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CONSOLIDATION ANALYSIS OF SYMMETRICALLY LOADED STRIP FOOTINGS ON A POROELASTIC LAYER

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ABSTRACT

The interaction effects between adjacent strip footings is of interest to the geotechnical design of structural foundations. The present paper examines the consolidation behaviour of interacting rigid strip footings resting on the surface of a soil layer of finite thickness. The rigid nature of the footings is simulated by constraining the vertical displacements under the footing using the multi-point constraint technique. The soil is modelled by composite-type finite and mapped infinite elements which can simulate the behaviour of an unbounded saturated soil medium. The numerical results presented in the paper indicate the manner in which the consolidation settlement response of the interacting foundations is influenced by the spacing between them and the drainage conditions at the various free surfaces and interfaces.

INTRODUCTION

The group influences of structural foundations are of considerable importance to their proper geotechnical design. Such group influences invariably occur in all foundation groups including piled foundations, footings and bases for off-shore structures, structural foundations in urban environments, etc. An assessment of the group interactions is a necessary prerequisite for the accurate prediction of the foundation behaviour. Although the influences of group interactions have been appreciated in the geotechnical context, a majority of the investigations of group interaction have focussed on the evaluation of the time independent response of such interaction [1–5]. To the knowledge of the author's, the study of the time dependent influences of viscoelastic, viscoplastic, and consolidation properties on the group behaviour has received little attention. In particular, the evaluation of the consolidation response of group interaction is important to the study of the long term behaviour of structural foundations.

The objective of this study is to evaluate the manner in which the time–dependent soil consolidation phenomenon manifest in such group interaction problems. To particularly focus

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on the assessment of the time-dependent effects, we restrict attention to the study of the plane strain response of symmetrically loaded footings of identical width which are loaded on the surface of a poroelastic layer. The solution to the problem can be achieved by a variety of techniques. The analytical methods can accurately model this class of problems. However, the existence of interaction effects, finite boundaries, mixed boundary conditions at the surface of the layer makes the analytical procedure very difficult if not impossible. It is therefore prudent to employ numerical schemes for the analysis of the problem. In this paper the finite element technique is utilized. Finite element techniques have been applied quite successfully for the analysis of soil consolidation problems [6-9].

In this paper we incorporate a further dimension to the finite element modelling whereby an infinite element is used to model accurately the layer of infinite extent. Various techniques have been proposed for the finite element modelling of unbounded media. Of all the available techniques for modelling such media, the coordinate ascent mapped infinite element method seems to be the most promising for general applications [10-13]. These elements are formulated in terms of standard shape functions using conventional Gauss-Legendre numerical integration which makes them suitable for incorporation in general purpose finite element codes. Selvadurai and Gopal (11), Selvadurai and Karpurapu (13) and Simoni and Schrefler (14) have extended the scope of the infinite elements to the analysis of the response of saturated soil media. A brief description of the formulation of the poro-elastic infinite element is made in this paper.

COMPOSITE INFINITE ELEMENT

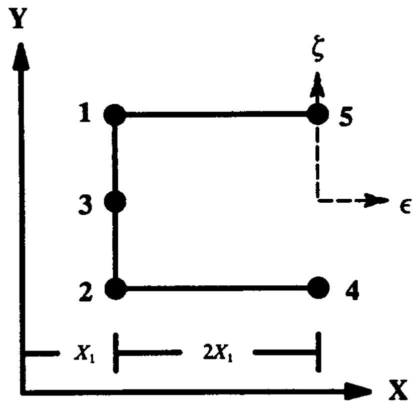
The finite element equations of consolidation can be derived from the governing consolidation equations based on variational principles and can be written as follows: [6-9,15].

$$\begin{bmatrix} K & C \\ C^T & -(E + \delta\Delta t H) \end{bmatrix} \begin{Bmatrix} u_{t+\Delta t} \\ p_{t+\Delta t} \end{Bmatrix} = \begin{Bmatrix} f_{t+\Delta t} \\ g_{t+\Delta t} \end{Bmatrix} \tag{1}$$

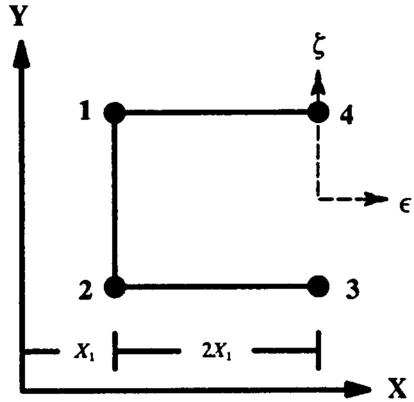
In Equation (1), {u} and {p} are the nodal displacements and pore pressures respectively. The finite element solution of the above equations is obtained by employing composite type elements which have both displacements and pore pressures as nodal variables. The composite element is obtained by superposition of an element which represents the nodal displacements and the other which represents pore pressures [6,9]. The same concept is extended to the derivation of composite infinite element.

The infinite elements are constructed using the mapping principles. The mapped infinite elements can be constructed to represent various orders of decay rates for the field variables. In the current investigation $1/r$ type decay rate is employed for the composite infinite element.

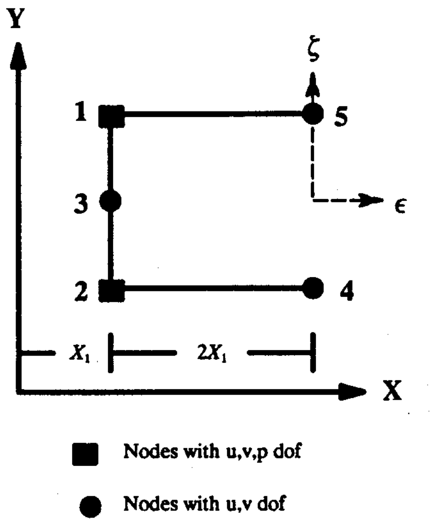
Figure 1 shows the five-noded composite infinite element employed in the numerical investigation. It represents an infinite medium in the local coordinate direction ϵ. This element is obtained by superposing a five noded serendipity infinite element representing quadratic variation for the displacement field and a two-noded superparametric infinite element representing linear variation for the pore pressure field.



a) 5-noded serendipity element for displacements



b) 2-noded superparametric element for pore pressures



c) Superposed Element

Figure 1: The Composite Infinite Element

Mapping functions for the element nodes are constructed separately in the infinite direction ϵ and the finite direction ζ and the global functions are obtained as their product. The mapping functions in the local coordinate direction ϵ are constructed with the singularity at the $\epsilon = +1$ face which corresponds to an infinite distance. The mapping functions in the finite direction ζ are constructed as for the isoparametric finite elements.

For the class of problems examined in this paper, the field variables are assumed to reduce to zero at an infinite distance from the loaded regions. This assumption is justified for most geotechnical problems. With this assumption, the interpolation for the field variables of infinite elements is carried out in terms of nodal variables at the finitely located nodes. By invoking this assumption, the five and two-noded infinite elements can be considered as analogues of eight and four-noded quadrilateral finite elements whose field variables on the nodes at the $\epsilon = +1$ face are zero. The shape functions of infinite element nodes are then obtained directly from the corresponding nodes of the finite elements.

The shape functions and mapping functions for the infinite element are reported by Selvadurai and Karpurapu (13). The readers are referred to that reference for more details on the formulation of the infinite element and its implementation in finite element programs.

NUMERICAL ANALYSIS

The advantages of modelling the unbounded media using the coupled finite and infinite elements have been described in detail in the investigations of Selvadurai and Gopal (11), Karpurapu and Bathurst (12), and Selvadurai and Karpurapu (13). Hence the numerical analysis in this section is only directed towards the current problem of interacting foundations.

We now focus on the plane-strain problem of two interacting rigid footings which are resting on the surface of a consolidating layer of finite thickness underlain by a rough impervious base. Figure 2 shows an overview of the problem considered for numerical analyses.

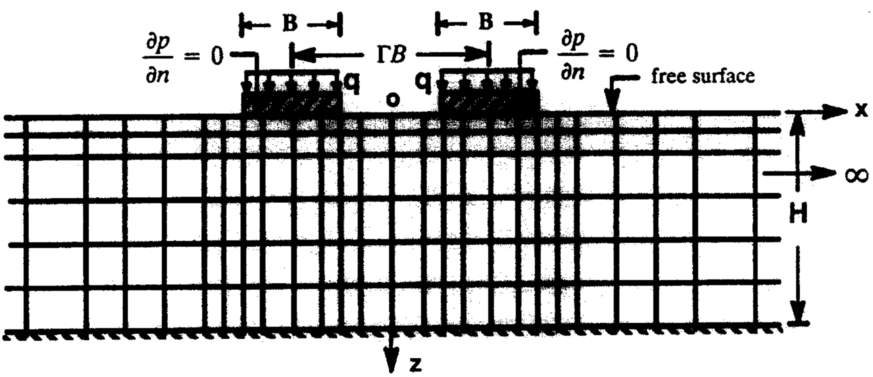


Figure 2: General overview of the problem

Soil medium is modelled by poro-elastic type finite and infinite elements. Isoparametric 8-noded quadratic finite elements are used to model the soil up to six times the footing width. The mapped infinite elements are used to model the soil beyond this region. The rigid nature of the footings was simulated by constraining the vertical displacements of nodes under the footing to move in a rigid manner. Multi-point constraint method proposed by Abel and Shephard (16) was used for this purpose. Uniform external pressure of equal magnitude is assumed to act on both footings. Equations 2.1–2.10 show the boundary and drainage conditions applicable to the problem.

$$u_x = u_z = 0; \quad z=H; \quad -\infty < x < \infty; \quad t > 0 \tag{2.1}$$

$$\sigma_z = q; \quad z=0; \quad \frac{(\Gamma-1)B}{2} \le x \le \frac{(\Gamma+1)B}{2}; \quad t > 0 \tag{2.2}$$

$$\sigma_z = q; \quad z=0; \quad \frac{-(\Gamma+1)B}{2} \le x \le \frac{-(\Gamma-1)B}{2}; \quad t > 0 \tag{2.3}$$

$$\sigma_{xz} = 0; \quad z=0; \quad -\infty < x < \infty \quad t > 0 \tag{2.4}$$

$$p=0; \quad z=0; \quad -\infty < x < \frac{-(\Gamma+1)B}{2}; \quad t > 0 \tag{2.5}$$

$$p=0; \quad z=0; \quad \frac{-(\Gamma-1)B}{2} < x < \frac{(\Gamma-1)B}{2}; \quad t > 0 \tag{2.6}$$

$$p=0; \quad z=0; \quad \frac{(\Gamma+1)B}{2} < x < \infty; \quad t > 0 \tag{2.7}$$

$$\frac{\partial p}{\partial n} = 0; \quad z=0; \quad \frac{(\Gamma-1)B}{2} \le x \le \frac{(\Gamma+1)B}{2}; \quad t > 0 \tag{2.8}$$

$$\frac{\partial p}{\partial n} = 0; \quad z=0; \quad \frac{-(\Gamma+1)B}{2} \le x \le \frac{-(\Gamma-1)B}{2}; \quad t > 0 \tag{2.9}$$

$$\frac{\partial p}{\partial n} = 0; \quad z=H; \quad -\infty < x < \infty; \quad t > 0 \tag{2.10}$$

Various cases of spacing between the centres of footings and the soil strata thicknesses were considered in the analysis. Both these distances were expressed in terms of the width of the footing (B). Table 1 presents the various analysis cases. Independent behaviour of footings is represented by the $\Gamma = \infty$ case. The Poisson's ratio values considered for the analysis are 0.30 and 0.45. By imposing appropriate boundary conditions at the central axis to ensure symmetry, only half of the region shown in Figure 2 is considered for finite element analysis.

Table 1
Analysis Cases

H	Γ				
B	∞	2	3	5	-
5B	∞	2	3	5	10
10B	∞	2	3	5	10

Results and Discussion

The Figures 3 and 4 show the variation of the normalized settlement at the centre of the footing as a function of the time factor for all the analyses cases. The time factor, T_v, has been defined as follows:

$$T_v = \frac{ct}{B^2} \tag{3}$$

in which t is the time, B is the width of the footing, and c is the coefficient of consolidation which is defined as follows:

$$c = \frac{2Gk}{\gamma_w} \tag{4}$$

in which G is the shear modulus of soil, k is the permeability coefficient, and γ_ω is the unit weight of pore fluid. The normalized settlement has been defined as (Gw_o/Bq), in which w_o is the settlement at the centre of the footing and q is the intensity of loading.

For H = B case, the maximum consolidation rate (indicated by the slope of the normalized settlement curve) is between the time factors of 0.1 and 1 for $\nu = 0.3$ and between time factors 0.06 and 0.7 for $\nu = 0.45$ for all the spacings. Much of the consolidation settlement occurs during this time period. For H = 5B, the maximum consolidation rate is between the time factors 1 and 10 for $\nu = 0.3$ and between time factors 0.6 and 5 for $\nu = 0.45$. For these two depths, there is a definite jump in the slope of the settlement curves. For H = 10B case, the slope is more gradual indicating that the total settlement occurs over a much longer time period. In general, it can be observed that for higher Poisson's ratio values, the difference between the initial and final settlements is smaller. This can be attributed to the decrease in the compressibility of the soil for higher Poisson's ratios.

From Figures 3 and 4, it could be inferred that the time for total consolidation is lesser for $\nu = 0.45$ soil than that for the $\nu = 0.30$ soil. This observation is true for all cases of spacings and depths. For higher Poisson's ratio values, the soil becomes less compressible causing the soil to have smaller volumetric changes. Hence a greater portion of the volumetric changes are associated with the expulsion of pore water from the soil skeleton which would in turn promote a faster dissipation of the pore fluid from the soil skeleton. This would result in faster consolidation of the medium. The influence of the [C] matrix in the stiffness Equation (1) becomes more significant for soils with higher Poisson's ratio values. For a constant value of Poisson's ratio, the time for consolidation is observed to be dependent only on the depth of soil stratum. The spacing between the footings was not observed to influence the time for total consolidation.

The interaction effects for various spacings is measured in terms of a normalized differential settlement factor. The differential settlement factor is defined as (Gw_d/Bq), in which w_d is the difference in settlement between the two ends of the footing. Positive rotation is the one in which the footings tilt towards each other and the negative rotation is in which the footings tilt away from each other. The Figures 5 and 6 show the variation of differential settlement with time factor. It could be observed from these figures that the maximum changes in differential settlements happen during the times of the highest consolidation rates as shown in Figures 3 and 4. The difference between the initial and final differential settlement is least for H = 10B case and in general decreased for higher Poisson's ratio values. For $\nu = 0.45$ soil, practically the full differential settlement occurred at the beginning of consolidation for all the analyses cases. This could again be attributed to the higher incompressibility of soil at higher values of Poisson's ratio.

The Figures 7 and 8 show the variation of pore pressures at the centre of the footings. The pore pressure (p) has been normalized with respect to the applied footing pressure (q). In general, for the isolated footings, the maximum pore pressure was highest at the beginning and decreased gradually with time. On the other hand, for the interacting footings, the pore pressures increased to a maximum followed by a decrease. The difference in this basic mech-

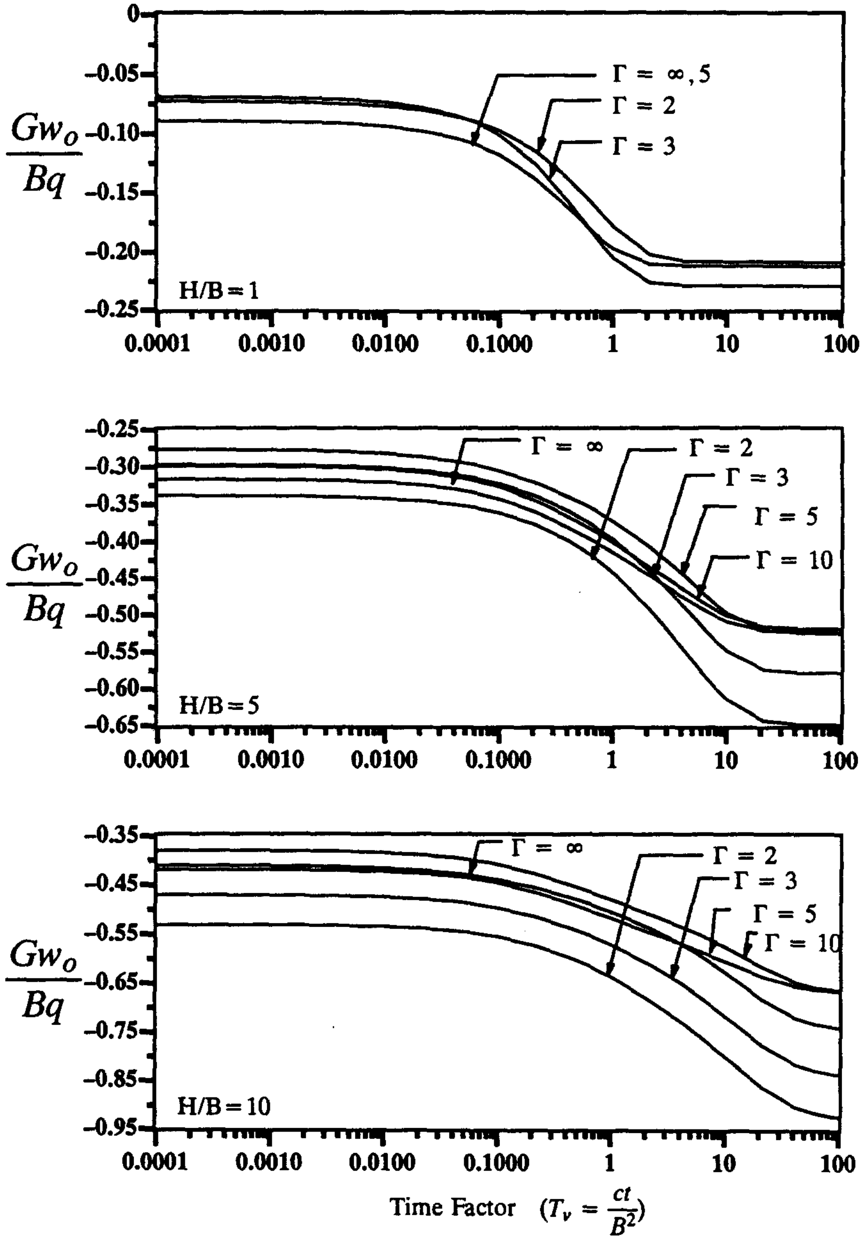


Figure 3: Normalized settlements $(\nu = 0.3)$

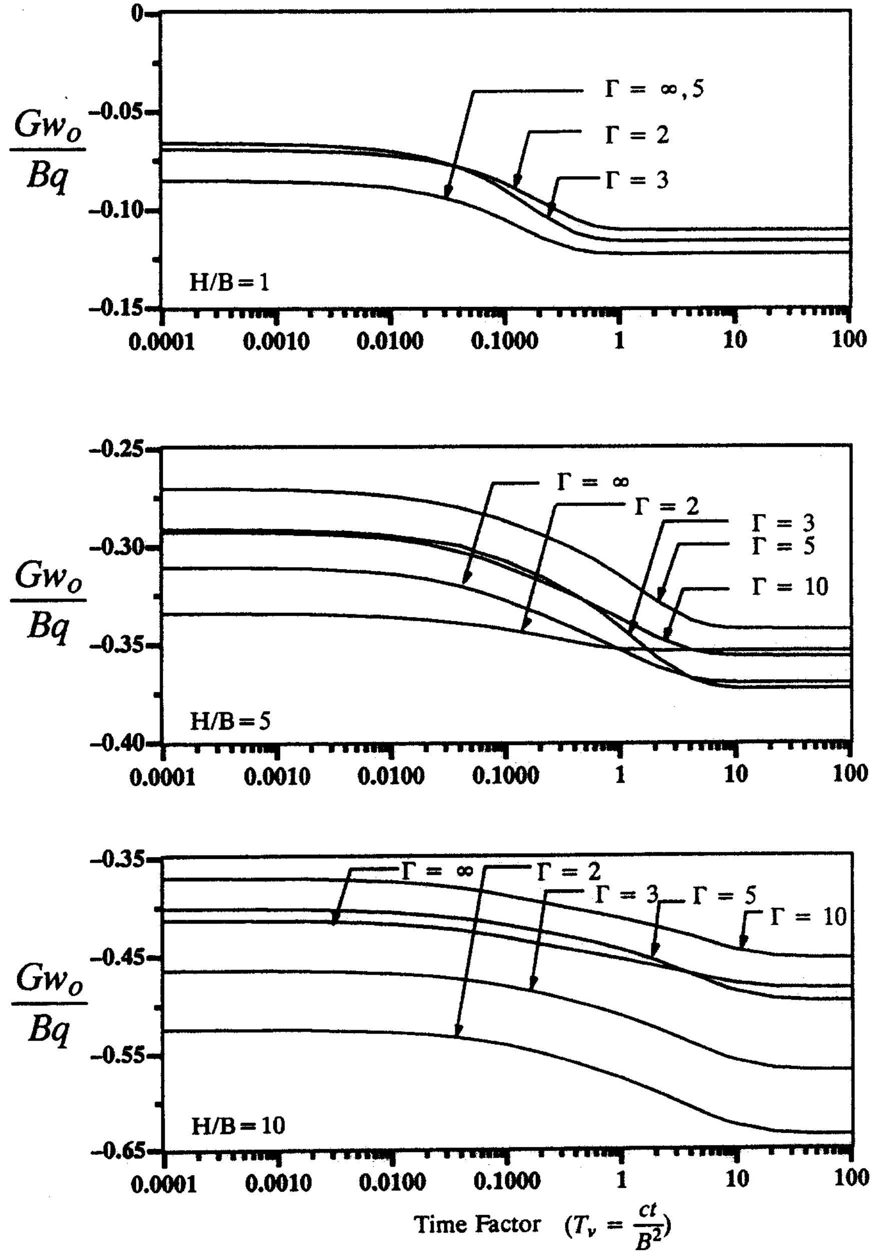


Figure 4: Normalized settlements $(\nu = 0.45)$

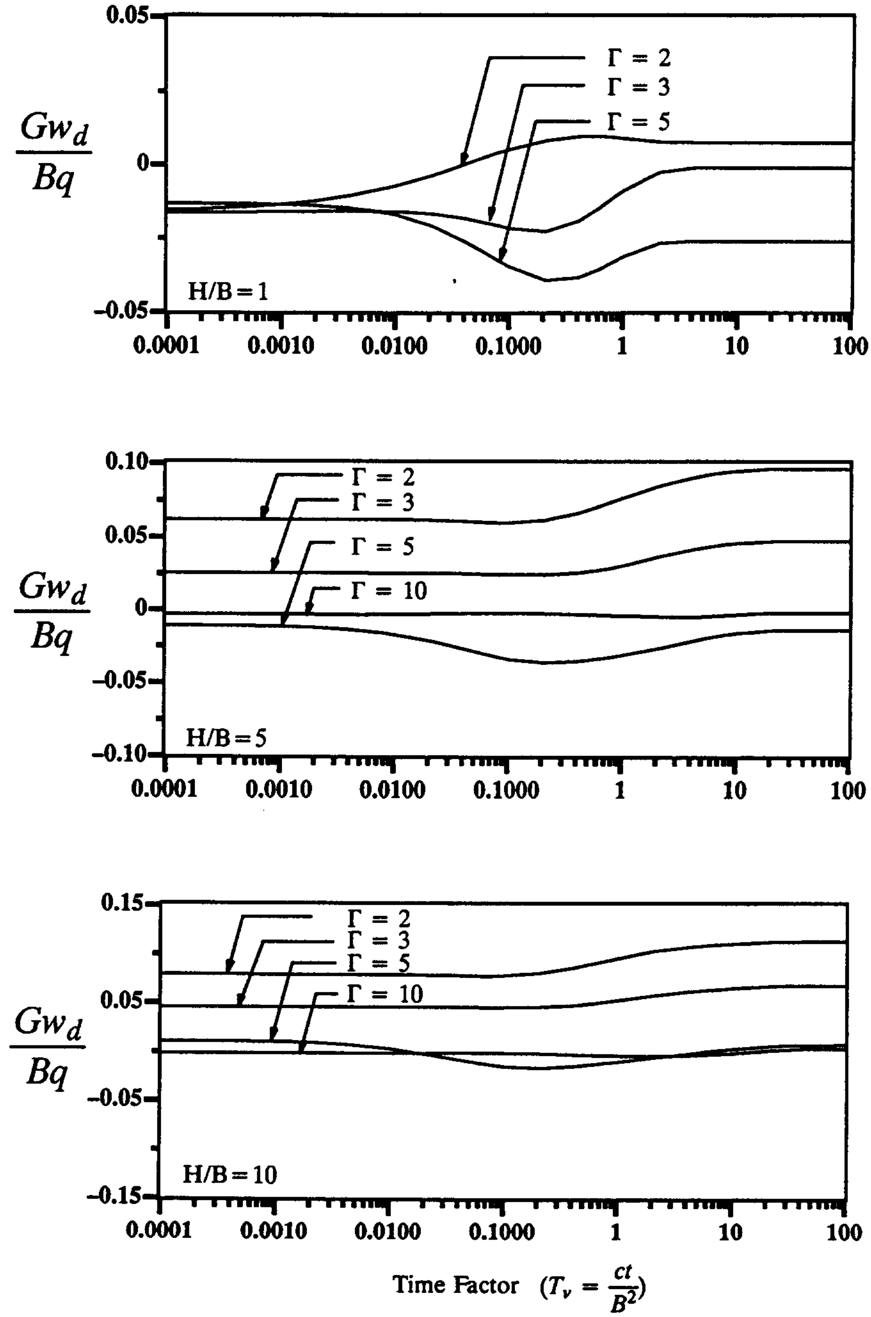


Figure 5: Normalized differential settlements $(\nu = 0.30)$

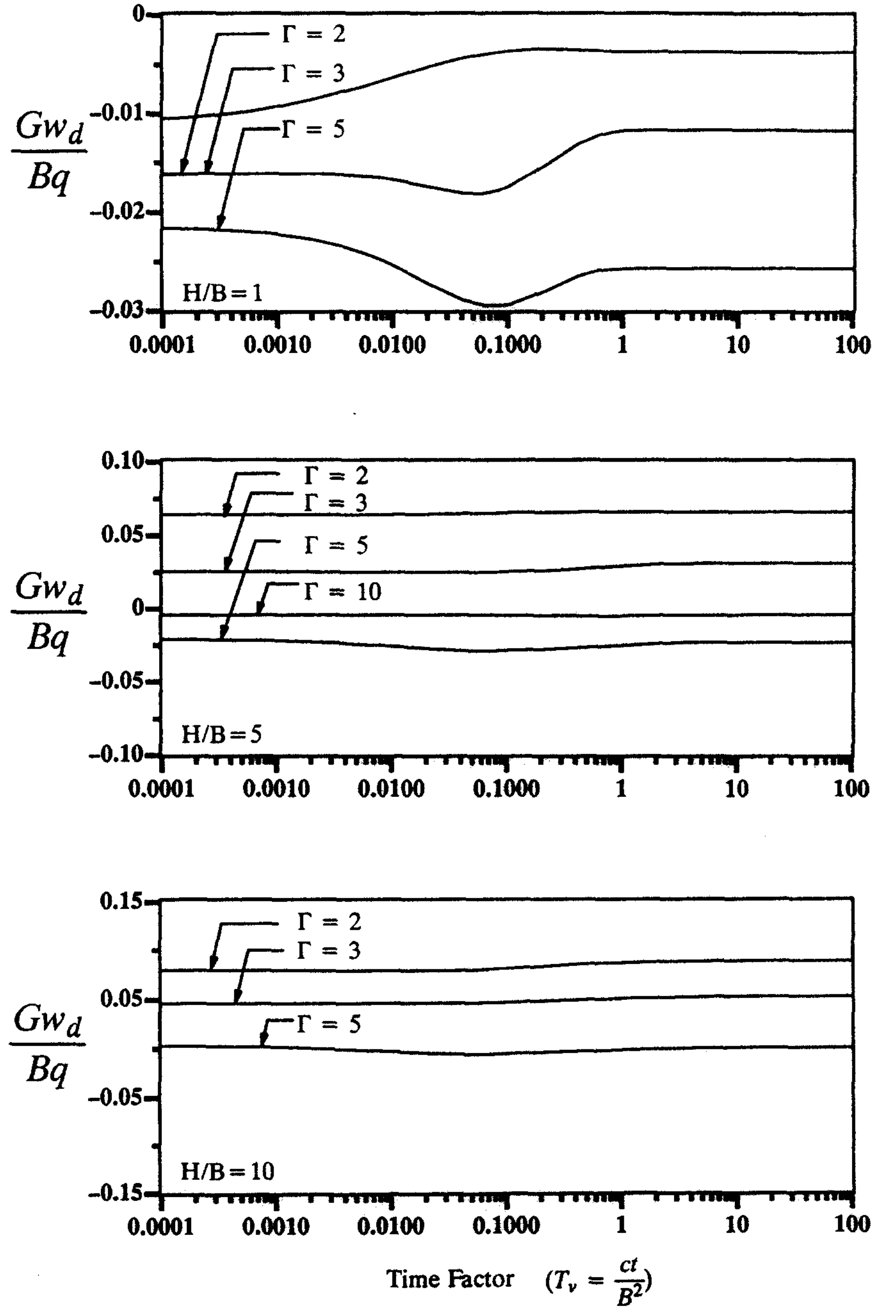


Figure 6: Normalized differential settlements (ν = 0.45)

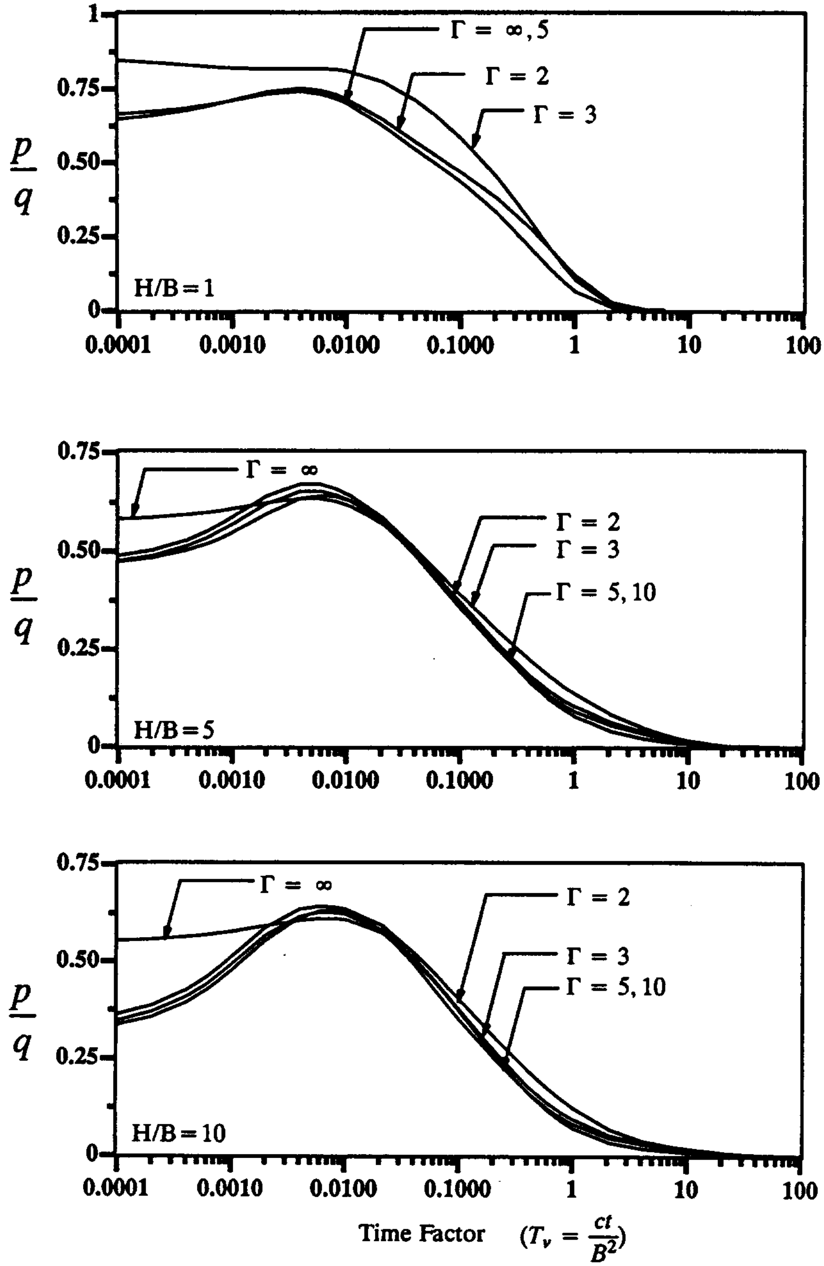


Figure 7: Pore pressure variation $(\nu = 0.30)$

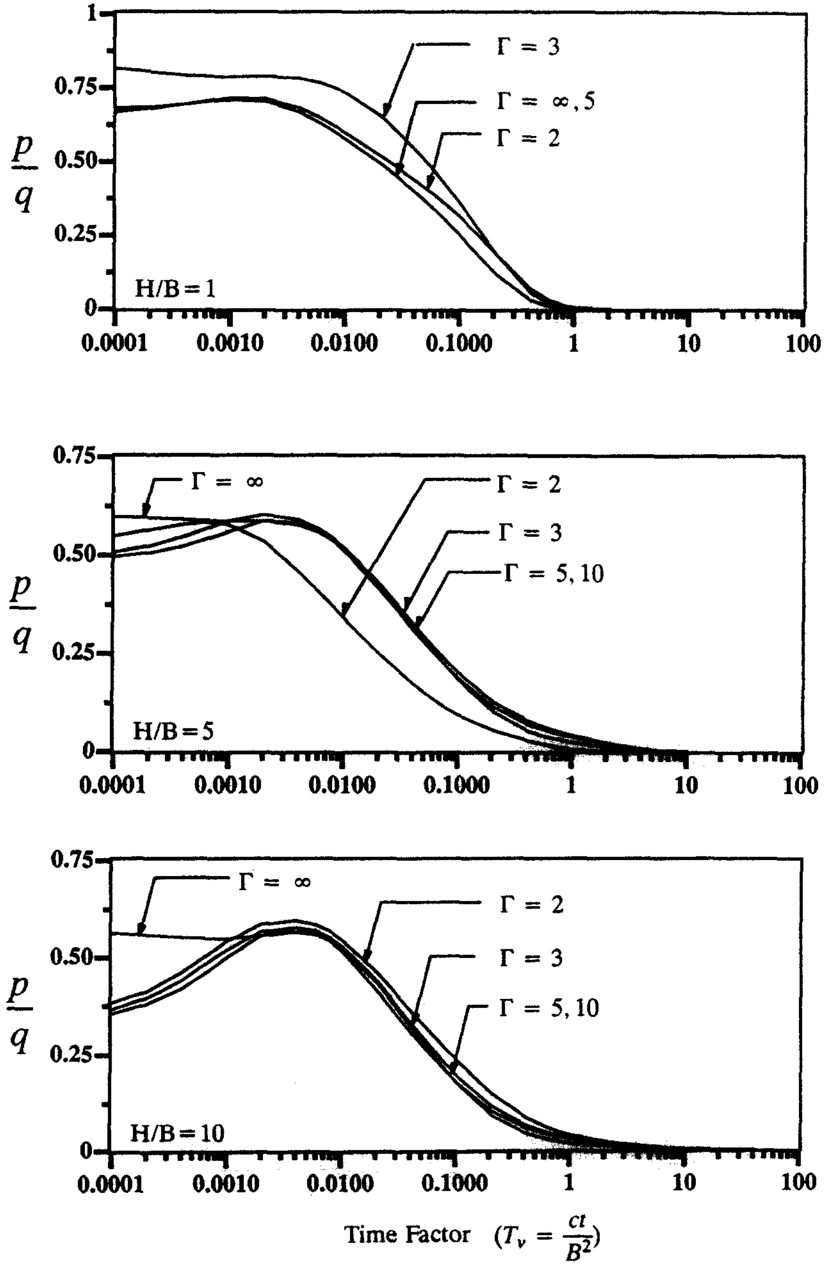


Figure 8: Pore pressure variation $(\nu = 0.45)$

anism of pore pressure increase can be attributed to the differences in the flow patterns under the isolated footing and the interacting footings. The maximum pore pressures for all spacings considered in this paper were observed to be higher than those for the isolated footings. The actual increase is dependent on the spacing between the footings and the depth of soil stratum. In general, the pore pressures were observed to be higher for $\nu = 0.3$ cases than those for $\nu = 0.45$. It can also be observed that the ratio between the maximum and initial pore pressures is higher for larger depths of soil layer.

The results presented in the paper point to the significant influence of the depth of the soil stratum and the Poisson's ratio on the interaction behaviour. The effect is appreciable if the footings are located closer to each other than the depth of soil layer and the effect rapidly decays as they are situated farther apart. In general, the duration for total consolidation was found to be dependent only on the depth of the stratum and it is not significantly influenced by the spacing between the footings.

APPENDIX-I: REFERENCES

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APPENDIX-II: NOTATION

The following symbols are used in the paper:

K	stiffness matrix of soil,
C	interaction stiffness between the soil and pore fluid,
E	compressibility matrix of the pore fluid,
H	permeability matrix governing the dissipation of pore fluid,
δ	convolution integration factor,
Δt	time step increment,
$u_{t+\Delta t}$	nodal displacement vector,
$p_{t+\Delta t}$	nodal pore pressure vector,
$f_{t+\Delta t}$	external traction and body force vector,
$g_{t+\Delta t}$	dissipation force vector,
α	porosity of the soil,
κ	permeability coefficient,
w_o, w_t, w_∞	settlements at the centre of footing at the instant of loading, at time t, and at the end of consolidation,
B	half-width of footing,
G	shear modulus of soil,
q	intensity of footing load,
c	coefficient of consolidation $= 2G\kappa/\gamma_w$, and
γ_w	unit weight of the pore fluid.

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