is the risk-averse entrepreneur who designs the contract and maximizes his expected utility -- while outside investors only receive a competitive return on investment.

We then quantify the deadweight losses brought about by an exogenous (legal) mandate of liability limits. These losses are significant -- roughly 0.6% of company assets at the calibrated parameter values. In several cases, limited liability rules render outright unprofitable some investments that would have been undertaken in their absence.

Those features of the optimal financing contract stand in sharp contrast to the demise of bonded labor and debtors' prison in modern economies. On the one hand, this apparent discrepancy between theory and corporate practice might indicate that basic financial agency models make counter-factual predictions. On the other hand, societal preferences may have led to the abolition of punishments deemed excessively harsh. Moreover, enforcing unlimited liability may be costly for investors (and for principals in general). Our results therefore suggest the costs associated with unlimited liability and debtors' prisons must be substantial to explain the ubiquity of limited liability rules in modern economies.

Given that limited liability rules are quite costly, a natural question is what qualitative impact they have on the optimal contract. We show that entrepreneurs will use debt financing and give debtholders absolute priority in an attempt to control the costs caused by limited liability. Merely using debt financing, however, is almost never optimal: we show that legal bounds on entrepreneurs' liability may have fostered the emergence of sophisticated modern financial instruments, such as warrants.

Appendix: Robustness to preferences

Exponential preferences over consumption imply that the agent's second-best monetary payoff, $t^*(y)$, is strongly concave in output. This concavity, together with the optimality of debtors' prisons in the second-best environment, explains why the third-best liability limits are binding. It is therefore natural to investigate whether such violations take place when $t^*(y)$ is convex. One way to do so is to assume that the entrepreneur has constant relative risk aversion (CRRA) and is not very risk averse:

(7) $$u(c) \equiv \frac{c^{1-s}-1}{1-s}, \ \frac{1}{2} < s < 1$$

Assuming CRRA preferences is common in finance and economics. Setting $s > 0.5$ guarantees the first-order approach's validity (Jewitt, 1988). We concentrate on the case where $I = 50$ and $q = 1.121$, and re-calibrate the intensity of managerial disutility from effort, n.

We find that the constraint that the entrepreneur's consumption be non-negative is always binding whenever the expected rate of return on assets I is reasonable. There do exist parameter choices such that the entrepreneur's liability is optimally limited to the available output. These choices, however, imply expected rates of return on assets in excess of 30%. Furthermore, many of these combinations lead to a second-best contract in which, once the project's output is sufficiently high, the entrepreneur receives money from outside investors over and above that output.

References

Akerlof, G. and L. Katz, 1989, "Workers' Trust Funds and the Logic of Wage Profiles", *Quarterly Journal of Economics*, 104, pp. 525-36.

Allen, F. and G. Gale, *Financial Innovation and Risk Sharing*, Cambridge: The M.I.T. Press (1994).

Basu, K., 1992, "Limited Liability and the Existence of Share Tenancy," *Journal of Development Economics*, 38, 1, pp. 203-20.

Benveniste, L., Busaba, W. and W. Wilhelm Jr., 1996, "Price Stabilization as a Bonding Mechanism in New Equity Issues," *Journal of Financial Economics*, 42, pp. 223-55.

Boyd, J. and B. Smith, 1994, "How Good are Standard Debt Contracts? Stochastic vs. Non-Stochastic Monitoring in a Costly State Verification Environment," *Journal of Business*, pp. 539-561.

Brealey, R. and S. Myers, 1991, *Principles of Corporate Finance (4th ed)*, McGraw Hill, NY.

Carmichael, H.L., 1989, "Self-Enforcing Contracts, Shirking, and Life Cycle Incentives," *Journal of Economic Perspectives*, 3, 4, pp. 65-83.

Carr, J. and G.F. Mathewson, 1988, "Unlimited Liability as a Barrier to Entry," *Journal of Political Economy*, 96, 4, pp. 766-84.

Chu, C.Y. and N, Jiang, 1993, "Are Fines More Efficient Than Imprisonment?" *Journal of Public Economics*, 51, 3, pp. 391-413.

Demougin, D. and D. Garvie, 1991, "Contractual Design with Correlated Information under Limited Liability," *Rand Journal of Economics*, 22, 4, pp. 477-89.

Demougin, D. and C. Fluet, 1997, "Monitoring versus Incentives: Substitutes or Complements?" Working Paper No.47, CREFE-UQAM (forthcoming in the *European Economic Review*).

Diamond, D., 1984, "Financial Intermediation and Delegated Monitoring," *Review of Economic Studies*, 51, 3, pp. 393-414.

Esty, B., 1998, "The Impact of Contingent Liability on Commercial Bank Risk Taking," *Journal of Financial Economics*, 47, 2, pp. 189-218.

Faynzilberg, P. and P. Kumar, 1997, "Optimal Contracting of Separable Production Technologies," *Games and Economic Behavior*, 21, 1-2 (Oct.-Nov.), pp. 15-39.

Ferrall, C. and B. Shearer, 1994, "Incentives, Team Production, Transactions Costs and the Optimal Contract: Estimation of an Agency Model Using Payroll Records," Working Paper, Queen's University (August).

Haubrich, J., 1994, "Risk Aversion, Performance Pay, and the Principal-Agent Problem," *Journal of Political Economy*, pp. 258-276.

Heaton, J. and D. Lucas, 1996, "Evaluating the Effects of Incomplete Markets on Risk Sharing and Asset Pricing," *Journal of Political Economy*, 104, pp. 443-487.

Holmström, B., 1979, "Moral Hazard and Observability," *Bell Journal of Economics*, pp. 74-91.

Innes, R., 1990, "Limited Liability and Incentive Contracting with Ex-ante Action Choices," *Journal of Economic Theory*, pp. 45-67.

Innes, R., 1993, "Debt, Futures and Options: Optimal Price-Linked Financial Contracts under Moral Hazard and Limited Liability," *International Economic Review*, pp. 271-95.

Jewitt, I., 1988, "Justifying the First-Order Approach to Principal Agent Problems," *Econometrica*, pp. 1177-90.

Mehra, R. and E.C. Prescott, 1985, "The Equity Premium: A Puzzle?" *Journal of Monetary Economics*, 15, pp. 145-161.

Mirrlees, J., 1976, "The Optimal Structure of Incentives and Authority within an Organization," *Bell Journal of Economics*, 7, 1, pp. 105-31.

Robe, M., 1999, "Optimal vs. Traditional Securities under Moral Hazard," *Journal of Financial and Quantitative Analysis* (forthcoming).

Rogerson, W., 1985, "The First-Order Approach to Principal-Agent Problems," *Econometrica*, 53, 1357-67.

de Sainte Croix, G.E.M., 1988, "Slavery and Other Forms of Unfree Labor;" in *Slavery and Other Forms of Unfree Labor*, L. Archer Ed., Routledge, NY, pp. 102-17.

Sappington, D., 1991, "Incentives in Principal Agent Relationships," *Journal of Economic Perspectives*, pp. 45-66.

Sappington, D., 1983, "Limited Liability Contracts between Principal and Agent," *Journal of Economic Theory*, pp. 1-21.

Stoughton, N., 1993, "Moral Hazard and the Portfolio Management Problem," *Journal of Finance*, 2009-28.

Stein, S. and S. Subrahmanyam, Eds. 1996, *Institutions and Economic Change in South Asia*, Oxford University Press, 314 pp.

Sung, J., 1995, "Linearity with Project Selection and controllable diffusion rate in continuous-time principal-agent problems," *Rand Journal of Economics*, pp. 720-743.

Telmer, Chris I., 1993, Asset pricing puzzles and incomplete markets, *Journal of Finance*, 48, pp.1803-32.

t'Kint, F., 1991, *Sûretés et Principes Généraux du Droit des Créanciers*, Bruxelles: Larcier.

Vigneron, R., 1998, *Cours de Droit Romain*, Université de Liège.

Weisbach, M., 1988, "Outside Directors and CEO Turnover," *Journal of Financial Economics*, 20, 431-60.

Welch, K.D., 1995, "Bankruptcy, Debtors' Prison, and Collateral in Optimal Debt Contracts," Working Paper, University of Chicago (December).

Table 1: Parameter Choices with exponential utility.

parameter	symbol	base-case value	range
manager's absolute risk aversion	*g*	0.5	0.1 -> 0.6
manager's disutility from effort	*n*	1.574	1.475 -> 1.825
intrinsic asset productivity (*t*=1)	*q*	1.121	1.05 -> 1.40
(rate of return on assets, in %)	*(q-1)*	*(12.1%)*	*(5% -> 40 %)*
marginal managerial effort productivity	1/*I*	2%	1% -> 10%

The scaling factor A is set equal to I^2. This ensures that levels of disutility from effort are in the same range as the agent's expected utility from consumption. The central value n=1.574 is chosen so that the rate of return on the investment I be equal to 12.1% when $g = 0.5$, $q = 1.121$ and $I = 50$.

Table 2: Welfare and Efficiency Losses from Liability Limits

Parameters	Levels in units (expected managerial utility) or as a fraction of assets (expected rate of return on assets)			Change from 2nd to 3rd best, in percent(s) of the change from 1st to 2nd best
parametrization	first best	second best (no liability limit)	third best (limited liability)	
Panel A: Expected Managerial Utility				
Worst-Case	-0.389446	-0.782255	-0.914277	33.6%
Base-Case	-0.21557	-0.512664	-0.558769	15.5%
Best-Case	-0.146516	-0.337287	-0.350143	6.7%
Panel B: Certainty Equivalent Consumption				
Worst-Case	24.2562	5.45569	3.09075	12.6%
Base-Case	9.03609	2.2537	1.94223	4.6%
Best-Case	9.96589	3.1452	2.99501	2.2%
Panel C: Expected Rate of Return on Assets				
Worst-Case	48.5%	24.8%	13.5%	47.7%
Base-Case	18.1%	12.1%	8.9%	54%
Best-Case	19.9%	14.1%	12.1%	34.5%

(*) Table 2 uses the following parametrizations. Firm size: I=50. Agent utility from consumption: $u(c) \equiv -e^{-gc}$. In the base- and best- cases, g= 0.5; in the worst-case scenario, g= 0.115. Technology: $f(y,a) = \frac{e^{y/(-aq)}}{aa}$, with q = 1.121. Agent disutility from effort: $v(a) = \frac{a^n}{I^2}$.

In the base-case, n=1.574 -- the value corresponding to a rate of return on investment of 12.1% in the absence of liability limits. In the best-case, n=1.472 -- which yields a rate of return on investment of 12.1% when the entrepreneur's liability cannot exceed the available output. In the worst-case, n=1.6.

Table 3: Welfare and Efficiency Losses from Liability Limits
(Traditional Contracting, Debt-and-Equity Financing)

Parameters	**Levels in units (expected managerial utility) or as a fraction of assets (expected rate of return on assets)**			**Change from 2nd to 3rd best, in percent(s) of the change from 1st to 2nd best**
parametrization	first best	no liability limit	limited liability	
Panel A: Expected Managerial Utility				
Worst-Case	-0.389446	-0.796715	-1	49.91%
Base-Case	-0.21557	-0.550597	-0.586087	10.59%
Best-Case	-0.146516	-0.371385	-0.373566	0.97%
Panel B: Certainty Equivalent Consumption				
Worst-Case	24.2562	5.03617	0	26.2%
Base-Case	9.03609	2.03051	1.80813	3.17%
Best-Case	9.96589	2.85464	2.81236	0.59%
Panel C: Expected Rate of Return on Assets				
Worst-Case	48.5%	21.0083%	0%	76.42%
Base-Case	18.1%	11.9789%	9.26941%	44.26%
Best-Case	19.9%	15.02%	13.263%	36%

(*) Table 3 uses the following parametrizations. Firm size: I=50. Agent utility from consumption: $u(c) \equiv -e^{-gc}$. In the base- and best- cases, g= 0.5; in the worst-case scenario, g= 0.115. Technology: $f(y,a) = \frac{e^{y/(-aq)}}{aa}$, with q = 1.121. Agent disutility from effort: $v(a) = \frac{a^n}{I^2}$. In the base-case, n=1.574. In the best-case, n=1.472. In the worst-case, n=1.6. Both with and without limited liability rules, contracts are piecewise linear with a single kink, and have slopes at most equal to 1. Such contracts, in a financing context, correspond to issuing straight debt and equity.

Table 4: Welfare and Efficiency Losses from Liability Limits
(Piece-wise Linear Contract with 2 Kinks: Debt, Equity and Warrants)

Parameters	**Levels in units (expected managerial utility) or as a fraction of assets (expected rate of return on assets)**			**Change from 2nd to 3rd best, in percent(s) of the change from 1st to 2nd best**
parametrization	first best	no liability limit	limited liability	
Panel A: Expected Managerial Utility				
Worst-Case	-0.389446	-0.785661	-0.920148	33.9%
Base-Case	-0.21557	-0.516623	-0.559668	14.3%
Best-Case	-0.146516	-0.340259	-0.350838	5.5%
Panel B: Certainty Equivalent Consumption				
Worst-Case	24.2562	5.35626	3.01003	12.4%
Base-Case	9.03609	2.22285	1.93798	4.2%
Best-Case	9.96589	3.11248	2.98985	1.8%
Panel C: Expected Rate of Return on Assets				
Worst-Case	48.5%	23.9308%	13.3076%	43.2%
Base-Case	18.1%	11.6937%	8.89854%	43.6%
Best-Case	19.9%	13.7678%	12.1727%	26%

(*) Table 4 uses the following parametrizations. Firm size: I=50. Agent utility from consumption: $u(c) \equiv -e^{-gc}$. In the base- and best- cases, g= 0.5; in the worst-case scenario, g= 0.115. Technology: $f(y,a) = \frac{e^{y/(-aq)}}{aa}$, with q = 1.121. Agent disutility from effort: $v(a) = \frac{a^n}{I^2}$. In the base-case, n=1.574. In the best-case, n=1.472. In the worst-case, n=1.6. Both with and without limited liability rules, contracts are piece-wise linear, have a maximum of two kinks, and have slopes at most equal to 1. Such contracts, in a financing context, correspond to issuing debt, equity and one class of warrants.

Table 5: Welfare and Efficiency Losses from Liability Limits
(Traditional Contracting, Debt-and-Equity Financing)

Parameters	**Levels in units (expected managerial utility) or as a fraction of assets (expected rate of return on assets)**			**Change from 2nd to 3rd best, in percent(s) of the change from 1st to 2nd best**
parametrization	first best	no liability limit	limited liability	
Panel A: Expected Managerial Utility				
Worst-Case	-0.3798	-0.7126	-1	86.37%
Base-Case	-0.3333	-0.6449	-0.7293	27.05%
Best-Case	-0.2077	-0.3881	-0.4063	10.04%
Panel B: Certainty Equivalent Consumption				
Worst-Case	7.6856	1.9430	0	33.84%
Base-Case	7.9960	2.2209	1.5944	10.85%
Best-Case	9.1255	3.2331	2.9971	4%
Panel C: Expected Rate of Return on Assets				
Worst-Case	15.37%	10.77%	0%	234.14%
Base-Case	15.99%	11.71%	7.89%	89.31%
Best-Case	18.25%	15.28%	12.63%	16.93%

(*) Table 5 uses the following parametrizations. Firm size: I=50. Agent utility from consumption: $u(c) \equiv - e^{-gc}$ with g= 0.5. Technology: $f(y,a) = 4\, y \exp(-2y/(aq))/(aq)^2$, with q = 1.121. Agent's disutility from effort: $v(a) = a^n/I^2$, with n=1.69 (base-case); n=1.564 (best-case); or n=1.725 (worst-case). Both with and without limited liability rules, contracts are piece-wise linear with a single kink, and have slopes at most equal to 1. Such contracts, in a financing context, correspond to issuing straight debt and equity.

Table 6: Welfare and Efficiency Losses from Liability Limits
(Piece-wise Linear Contract with 2 Kinks: Debt, Equity and Warrants)

Parameters	**Levels in units (expected managerial utility) or as a fraction of assets (expected rate of return on assets)**			**Change from 2nd to 3rd best, in percent(s) of the change from 1st to 2nd best**
parametrization	first best	no liability limit	limited liability	
Panel A: Expected Managerial Utility				
Worst-Case	-0.3798	-0.6855	-0.9227	77.61%
Base-Case	-0.3333	-0.5968	-0.7016	39.78%
Best-Case	-0.2077	-0.3613	-0.3873	16.93%
Panel B: Certainty Equivalent Consumption				
Worst-Case	7.6856	2.1118	0.9676	20.53%
Base-Case	7.9960	2.4117	1.7206	12.37%
Best-Case	9.1255	3.5008	3.1625	6.01%
Panel C: Expected Rate of Return on Assets				
Worst-Case	15.37%	11.45%	5.33%	155.86%
Base-Case	15.992%	12.1%	7.87%	108.57%
Best-Case	18.251%	14.48%	12.15%	61.74%

(*) Table 6 uses the following parametrizations. Firm size: I=50. Agent utility from consumption: $u(c) \equiv -e^{-gc}$ with g= 0.5. Technology: $f(y,a) = 4\, y \exp(-2y/(aq))/(aq)^2$, with q = 1.121. Agent's disutility from effort: $v(a) = a^n/I^2$, with n=1.69 (base-case); n=1.564 (best-case); or n=1.725 (worst-case). Both with and without limited liability rules, contracts are piece-wise linear, have a maximum of two kinks, and have slopes at most equal to 1. Such contracts, in a financing context, correspond to issuing straight debt, equity and one class of warrants.

Figure 1: Optimal Sharing Rule with and without Wealth Constraints

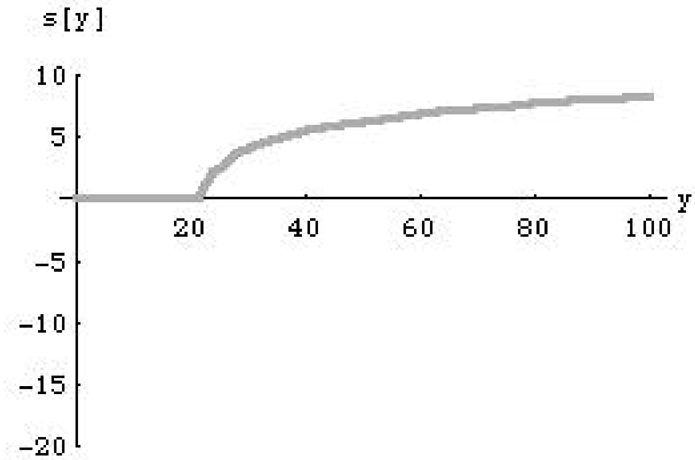

Figure 1.a

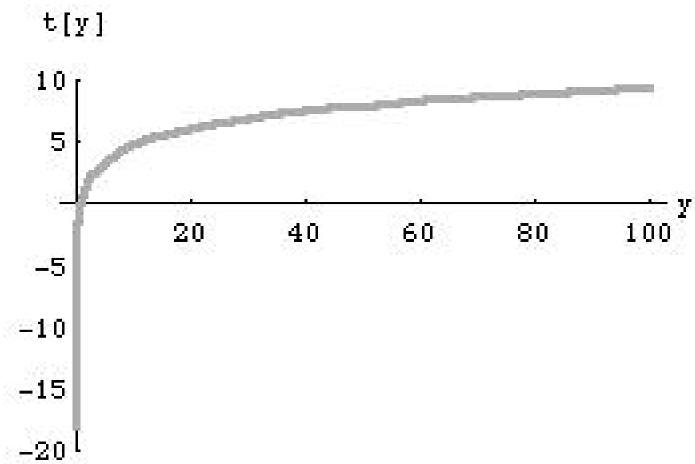

Figure 1.b

Figure 1.a shows the payoff under the exogenous requirement of limited liability, $s^*(y) \geq 0$. Figure 1.b shows the agent's monetary payoff as a function of the project's output in the absence of liability restrictions, $t^*(y)$. By construction, $t(y)$ and $s(y)$ are continuous functions. The second-best intercept, $t^*(0)$, is -17.90.

Figures 1.a and .b uses the following parametrization. Firm size: I=50. Entrepreneur's preferences over consumption: $u(c) \equiv -e^{-gc}$, $g = 0.5$. Entrepreneur's disutility from effort: $v(a) = a^n/I^2$, n = 1.574. Technology: $f(y,a) = \frac{e^{y/(-aq)}}{aa}$, $q = 1.121$.

Figure 2: Maximum Monetary Punishment in % of the Agent's Expected Income

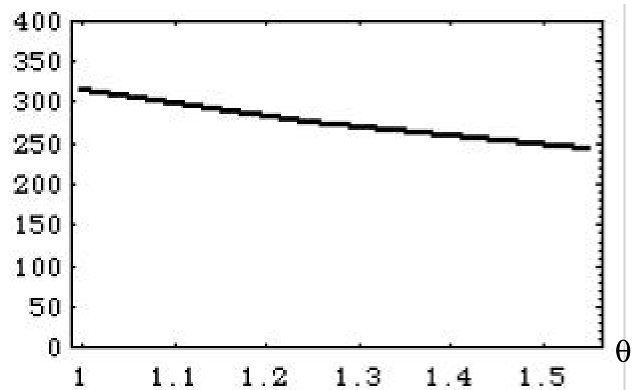

Figure 2.a

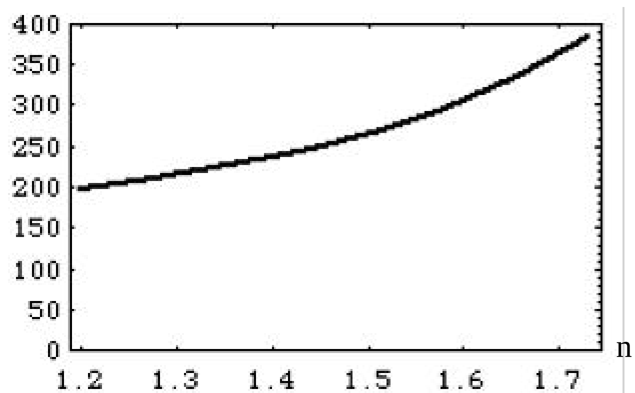

Figure 2.b

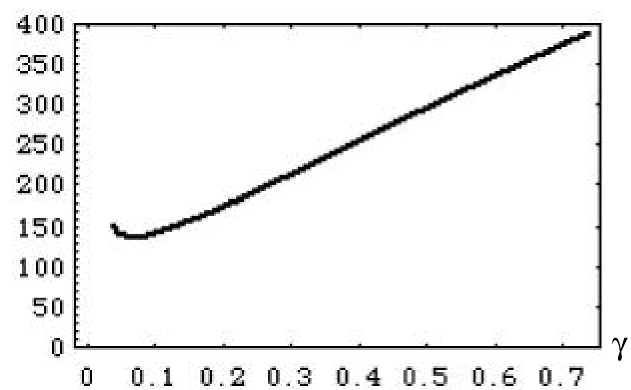

Figure 2.c

Figure 2 shows the maximum monetary penalty, $t^*(0)$, as a fraction of the entrepreneur's expected income from the project, E[*y*]-*I*. Figure 2 uses same base-case parametrization as Figure 1. Firm size: *I*=50. Disutility from effort: $v(a) = a^n/I^2$, $n = 1.574$ (except Figure 2.b). Entrepreneur's utility from consumption: $u(c) \equiv -e^{-gc}$, $g=0.5$ (except Figure 2.c). Technology: $f(y,a) = e^{y/(-aq)}/aq$, $q = 1.121$ (except Figure 2.a).

Figure 3: Maximum Monetary Punishment in % of the Expected Output

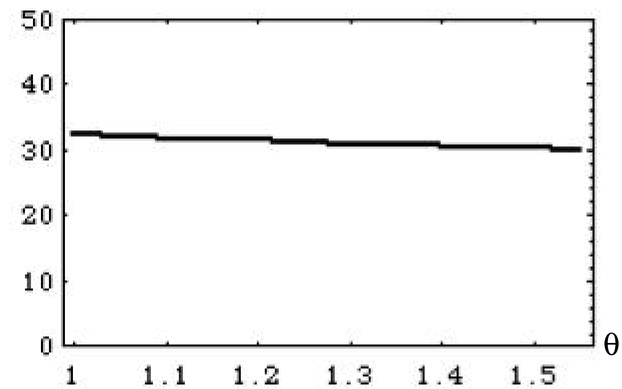

Figure 3.a

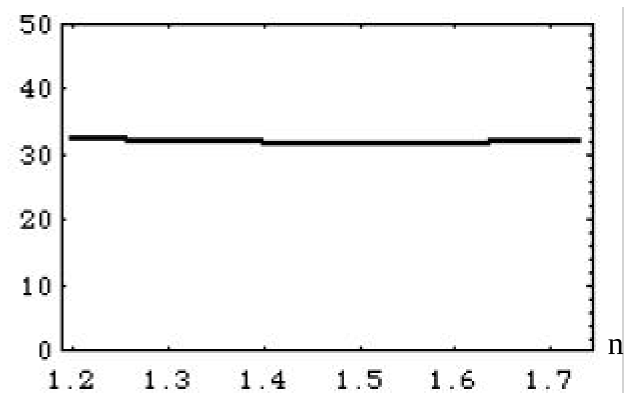

Figure 3.b

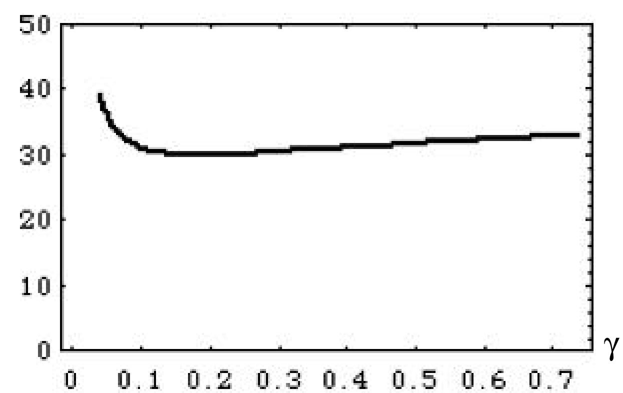

Figure 3.c

Figure 3 shows the maximum monetary penalty, $t^*(0)$, as a fraction of the expected end-of-period value of the project, E[*y*]. Figure 3 uses same base-case parametrization as Figure 1. Firm size: *I*=50. Disutility from effort: $v(a) = a^n/I^2$, $n = 1.574$ (except Figure 3.b). Entrepreneur's utility from consumption: $u(c) \equiv -e^{-gc}$, g=0.5 (except Figure 3.c). Technology: $f(y,a) = e^{y/(-aq)}/aq$, $q = 1.121$ (except Figure 3.a).

Figure 4: Maximum Monetary Punishment in % of the Agent's Expected Income

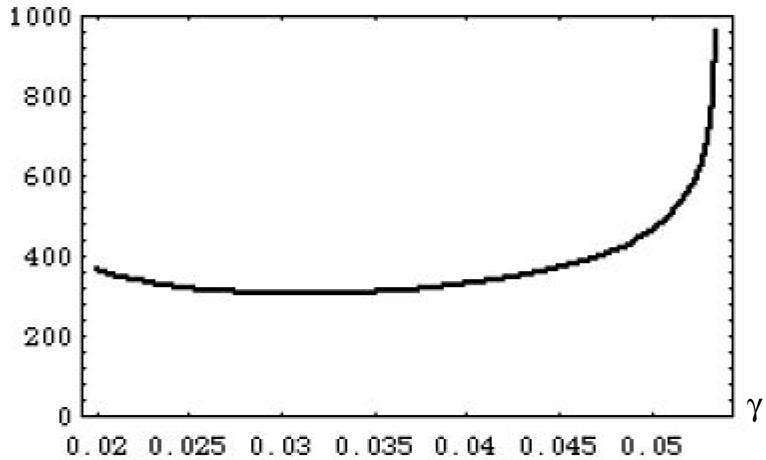

Figure 4 shows the maximum monetary penalty, $t^*(0)$, as a fraction of the entrepreneur's expected income from the project, E[y]-I. Figure 4 uses the same parametrization as Figures 1 to 3, except for the technology. Firm size: $I = 50$. Disutility function from effort: $v(a) = a^n/I^2$, $n = 1.65$. Entrepreneur's preferences over consumption: $u(c) \equiv -e^{-gc}$. Technology: $f(y,a) \equiv y\, e^{-y/(a q)}/(a q)^2$, $q = 0.5605$.