Computer Adaptation of Newmark's Charts

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The present paper describes the computer adaptation of the classical graphical technique developed by Newmark in 1942, for the computation of the axial stresses within an elastic soil mass. The paper describes the essential features of the computer implementation procedure and presents results for certain specific cases in which the surface of the elastic half-space region is subjected to uniformly distributed loads of finite extent.

INTRODUCTION

The determination of axial stresses induced within soil media due to externally applied loadings is of importance to many geotechnical design calculations (see e.g., Harr [2]; Winterkorn and Fang [7]; Bell [1]). In these calculations the mechanical behavior of the soil mass is usually idealized by an isotropic elastic response and the soil region is modelled by a half-space region. When these assumptions are invoked, the state of stress within the elastic soil mass can be calculated by employing Boussinesq's fundamental solution for the loading of a half-space region by a concentrated force acting normal to the plane surface of the region (Timoshenko and Goodier [6]). In particular, the stress component of interest to geotechnical calculations is the axial stress which acts within the loaded half-space region. From the solution of Boussinesq's problem, it is noted that when the elastic soil mass is homogeneous and isotropic, the axial normal stress σ_{zz} which acts in the vertical direction is independent of the elastic constants of the soil mass. Consequently, an integration procedure can be used to develop results for situations in which the surface of the half-space region is subjected to loads with an arbitrary planform. From a purely formal analytical perspective, the determination of the vertical

stress (σ_{zz}) at the interior of the half-space region (Figure 1) can be relegated to a simple exercise in integration where the stresses at the location (x, y, z) due to normal loading of intensity $p(\xi, \eta)$ can be written in the form

$$\sigma_{zz}(x, y, z) = \iint_A K(\xi, \eta, x, y, z)p(\xi, \eta)d\xi d\eta, \qquad (1)$$

where $K(\xi, \eta, x, y, z)$ is an influence or kernel function and A is the region of application of $p(\xi, \eta)$. Although the integration procedure is formally straightforward, convenient techniques for the evaluation of the above result can be developed only for a limited number of situations involving regular regions such as square regions, rectangular regions, circular regions, etc. When the loaded area has an irregular planform, it becomes necessary to adopt approximate numerical schemes to evaluate the integral encountered in Eq. (1). Recognizing this limitation, Newmark [3, 4] proposed a procedure for determining, graphically, the vertical stresses induced at the interior of a half-space region by the action of a distributed loading of constant intensity and arbitrary plan shape. Newmark's method essentially develops an influence chart in the form of contours of centered curvilinear squares which can be utilized in conjunction with a prescribed length scale to evaluate the vertical stress at a desired depth. One of the primary drawbacks of the method is the inordinate amount of repetitive calculations and graphical procedures that are entailed when stresses are computed at arbitrary locations and at arbitrary depths within the soil mass. The objective of this study is to investigate whether the basic features of Newmark's procedure can be adopted to computer-aided manipulations. The paper shows that computer-aided graphical techniques can be utilized to great advantage in achieving the desired objective. The paper develops a computer graphics scheme which takes into consideration (1) the change of scale of the influence chart as dictated by the depth of location at which the stresses are to be computed; (2) the arbitrary orientation of the loading area or areas in relation to the geometric center of the influence chart; and (3) the automatic eval-
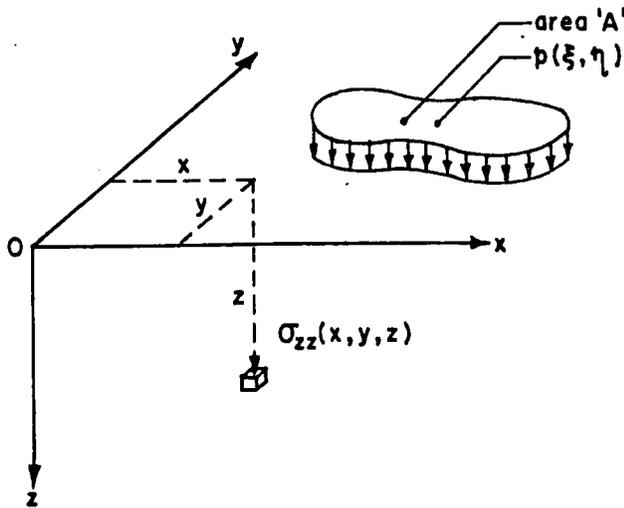
FIGURE 1. Loading of a half-space region.

uation of the subset of curvilinear squares and their fractions which are enclosed within the boundary geometry of the loaded regions.

NEWMARK'S GRAPHICAL PROCEDURE

In Newmark's graphical procedure, the influence charts are constructed by considering the state of stress induced in the half-space region due to a uniform loading in the shape of a sector (Figure 2). Boussinesq's fundamental solution for the concentrated normal force acting at the surface of a half-space region can now be integrated to determine the vertical stress σ_{zz} at any location beneath the apex of the loaded region. For a loading of unit intensity, the vertical stress can be evaluated in the form

$$\sigma_{zz} = \frac{\beta}{2\pi}(1 - \cos^3 \alpha), \tag{2}$$

FIGURE 2. Uniformly loaded sector region.
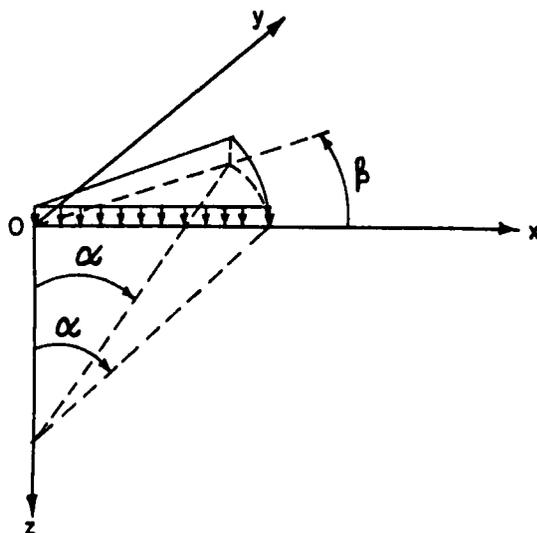
where β is expressed in radians. The surface of the half-space region can now be discretized into (1) sector regions bounded by one circular boundary, and (2) annular sector regions bounded by two circular boundaries, such that the vertical stress induced at the specific location due to each discretized region (which is subjected to the same intensity of external loading) is identical. Figure 3 shows a typical discretization pattern for the influence chart. The chart is also characterized by a scaling dimension which is indicated in Figure 3. The procedure for utilizing Newmark's graphical technique can be summarized as follows:

1. The outline of the loaded region (regular or irregular; single or multiple loaded areas of uniform stress) is drawn to scale. The scaling is defined in terms of the depth at which the vertical stress is to be evaluated, i.e., the scale in the influence chart is set equal to this depth.
2. The projected surface location of the point at which the stresses are to be calculated is placed at the origin of the influence chart. The orientation of the outline of the loaded regions is immaterial since the influence chart exhibits radial symmetry with respect to the origin.
3. The number of curvilinear squares and proportions thereof (m) enclosed within the boundary of the loaded regions is enumerated. The vertical stress induced at the point of interest is given by $\sigma_{zz} = mNp$, where N is the influence coefficient applicable for the particular influence chart and p is the intensity of the loading. In general, if m_i $(i = 1,2 . . N)$ curvilinear squares and proportions thereof are respectively subjected to loads of stress intensities p_i, then the stress induced at the point of interest is given by

$$\sigma_{zz} = N\sum_{i=1}^{s} m_i p_i. \tag{3}$$

Although the procedure of using Newmark's charts is extremely straightforward, it entails a tedious amount of manipulative effort. The effort is compounded in situations where either spatial or in-depth distributions of the vertical stresses are required for purposes of geotechnical engineering analysis. The procedures entailed in the calculation of vertical stresses by using Newmark's graphical technique are largely repetitive; for this reason it is natural to enquire whether a computer graphics-based procedure can be implemented to facilitate the application of the technique.

COMPUTER ADAPTATION OF NEWMARK'S GRAPHICAL PROCEDURE

In this section, we shall present a brief outline of the general procedure that will be adopted in the computer implementation of Newmark's procedure. The imple-
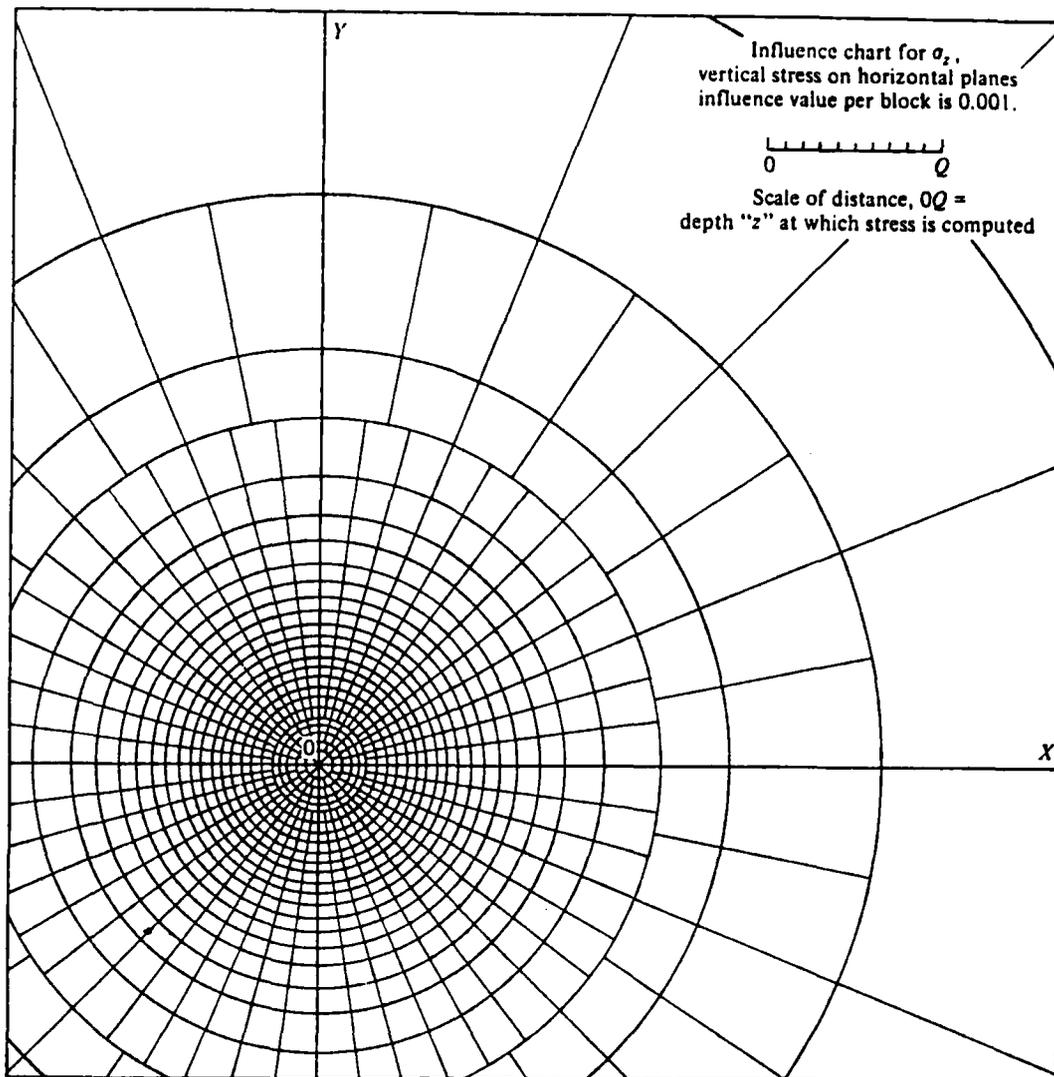
FIGURE 3. Influence chart for vertical stresses on horizontal plane (after Newmark [4]).

mentation of the graphical procedure takes into account two aspects. The first focusses on the graphical representation of the influence chart itself, and the second focusses on the development of a generalized algorithm which can be used to enumerate the number of curvilinear squares and proportions thereof which may be contained within the boundary of a loaded region. From the discussions in the preceding section, it is evident that the geometry of the influence chart is invariant of the specific problem examined. Consequently, the information concerning the graphical illustration of an influence chart can be included permanently in the program. The details of the implementation procedure are given elsewhere (Sutcliffe [5]). The second aspect of the software development, namely, the evaluation of the number of curvilinear squares enclosed within the foundation outline, can be approached by focussing on the estimation of the area enclosed within the segments of

the foundation outline and the appropriate annular region (Figure 4). It can be shown that the number of curvilinear squares and proportions thereof (m_i^*) contained within the intersection of the loaded region and the annular region are given by

$$m_i^* = M_i^* \frac{a_i^*}{A_i^*},\qquad(4)$$

where M_i^* are the number of curvilinear squares in the complete annulus; a_i^* is the area contained within the interaction between the loaded region and the annulus, and A_i^* is the area of the complete annulus. Since both M_i^* and A_i^* depend purely upon the basic influence chart, the problem of evaluating m_i^* is essentially reduced to evaluating the relevant area a_i^*. To evaluate a_i^* for a particular annulus, it is convenient to consider three regions, namely:
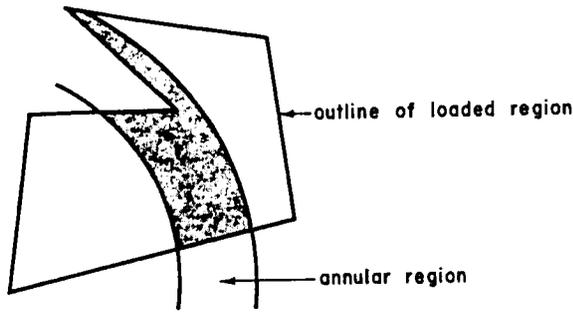
FIGURE 4. Intersection of annular region and loaded region.

1. the region inside the inner radius;
2. the region outside the outer radius;
3. the region contained within the annulus.

Each side of the polygon may pass through any or all of these regions. Any side of a polygonal loaded region may be associated with the three separate regions identified by the exterior, interior, and annular regions. For the purposes of identifying the location of the various segments, it is necessary to find the intersection of the line segment and the circles bounding the annulus. The software developed in connection with the exercise contains such a generalized subroutine (see, e.g., Sutcliffe [5]). In the evaluation of the intersection area a_i^*, it is convenient to proceed in a systematic way by calculating the areas subtended by parts of the line segments (applicable to the boundary of the loaded region) or curves (applicable to the boundaries of the annular region) applicable to each region. The following summarizes the procedure adopted.

1. For segments in the first region, the area of the sector of a circle shown in Figure 5 is calculated.
2. For segments in the second region, the area of the triangular region shown in Figure 6 is calculated.
3. For segments in the third region, the area of the sector of a circle shown in Figure 7 is calculated.

These areas are summed over all the sides of the polygon. When the operation of selecting the segments of

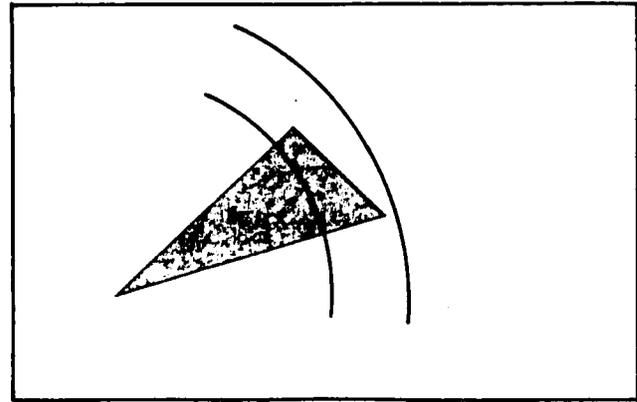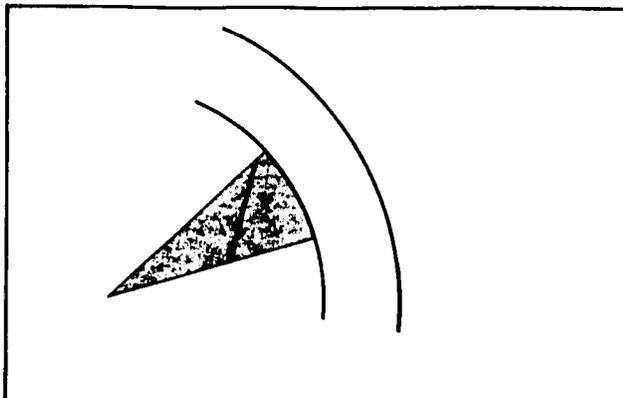FIGURE 5. Segments of the boundary in the first region.

FIGURE 6. Segments of the boundary in the second region.

the boundary of the loaded region is done in a counterclockwise sense, the areas around the point at which the stresses are to be determined (which corresponds to the origin of the chart) are added. The areas are subtracted for a clockwise sequence of operations. If the point at which the stress is required is outside the polygon, the result would be as shown in the sequence of Figure 8. In contrast, when the point of interest occurs within the boundary of the loaded region, the computation of the areas will also yield the area within the complete inner circular boundary of the annulus. This is illustrated in the sequence of Figure 9. For this reason, it is necessary to distinguish a priori whether the point of interest occurs either within or exterior to the loaded region. The program developed in connection with the study has appropriate algorithms to distinguish between the nature of the location at which the stresses are to be computed.

PROGRAM DESCRIPTION AND TYPICAL RESULTS

The procedures described in the preceding section have been implemented into the form of a program written in BASIC. The program contains 1033 lines and is written in

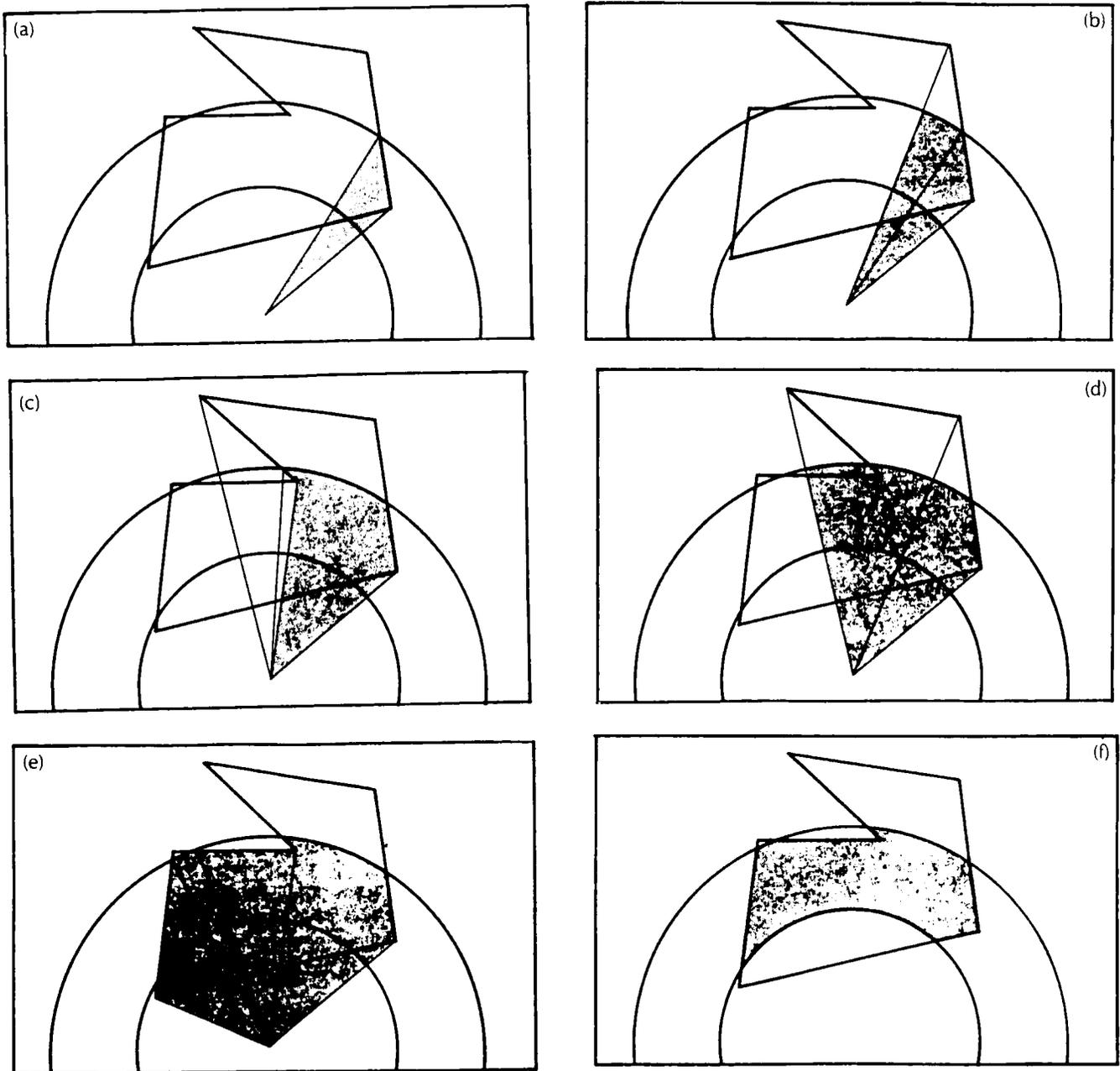FIGURE 7. Segments of the boundary in the third region.
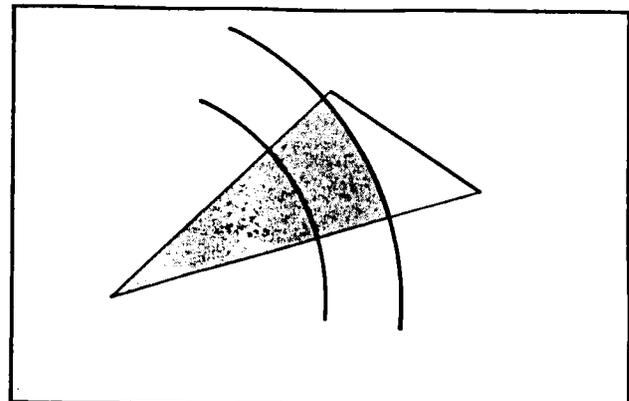
FIGURE 8. Computation of area at the intersection between the annular region and the loaded region. Point of interest outside the loaded region.

a user-friendly format. For example, to describe the polygonal load, the user inputs the coordinates of the corners of each polygon under uniform load. There may be up to 50 polygons of different load intensities that can be accommodated. Each polygon may contain up to 50 sides. Using such a discretization, it is possible to simulate relatively conveniently loaded regions with curved boundaries. The user must input the load intensity on each polygon as well as the coordinate of the location at which the stress is to be computed. The data

may be entered either manually or through an input file. An example of a data input sequence is a follows.

* number of areas

* loading in the first area

* number of corners in the first area

* loading in second area

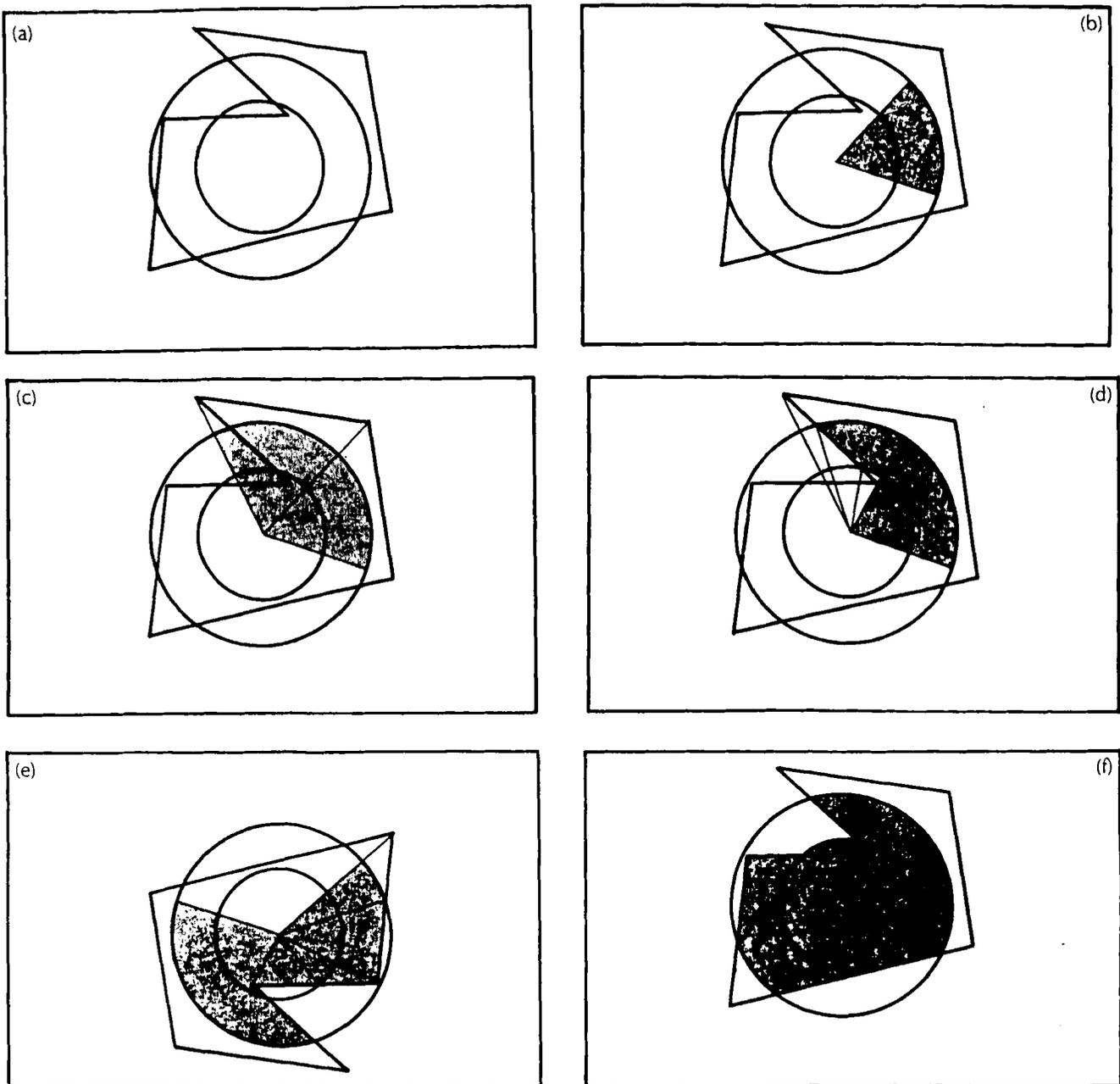*number of corners in the second area

:

FIGURE 9. Computation of area at the intersection between the annular region and the loaded region. Point of interest inside the loaded region.

⋮

etc.

* loading in the last area

* number of corners in the last area

* x, y for first corner in first area

* x, y for second corner in first area

⋮

etc.

* x, y for last corner in first area

* x, y for first corner of second area

* x, y for second corner in second area

⋮

etc.

* x, y for last corner in last area

* x, y for point for stress is to be determined

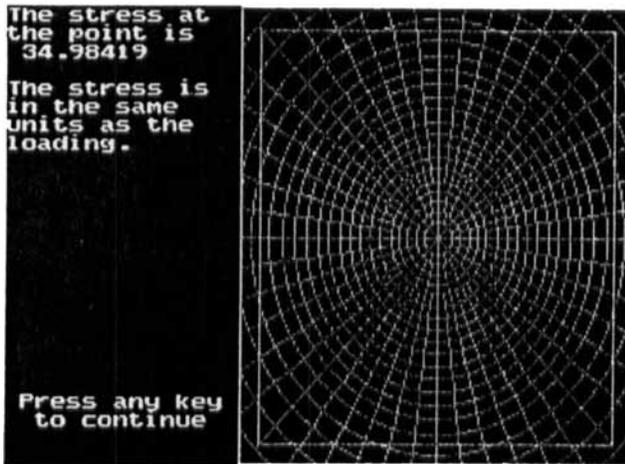* depth at which the stress is to be determine

FIGURE 10. Computation of vertical stress beneath the center of a uniformly loaded square region. The stress is computed at a depth of $z = b/2$, where b is the width of the loaded area. The applied stress = 50 kN/m^2. Result obtained from computerized Newmark's chart = 34.984 kN/m^2. The theoretical result = 35.043 kN/m².
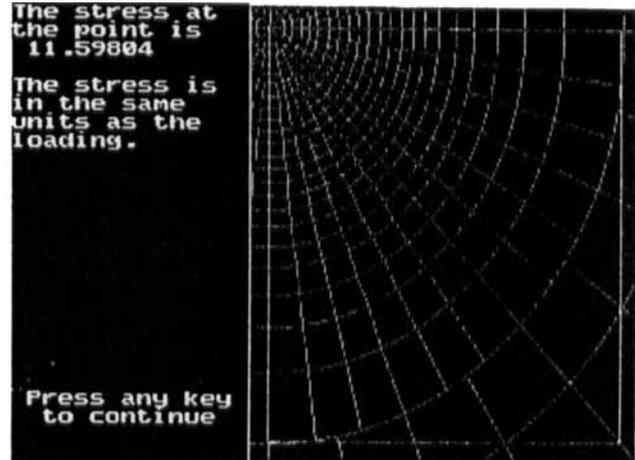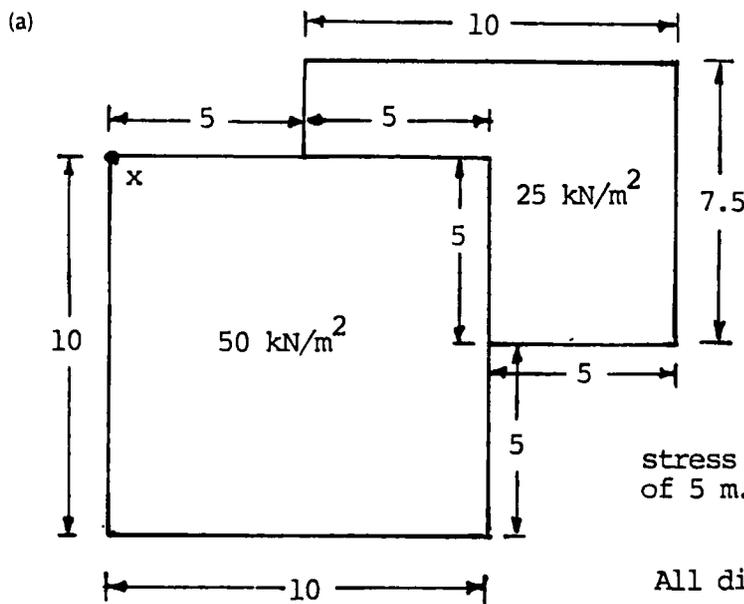

FIGURE 11. Computation of vertical stress beneath the corner of a uniformly loaded square region. The stress is computed at a depth of $z = b/2$, where b is the width of the loaded area. The applied stress = 50 kN/m^2. Result obtained from the computerized Newmark's chart = 11.598 kN/m². The theoretical result = 11.623 kN/m^2.

(a)

stress is computed at x at a depth of 5 m.
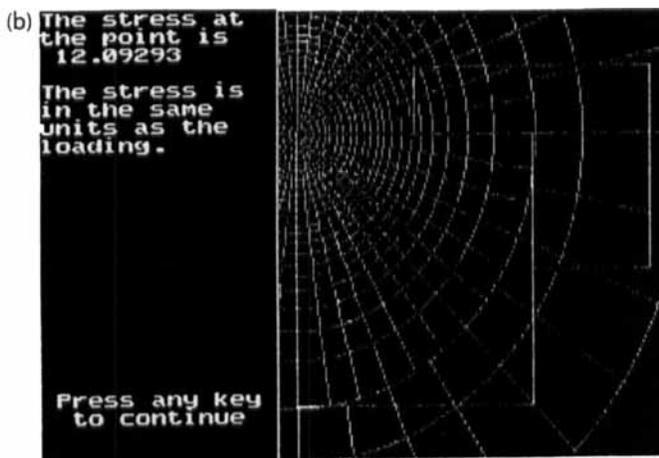
All dimensions are in metres.

(b)

FIGURE 12. Computation of vertical stress beneath the corner of an irregular and non-uniformly loaded region. The stress is computed at a depth of 5 m. Result obtained from the computerized Newmark's chart = 12.09 kN/m^2.

If any quantity is abnormally large or small, the program will not accept the input data. If an input file is used, the program is stopped and the message describing the problem appears on the screen. If the data are entered manually, the user is simply prompted again for the same information. An additional feature of the program is the graphical display of the information during manual data entry. At certain points during the data entry, the data are plotted on the screen so that the user may check the accuracy of the coordinates visually. If the display shows an error, the data can be changed. The program is simple to use, and the data may be prepared by sketching a plan view of the loading pattern on grid paper. All the coordinates can then be read from the grid while the program is running. The units used in preparing the coordinates are not important, provided they are consistent with those used to define the location of the point at which the stress is to be computed. The resulting stress output is in the same units as those used for data input concerning loading in each area. If the loading pattern is nonuniform, a discretization into uniformly loaded regions can be invoked.

Once all the data are entered, the program shows the loading areas on the screen along with the display of Newmark's charts, all drawn to scale. The stress at the desired point is calculated and displayed on the screen. At this point, any or all of the data may be changed and the calculations repeated.

The procedure described in the paper has been applied to a number of typical examples which are illustrated in Figures 10–12. In each case, the results derived from the developed computer algorithm are compared with equivalent results derived from manual calculations or exact analytical results. The results indicate very favorable and accurate comparisons.

CONCLUSIONS

The methodology outlined in this paper extends the use of Newmark's classical graphical procedure to computer implementation. This approach presents a simple and effective means of evaluating the vertical stresses that are induced in elastic earth masses subjected to surface loadings which can have discrete plan shapes and non-uniform intensities. The software is developed for implementation on a microcomputer which extends its use to geotechnical engineering design applications. The methodologies proposed in this paper can be further applied to evaluate other types of stresses and displacements in half-space regions which can be described in the form of Newmark-type influence charts.

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