Large Strain and Dilatancy Effects in Pressuremeter

By A. Patrick S. Selvadurai,[1] M. ASCE

Abstract: Herein is developed a fundamental solution to the problem of the expansion of a cylindrical cavity in a dilatant soil which exhibits effects of large strains. A result of some importance to geomechanics concerns the development of a shear stress-strain curve for the tested soil by using the measured pressure-cavity wall displacement response. It is shown that in order to develop the desired stress-strain relationship for a dilatant soil it is necessary to introduce a kinematic constraint which relates the finite volumetric strain in the soil to the components of the Lagrangian strain matrix. Particular results are developed for the case where the volumetric strains vary linearly with the radial and circumferential Lagrangian strain components.

Introduction

The class of problems which examines the expansion of a cylindrical cavity in a soil mass is of some interest to geotechnical engineering. Solutions developed for the expansion of a cavity in an unbounded soil have been used quite extensively in the examination of the pressuremeter problem and in the study of the state of stress around driven piles. Articles by Gibson and Anderson (5), Smith (23), Smith and Kay (24), Ladanyi (11–13), Vesic (27), Palmer (16), Baguelin, et al. (1), Prevost and Hoeg (18), Banerjee and Stipho (3), and Carter, et al. (4), investigate the cylindrical cavity expansion problem by incorporating a diversity of material properties which range from ideal or work hardening plastic behavior to strain softening. The literature, both theoretical and experimental, devoted to cavity expansion problems associated with soil media is quite extensive and no attempt will be made to provide a comprehensive bibliography. Accounts of such studies are given by Wroth (28) and Baguelin, et al. (2).

The pressuremeter problem in in situ testing consists of the application of a uniform pressure to a section of a borehole which is either preformed or created by a self-boring pressuremeter. The dimensions of the pressurized region are arranged in such a way that an approximate state of plane strain exists in the central section of the pressurized length [e.g., Laier, et al. (14), Livneh, et al. (15)]. With currently used pressuremeters, the displacement of the cavity wall is measured as a function of the applied pressure. The primary objective of the test is therefore to determine the constitutive relationships for the tested soil by examining the pressure-cavity wall displacement response.

In earlier theoretical developments of the pressuremeter problem, it has been recognized that the expansion of the cavity induces large strains in soil regions in the vicinity of the cavity wall. Figs. 1 and 2 show typ-

[1]Prof. and Chmn., Dept. of Civ. Engrg., Carleton Univ., Ottawa, Ontario, Canada, K1S 5B6.

Note.—Discussion open until August 1, 1984. To extend the closing date one month, a written request must be filed with the ASCE Manager of Technical and Professional Publications. The manuscript for this paper was submitted for review and possible publication on February 3, 1983. This paper is part of the *Journal of Geotechnical Engineering*, Vol. 110, No. 3, March, 1984. ©ASCE, ISSN 0733-9410/84/0003-0421/$01.00. Paper No. 18637.

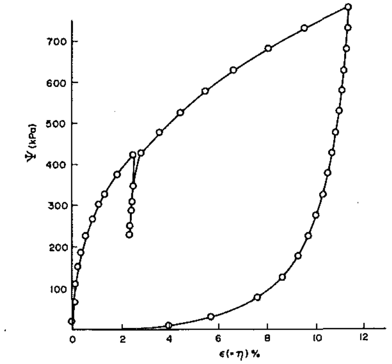


FIG. 1.—Results of Pressuremeter Tests Conducted in Sand [Hughes, et al. (8)]

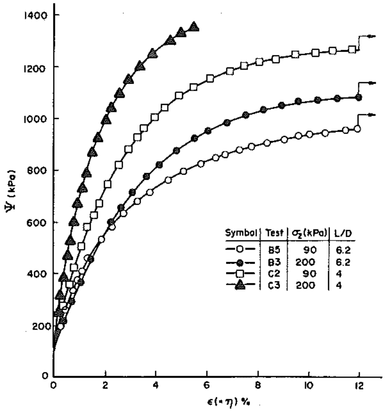


FIG. 2.—Results of Laboratory Pressuremeter Tests in Sand [Jewell, et al. (10)]

ical results obtained by Hughes, et al. (8) and Jewell, et al. (10) for the pressure versus cavity wall displacement during tests conducted on sands. It is evident that substantial displacements are induced at the boundary of the pressurized cavity. [Test results by Hughes, et al. (8), record cavity boundary displacements of the order of 14% of the cavity radius, and the test results by Jewell, et al. (10) record values of the order of 24%.] These in turn would lead to the development of large strains in the tested soil mass. Several authors have therefore attempted to examine the pressuremeter problem without placing any restriction on the magnitude of the strains developed. Separate investigations by Palmer (16), Ladanyi (13), and Baguelin, et al. (1) have considered the problem related to the expansion of a cylindrical cavity in a clay soil mass which exhibits undrained or incompressible response. The assumption of incompressibility greatly reduces the mathematical analysis of the large strain problem. In particular, these investigations assume that the relationship between the stress difference, $\sigma_{rr} - \sigma_{\theta\theta}$ (where $\boldsymbol{\sigma}_{ij}$ is the Cauchy stress matrix in the deformed state), can be expressed as a function, $\Phi(\epsilon_{\theta\theta})$, of the circumferential component, $\epsilon_{\theta\theta}$, of the finite strain matrix. Furthermore, the form of the function, $\Phi(\eta)$, can be directly determined from the pressure ψ-cavity wall displacement, ηa, relationship (in which a = the radius of the cavity in the undeformed state).

The analogous problem which relates to the expansion of a cylindrical cavity in a compressible or dilatant soil is particularly relevant to the laboratory or in situ determination of the mechanical properties of granular soils. The presence of soil compressibility or dilatancy makes the large strain analysis of the cavity expansion problem much more complicated. The investigations of Ladanyi (11), Palmer and Mitchell (17), Wroth and Windle (29), Hughes, et al. (8), and Baguelin, et al. (2), concentrate on the treatment of the small strain cavity expansion problem for compressible or dilatant soils. These articles indicate that a solution for the cavity expansion problem, for the small strain formulation, can be attempted only by imposing certain a priori assumptions with regard to the constitutive behavior of the tested soil.

This paper is concerned with the analysis of the large strain response of a cylindrical cavity in a soil mass which exhibits incompressible, compressible, or dilatant characteristics. The general form of the constitutive relationship is assumed to be such that the stresses in the deformed configuration depend on the large strains in the current configuration. Such an assumption seems to be appropriate for monotonic loading paths that are associated with the virgin or pseudo-elastic curve of a cavity expansion problem.

In the first part of the paper it is shown that the undrained cavity expansion problem for large strains currently quoted in the literature (1,13,16), is *mathematically similar* to the finite deformation problem associated with the expansion of a cylindrical cavity in an isotropic elastic solid treated originally in the classical studies of Rivlin (19) [see also Rivlin (21), Truesdell and Noll (26), Green and Zerna (6), Green and Adkins (7), and Spencer (25)]. The function, Ψ, described previously bears a formal mathematical resemblance to a combination of response functions used to characterize isotropic elastic materials which experience large strains. The remainder of the paper concentrates on the development of a solution to the finite strain cavity expansion problem for a compressible or dilatant soil medium. In order to develop such a solution, a "kinematic constraint" is introduced in the form of a relationship between the large volumetric strain, $\Delta V/V_0$, and the components of the Lagrangian strain matrix. When this kinematic constraint has a linear form, the large strain cavity expansion problem can be solved in an exact fashion. In this sense, the functional relationship for $\Phi(\eta)$ for the compressible soil can be derived from the pressure-cavity wall displacement response. The a priori assumptions with regard to constitutive behavior of the tested soil (i.e., the kinematic constraints) limits the scope of the pressuremeter test as a means for determining all possible mechanical parameters of dilatant or compressible granular soils. Nonetheless, the test itself is a useful guide for the verification of constitutive parameters derived from other laboratory or in situ tests.

Fundamental Equations

In order to describe the large deformation, it is convenient [see e.g., Green and Zerna (6), Selvadurai and Spencer (22)] to introduce a system of rectangular Cartesian coordinates and denote by X_A (A = 1, 2, 3) the coordinates of a generic particle in the reference configuration, and by $x_i(X_A)$ (i = 1, 2, 3) the coordinates of the same particle in the deformed configuration (i.e., x_i are functions of X_A). The cylindrical polar coordinates (R, Θ, Z) in the reference configuration and r, θ, z in the deformed configuration are given by

$$X_1 = R\cos\Theta; \quad X_2 = R\sin\Theta; \quad X_3 = Z$$

$$x_1 = r\cos\theta; \quad x_2 = r\sin\theta; \quad x_3 = z \qquad (1)$$

For plane strain deformations which exhibit cylindrical symmetry, r depends solely on R and θ, z are independent of R: i.e.

$$r = r(R); \quad \theta = \Theta; \quad z = Z \qquad (2)$$

With Eq. 2, the matrix of deformation gradients in the (R, Θ, Z) directions is

$$\mathbf{F} = \begin{bmatrix} \frac{dr}{dR} & 0 & 0 \\ 0 & \frac{r}{R} & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (3)$$

From the geometry of deformation and from conservation of mass

$$\det \mathbf{F} = \frac{V}{V_0} \qquad (4)$$

in which V corresponds to the volume of an element in the deformed configuration whose initial volume is denoted by V_0. The Cauchy-Green strain matrix refers to the r, θ, z system given by

$$\mathbf{B} = \mathbf{F}\mathbf{F}^T = \begin{bmatrix} \left(\frac{dr}{dR}\right)^2 & 0 & 0 \\ 0 & \left(\frac{r}{R}\right)^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (5)$$

$$\text{Also} \quad \mathbf{B}^{-1} = \begin{bmatrix} \left(\frac{dR}{dr}\right)^2 & 0 & 0 \\ 0 & \left(\frac{R}{r}\right)^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (6)$$

The Lagrangian strain matrix, $\boldsymbol{\epsilon}$, is related to $\mathbf{B}$ according to

$$2\boldsymbol{\epsilon} = \mathbf{B} - \mathbf{I} \qquad (7)$$

in which $\mathbf{I}$ = the identity matrix. In keeping with the notation in soil mechanics, compressive strains are taken as positive. The invariants of $\mathbf{B}$ are

$$I_1 = \operatorname{tr} \mathbf{B} = 1 + \left(\frac{r}{R}\right)^2 + \left(\frac{dr}{dR}\right)^2 \qquad (8)$$

$$I_2 = \operatorname{tr} \mathbf{B}^{-1} = 1 + \left(\frac{R}{r}\right)^2 + \left(\frac{dR}{dr}\right)^2 \qquad (9)$$

$$I_3 = \det \mathbf{B} = \frac{r^2}{R^2}\left(\frac{dr}{dR}\right)^2 \qquad (10)$$

To define the state of stress in the soil mass, the Cauchy measure of stress (which is given by resolving the force per unit area in the deformed body referred to the r, θ, z system) is introduced. In pressuremeter type problems, the nonzero components of the Cauchy stress matrix, $\boldsymbol{\sigma}$, are given by

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{rr} & 0 & 0 \\ 0 & \sigma_{\theta\theta} & 0 \\ 0 & 0 & \sigma_{zz} \end{bmatrix} \qquad (11)$$

For consistency, compressive stresses are taken to be positive. The state of cylindrically symmetric deformation characterized by Eq. 2 will be realized only in materials which exhibit either complete isotropy or transverse isotropy in the X_1-X_2 plane. In the following, attention will be restricted to isotropic soils. The state of deformation is such that there is no rotation of principal axes of the strain components. The elements of the strain matrix in the finitely deformed state can be derived from the summation of the strain increments taken over a history of monotonic loading. Consequently, for virgin loadings associated with a pressuremeter test, the stress-strain relationship can be expressed in the functional form

$$\boldsymbol{\sigma} = F(\mathbf{B}) \qquad (12)$$

From a representation theorem for isotropic functions [e.g., Rivlin and Ericksen (20), Jaunzemis (9), and Spencer (25)] it can be shown that Eq. 12 has the explicit form

$$\boldsymbol{\sigma} = \chi_0 I + \chi_1 \mathbf{B} + \chi_{-1} \mathbf{B}^{-1} \qquad (13)$$

in which $\chi_n = \chi_n(I_r)$ ($n = 0, 1, -1$; $r = 1, 2, 3$) = scalar functions of the invariants of $\mathbf{B}$.

The equations of equilibrium in the deformed configuration can be expressed in terms of the components of $\boldsymbol{\sigma}$. For zero body forces, the nontrivial equation of equilibrium reduces to

$$\frac{d\sigma_{rr}}{dr} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0 \qquad (14)$$

The traction boundary conditions in the deformed configuration can be expressed in the form

$$\mathbf{T} = \boldsymbol{\sigma} \underset{\sim}{n} \qquad (15)$$

in which $\mathbf{T}$ = the tractions in the deformed surface which has direction cosines, $\underset{\sim}{n}$.

Pressuremeter Problem for Incompressible Soil

The solution to the large strain undrained cavity expansion problem is presented in current geotechnical literature [Palmer (16), Landanyi (13), and Baguelin, et al. (2)]. The purpose of this section is to illustrate the fact that the derivation of the large strain undrained cavity expansion problem can be recovered from the formal mathematical analysis of the associated finite elasticity problem in nonlinear continuum mechanics. For incompressible soil behavior det $\mathbf{F}$ = det $\mathbf{B}$ = I_3 = 1, and the remaining strain invariants of **B** reduce to the single invariant

$$I_1 = I_2 = I = 1 + \left(\frac{r}{R}\right)^2 + \left(\frac{R}{r}\right)^2 \quad (16)$$

Considering plane strain deformations involving incompressible materials, the nonlinear constitutive relation (Eq. 13) reduces to the form

$$\boldsymbol{\sigma} = -pI + \chi_1 \mathbf{B} + \chi_{-1}\mathbf{B}^{-1} \quad (17)$$

in which $\chi_n = \chi_n(I)$ $(n = 1, -1)$; and p = an isotropic stress which is to be determined by satisfying the boundary conditions of the cavity expansion problem. From Eq. 17

$$\sigma_{rr} - \sigma_{\theta\theta} = \Gamma(I)\left[\left(\frac{R}{r}\right)^2 - \left(\frac{r}{R}\right)^2\right] \quad (18)$$

in which $\Gamma(I) = \chi_1(I) - \chi_{-1}(I)$ .... (19)

The variable R can be eliminated from Eq. 18 by an integration of the incompressibility condition det $\mathbf{F} = 1$, i.e.

$$\int_{a(1+\eta)}^{r} r\,dr = \int_{a}^{R} R\,dR \quad (20)$$

in which a = the radius of the cavity in the undeformed state; and ηa = the displacement at the cavity wall boundary (Fig. 3).

Evaluation of Eq. 20 yields

$$\frac{r}{R} = \left[1 - \frac{a^2\eta(2+\eta)}{r^2}\right]^{-1/2} \quad (21)$$

Using this relationship, Eq. 18 can be recast in the form

$$\sigma_{rr} - \sigma_{\theta\theta} = \Phi^{i*}(r, \eta) \quad (22)$$

In Eq. 22 the superscript 'i' is intended to signify the incompressible case and

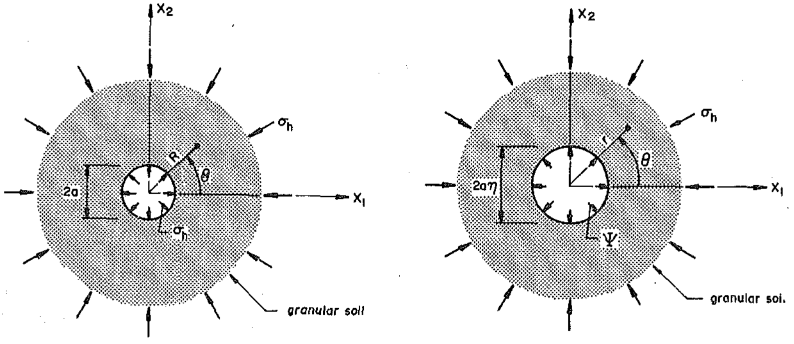


FIG. 3.—Geometry of Cavity in Undeformed and Deformed Configurations

$$\Phi^{i*}(r,\eta) = \Gamma(r,\eta)\left\{\frac{a^2\eta(2+\eta)[a^2\eta(2+\eta)-2r^2]}{r^2[r^2-a^2\eta(2+\eta)]}\right\} \quad (23)$$

The equation of equilibrium can now be integrated, within appropriate limits, to yield the following integral equation for the response function $\Phi^{i*}(\eta)$, i.e.

$$\int_{\Psi(\eta)}^{\sigma_h} d\sigma_{rr} = \int_{(1+\eta)a}^{\infty} \Phi^{i*}(r,\eta)\frac{dr}{r} \quad (24)$$

in which Ψ = the radial component of the Cauchy stress as the cavity wall boundary; and σ_h = the horizontal component of the in situ stress state (Fig. 3). The analysis of the large strain undrained cavity expansion problem is reduced to the solution of Eq. 24 for the constitutive functions $\chi_n(I)$. It becomes clear that the determination of both these response functions, $\chi_n(I)$, from a single test observation is an impossible task. The equivalent response function, $\Gamma(I)$, defined by Eq. 19 can be evaluated only by adopting numerical techniques for the solution of the integral equation

$$\Psi(\eta) - \sigma_h = \int_{\infty}^{a(1+\eta)} \Phi^{i*}(r,\eta)\frac{dr}{r} \quad (25)$$

The approach adopted by Palmer (16) and others in the treatment of the undrained cavity expansion problem is to identify $\Phi^{i*}(r,\eta)$ as a separate constitutive function which describes the "shear stress-strain" relationship for the tested soil. This is evident from Eq. 22. Introducing a change of variable defined by

$$\Omega = \frac{r}{R} - 1 \quad (26)$$

the response function (Eq. 25) can be expressed in the form $\Phi^{i*}(r,\eta) = \Phi^i(\Omega)$ or

$$\sigma_{rr} - \sigma_{\theta\theta} = \Phi^i(\Omega) \quad (27)$$

Using these reductions, the result (Eq. 25) can be expressed in the form

$$\Psi(\eta) - \sigma_h = \int_0^{\eta} \frac{\Phi^i(\Omega)d\Omega}{\Omega(1+\Omega)(2+\Omega)} \quad (28)$$

The explicit form for $\Phi^i(\eta)$ can be obtained by differentiating Eq. 28 with respect to η, i.e.

$$\Phi^i(\eta) = \eta(1+\eta)(2+\eta)\frac{d\Psi}{d\eta} \quad (29)$$

It is important to note that currently available pressuremeter devices, such as the Camkometer or the Autoforeur, measure the function $\Psi(\eta)$ directly in the test. The result (Eq. 29) is given in the geotechnical literature cited previously (2,13,16). It should, however, be noted that the methodology adopted in the present derivation uses a rigorous formulation based on principles of nonlinear continuum mechanics. It illustrates the fact that the large strain undrained constitutive property, derived from a pressuremeter test $[\Phi^i(\eta)]$ is essentially a combination of the response functions (χ_n) required to correctly describe an invariant nonlinear constitutive relationship applicable for isotropic soils. Such separate descriptions of χ_n are of course necessary for the analytical or numerical solution of two or three-dimensional boundary value problems.

Pressuremeter Problem for Dilatant Soil

When examining the pressuremeter problem for a dilatant (or compressible) soil it is important to realize that both the density of the soil in the finitely deformed state and the deformation [i.e., $r(R)$] constitute unknowns of the problem. Consequently, the large strain problem related to the expansion of a cylindrical cavity in a dilatant soil cannot be attempted without imposing certain a priori constitutive constraints on the deformation characteristics of the soil. [In contrast, the stringent constraints of incompressible behavior essentially fixes the possible mode of deformation, $r(R)$, independent of the stress-strain characteristics of the soil.] Therefore, to examine the pressuremeter problem related to a dilatant soil it is, unfortunately, necessary to impose a plausible kinematic constraint on the deformation. For granular soils such as dense sands which exhibit dilatancy effects, these constraints will usually be derived from experimental observations. For example, Ladanyi (11) introduces a bilinear form of the volumetric strain-shear strain relationship in his classical studies of the compressible small strain response of the pressuremeter test. Hughes, et al. (8) have assumed a linear relationship between the same variables in their analysis of the small strain response of the pressuremeter tests on cohesionless soils. The work by Wroth and Windle (29) employs a linearized small strain relationship between the volumetric strain and the circumferential strain. Any kinematic assumption, despite its limitations, provides an essential first approximation for the examination of the pressuremeter results derived from tests conducted on dilatant or compressible soils. In this paper these concepts are extended to include effects of large strains. In particular it is assumed that the dilatant characteristics of the soil are such that the dilatant volumetric strain $(\Delta V/V_0)$ is related to the components of the Lagrangian strain matrix, $\boldsymbol{\epsilon}$, in a linear fashion, i.e.,

$$\frac{\Delta V}{V_0} = -\sin \nu(\lambda_1 \epsilon_{rr} - \lambda_2 \epsilon_{\theta\theta}) \quad \text{(30)}$$

in which ν = a dilatancy angle; and λ_1 and λ_2 = positive constants. Hughes, et al. (8) have used a linearized equivalent of Eq. 30 (with $\lambda_1 = \lambda_2 = 1$) to examine the results of pressuremeter tests conducted on sand. The approximation used by Wroth and Windle (29) essentially corresponds to a linearized or small strain equivalent of Eq. 30 with $\lambda_1 = 0$ and $\lambda_2 =$ 1. The restrictions imposed on the constants encountered in Eq. 30 will be examined later (see Eqs. 34–36). In terms of the finite strain measures, Eq. 30 can be written in the form

$$\frac{\sin \nu}{2}\left[\lambda_1\left(\frac{dr}{dR}\right)^2 - \lambda_2\left(\frac{r}{R}\right)^2 + \frac{r}{R}\frac{dr}{dR}\right] = \frac{\sin \nu}{2}(\lambda_1 - \lambda_2) + 1 \quad \text{(31)}$$

By introducing suitable substitutions, the nonlinear differential Eq. 31 can be reduced to an ordinary differential equation of the first order. The solution of Eq. 31 is

$$r = R\left(1 + \frac{D}{\rho^{\mu+2}}\right)^{1/2} \quad \text{(32)}$$

$$\text{in which} \quad \mu = 2\left(\frac{1 - \sqrt{1 + \lambda_1\lambda_2 \sin^2 \nu}}{\lambda_1 \sin \nu}\right); \quad \rho = \frac{R}{a} \quad \text{(33)}$$

and D = a constant which can be evaluated by making use of the displacement boundary conditions at the cavity wall. Since $r = a(1 + \eta)$ at $R = a$, it is evident that $D = \eta(2 + \eta)$ (Fig. 3).

We shall now examine the constraints imposed on the constitutive assumption (Eq. 30) by virtue of the deformation induced in the infinite soil mass containing the cylindrical cavity:

1. We observe that in Eq. 30 $\Delta V/V_0$ is a volume increase, ϵ_{rr} is a compressive stress, $\epsilon_{\theta\theta}$ is a tensile strain, and λ_1 and λ_2 are considered to be positive constants. For the relationship (Eq. 30) to be consistent $\sin \nu > 0$, i.e.

$$\nu > 0 \quad \text{(34)}$$

2. Since the soil mass is of infinite extent it is evident that the displacements derived from Eq. 32 should reduce to zero as $R \to \infty$, i.e., as $R \to \infty$, $(r/R) \to 1$. This requirement will be satisfied by Eq. 32 if and only if $(\mu + 2) > 0$. For dilatant soils this condition is equivalent to the constraint

$$\left[1 - \frac{(\sqrt{1 + \lambda_1\lambda_2 \sin^2 \nu} - 1)}{\lambda_1 \sin \nu}\right] > 0 \quad \text{(35)}$$

The constraint (Eq. 35) is always satisfied when $\nu \to 0$ or $\lambda_2 = 0$. When λ_1 and λ_2 are nonzero, various combinations of λ_i and ν are admissible provided Eq. 35 is satisfied. [As an example, consider the specific case of a dilatant soil for which $\nu = 20°$. If $\lambda_1 = \lambda_2 = 0.1$, the right-hand side (R.H.S.) of the inequality (Eq. 35) gives a value of 0.983. Similarly if $\lambda_1 = \lambda_2 = 10$, the R.H.S. of the inequality (Eq. 35) gives a value of 0.250.]

3. For the deformation field (Eq. 32) to be physically admissible, det $\mathbf{F} \neq 0$. Using Eq. 3 and Eq. 32, this constraint can be rewritten in the form of a more stringent inequality:

$$\left[1 + \eta(2 + \eta)\left(\frac{\sqrt{1 + \lambda_1\lambda_2 \sin^2 \nu} - 1}{\lambda_1 \sin \nu}\right)\right] > 0 \quad \text{(36)}$$

Thus the large strain solution to the pressuremeter problem related to a dilatant soil is expected to be valid for cavity wall displacements which do not violate the inequality (Eq. 36). Since λ_i and ν are considered to be positive, Eq. 36 will be satisfied for all choices of $\eta > 0$.

Following the analysis presented in the previous sections, it may be assumed that the desired shear stress strain relationship for the dilatant soil can also be expressed in the form

$$\sigma_{rr} - \sigma_{\theta\theta} = \Phi^d(\Omega) \quad \text{(37)}$$

in which $$\Omega = \left(\frac{r}{R} - 1\right) = \left(1 + \frac{D}{\rho^{\mu+2}}\right)^{1/2} - 1 \quad \text{(38)}$$

By substituting the result (Eq. 37) in the equation of equilibrium, and performing the necessary change of variables and integrations, one obtains

$$\Psi(\eta) - \sigma_h = \int_0^{\eta} \frac{\Phi^d(\Omega)[2 - \mu\Omega(2 + \Omega)]}{\Omega(\mu + 2)(1 + \Omega)(2 + \Omega)} d\Omega \quad \text{(39)}$$

The explicit form for $\Phi^d(\eta)$ is obtained by a differentiation of Eq. 39, i.e.

$$\Phi^d(\eta) = \frac{\eta(1 + \eta)(2 + \eta)(\lambda_1 \sin\nu + 1 - \sqrt{1 + \lambda_1\lambda_2 \sin^2\nu})}{[\lambda_1 \sin\nu - (1 - \sqrt{1 + \lambda_1\lambda_2 \sin^2\nu})\,\eta(2 + \eta)]} \frac{d\Psi}{d\eta} \quad \text{(40)}$$

The preceding result represents the large strain shear stress-strain relationship for a dilatant soil which exhibits volume change characteristics defined by Eq. 30.

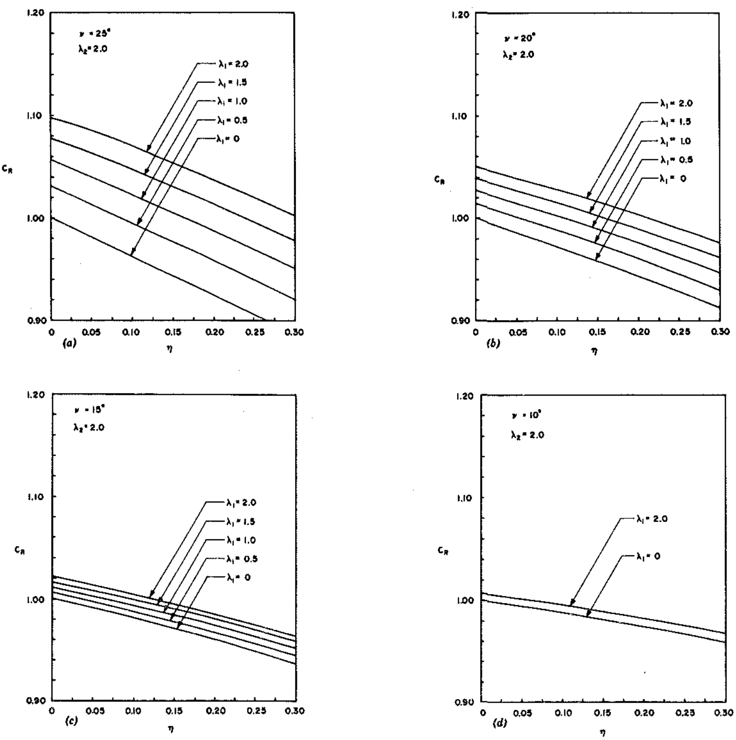


FIG. 4.—Correction Factor, C_R, for Pressuremeter Response—

Limiting Cases

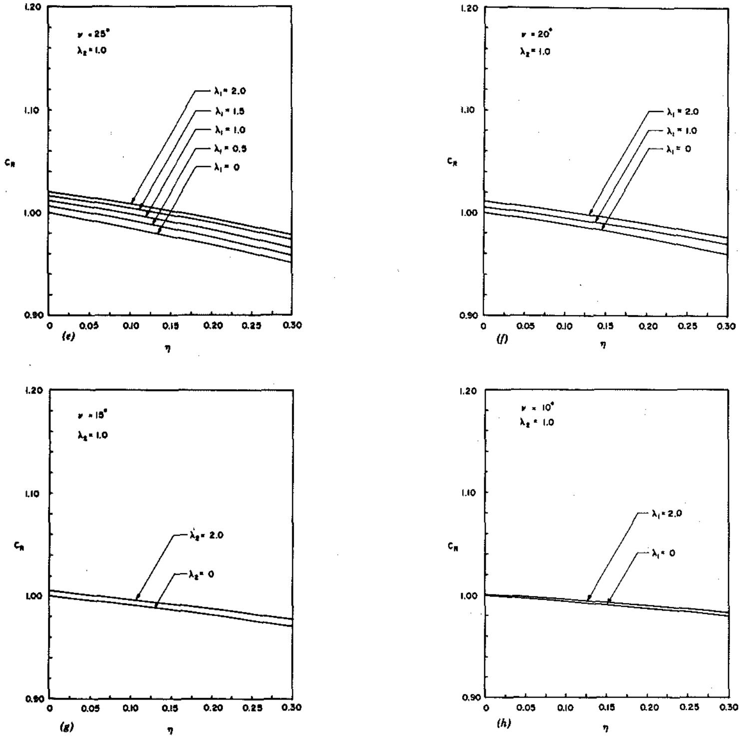
1. In the limiting case when the soil is incompressible, $\nu \rightarrow 0$ and Eq. 40 reduce to the result given by Palmer (16) (see, e.g., Eq. 29).
2. In the case when $\lambda_1 \rightarrow 0$, Eq. 40 gives

$$\Phi_s^d(\eta) = \frac{\eta(1+\eta)(4+2\eta-2\lambda_2\sin\nu-\eta\lambda_2\sin\nu)}{(2+2\eta\lambda_2\sin\nu+\eta^2\lambda_2\sin\nu)}\frac{d\Psi}{d\eta} \quad \text{.............. (41)}$$

The expression for $\Phi_s^d(\eta)$ given by Eq. 41 extends, to the large strain case, the results developed by Wroth and Windle (29) for the small strain response of a cavity expansion problem in which the kinematic constraint is a linearized or small strain equivalent of Eq. 30. The result developed by Wroth and Windle (29) can be rewritten in the form

$$\Phi_w^d(\eta) = \frac{\eta(1+\eta)(4+2\eta-2\lambda_2\sin\nu)}{(2+2\eta\lambda_2\sin\nu)}\frac{d\Psi}{d\eta} \quad \text{........................ (42)}$$

3. When $\lambda_1 = \lambda_2 = 1$, the expression for $\Phi^d(\eta)$ gives the large strain pressuremeter response that corresponds to a linearized volume change-shear strain response similar to that proposed by Hughes, et al. (8) for the small strain domain.



4. The individual stress components at the boundary of the cylindrical cavity are given by

$$\sigma_{rr}(\eta) = \Psi(\eta); \quad \sigma_{\theta\theta}(\eta) = \Psi(\eta) - \Phi^d(\eta) \quad (43)$$

Numerical Results

The purpose of this section is to examine the manner in which large strain and dilatancy effects can influence the shear stress-strain response of a soil as predicted from the result (Eq. 40). Usually in such an examination it is desirable to use actual data derived from pressuremeter tests. Such results are, however, unavailable to the writer. Furthermore, actual pressuremeter data rather than data recovered from published papers (e.g., Figs. 1–2) should be used in any mathematical computations. Under these circumstances it is appropriate to adopt a numerical illustration which is essentially independent of the measured pressuremeter response curve, $\Psi(\eta)$. We note that the work of Wroth and Windle (29) does take into account dilatant effects in the soil in the small strain range, particularly for the situation where the volumetric strain depends only on the circumferential strain, $\epsilon_{\theta\theta}$. In the current investigation the large strain effects of both ϵ_{rr} and $\epsilon_{\theta\theta}$ are incorporated. The relative influences of the additional variable ϵ_{rr} and large strain effects can be estimated by comparing Eq. 40 with the result developed by Wroth and Windle (29). From Eqs. 40 and 42 we obtain

$$\frac{\Phi^d(\eta)}{\Phi^d_w(\eta)} = \frac{(2+\eta)(1+\eta\lambda_2 \sin\nu)(\lambda_1 \sin\nu + 1 - \sqrt{1+\lambda_1\lambda_2 \sin^2\nu})}{(2+\eta-\lambda_2 \sin\nu)[\nu_1 \sin\nu + (\sqrt{1+\lambda_1\lambda_2\sin^2\nu} - 1)\,\eta(2+\eta)]} = C_R \quad (44)$$

The preceding nondimensional expression for correction factor C_R $[= C_R(\lambda_1, \lambda_2, \nu, \eta)]$ can be evaluated for specific choices of λ_i, ν, and η to assess the influences of ϵ_{rr} and large strain dilatancy effects on the pressuremeter response.

Figs. 4(*a*–*h*) shows the manner in which the parameters λ_i, ν, and η influence the large strain response in the pressuremeter test in which the volume change characteristics are defined by the kinematic constraint (Eq. 30). When $\lambda_1 = 0$, the kinematic constraint (Eq. 30) takes into account the influence of only $\epsilon_{\theta\theta}$. Therefore, the curves corresponding to $\lambda_1 = 0$ give estimates of only the influence of large strains on the pressuremeter response. The parameter $\lambda_2 \sin\nu$ can be identified with the constant, l, used in the small strain analysis carried out by Wroth and Windle (29). For example, when $\lambda_1 = 0$; $\lambda_2 = 2$; $\nu = 20°$ [or $\lambda_2 \sin \nu \simeq 0.68$; this can be compared with the value of $l = 0.50$ used by Wroth and Windle (29)], the value of C_R varies from 1.00 to 0.913 for η in the range 0 to 0.30. In general, the effects of large strains become dominant only with increasing dilatancy angle ν and λ_i. For values of $\nu < 10°$, the effects of large strains are relatively small. The incorporation of the variable ϵ_{rr} in the volume change characteristics leads to similar conclusions. In general, the correction factor, C_R, becomes appreciable only for large values of $\lambda_i \sin\nu$.

Conclusions

Experimental studies in pressuremeter testing indicate that large strain effects are of importance to the examination of these results. The influences of large strains can be examined quite conveniently in situations where cohesive soils exhibit incompressible or undrained behavior. This paper presents a theoretical assessment of the large strain response of a pressuremeter test conducted in a dilatant soil. It is shown that the mathematical analysis of the problem is facilitated by the introduction of a kinematic constraint which relates the finite volumetric strain to the components of the Lagrangian strain matrix in a linear fashion. This procedure essentially extends the techniques adopted by Wroth and Windle (29) and Hughes, et al. (8) to include effects of large strains. Despite these simplifying assumptions, it is difficult to determine all the constitutive functions which characterize the monotonic large strain response of a dilatant soil from the single response measured in a pressuremeter test. Some limited insight to the stress-strain characteristics of the tested soil can be gained by restricting attention to the derivation of a shear stress-strain curve similar to that derived from an undrained pressuremeter test. It is shown that this shear stress-strain curve for a dilatant soil can be derived from a knowledge of the pressuremeter curve and the linear form of the kinematic constraint for the soil. The numerical results presented in the paper examine the influence of large strain effects on the pressuremeter response in a dilatant soil. A comparison is made with the analytical estimates given by Wroth and Windle (29) for the pressuremeter response in a dilatant soil with linearized or small strain kinematic constraints. It is shown that the influences of large strains become appreciable only for soils which exhibit high ($\nu > 20°$) dilatancy angles or large λ_i (>2). When $\nu < 10°$, the results given by Wroth and Windle (29) can be used, quite accurately, to examine the pressuremeter response for a wide range of cavity displacements ($\eta < 0.30$). The analysis presented in this paper and that given by Wroth and Windle (29) assume that the dilatancy angle remains constant for the entire range of strains considered. This is essentially an approximation of the real response in which the dilatancy angle can decrease with increasing strain. Therefore, the analysis presented in this paper can only be visualized as an upper limit for the shear stress-strain response derived from a pressuremeter test conducted on a dilatant soil. The factor C_R can be applied as a correction factor to account for effects of large strains. The essential features of the method outlined here can be applied to examine other nonlinear forms of kinematic constraints. Such kinematic constraints should be derived from experiments which involve the measurement of finite volumetric strains and both major and minor principal strains.

Acknowledgments

The work described in this paper was supported by Natural Sciences and Engineering Research Council of Canada Grant A3866. The writer is indebted to the referees for their valuable comments which led to considerable improvements in the presentation of the theoretical developments.

Appendix I.—References

1. Baguelin, F., et al., "Expansion of Cylindrical Probes in Cohesive Soils," *Jour-*

nal of the Soil Mechanics and Foundations Division, ASCE, Vol. 98, 1972, pp. 1129–1142.
2. Baguelin, F., Jezequel, J.-F., and Shields, D. H., *The Pressuremeter and Foundation Engineering*, Trans. Tech. Publ., Clausthal, West Germany, 1978.
3. Banerjee, P. K., and Stipho, A. S., "Associated and Non-Associated Constitutive Relations for Undrained Behaviour of Isotropic Soft Clays," *International Journal for Numerical and Analytical Methods in Geomechanics*, Vol. 2, 1978, pp. 35–36.
4. Carter, J. P., Randolph, M. F., and Wroth, C. P., "Stress and Pore Pressure Changes in Clay During and After the Expansion of a Cylindrical Cavity," *International Journal for Numerical and Analytical Methods in Geomechanics*, Vol. 3, 1979, pp. 305–322.
5. Gibson, R. E., and Anderson, W. F., "In Situ Measurement of Soil Properties with the Pressuremeter," *Civil Engineering and Public Works Review*, Vol. 56, 1961, pp. 615–618.
6. Green, A. E., and Zerna, W., *Theoretical Elasticity*, Clarendon Press, Oxford, England, 1968.
7. Green, A. E., and Adkins, J. E., *Large Elastic Deformations and Nonlinear Continuum Mechanics*, Clarendon Press, Oxford, England, 1971.
8. Hughes, J. M. O., Wroth, C. P., and Windle, D., "Pressuremeter Tests in Sand," *Geotechnique*, Vol. 27, 1977, pp. 455–477.
9. Jaunzemis, W., *Continuum Mechanics*, Macmillan, New York, N.Y., 1967.
10. Jewell, R. J., Fahey, M., and Wroth, C. P., "Laboratory Studies of the Pressuremeter Test in Sand," *Geotechnique*, Vol. 30, 1980, pp. 507–531.
11. Ladanyi, B., "Evaluation of Pressuremeter Tests in Granular Soils," *Proceedings of the 2nd Pan American Conference on Soil Mechanics and Foundation Engineering*, Vol. 1, 1963, pp. 3–20.
12. Ladanyi, B., "Expansion of a Cavity in a Saturated Clay Medium," *Journal of the Soil Mechanics and Foundations Division*, ASCE, Vol. 89, 1963, pp. 127–161.
13. Ladanyi, B., "In Situ Determination of Undrained Stress-Strain Behaviour of Sensitive Clays with the Pressuremeter," *Canadian Geotechnical Journal*, Vol. 9, 1972, pp. 313–319.
14. Laier, J. E., Schmertmann, J. H., and Schaub, J. H., "Effect of Finite Pressuremeter Length on Dry Sand," *Proceedings of the ASCE Specialty Conference on In Situ Testing*, Raleigh, N.C., Vol. 1, 1975, pp. 241–259.
15. Livneh, M., Gellert, M., and Uzan, S., "Determination of the Elastic Modulus of Soil by the Pressuremeter Interpretation of the Pressuremeter Test—Theoretical Background," *Journal of Materials*, Vol. 6, 1971, pp. 348–355.
16. Palmer, A. C., "Undrained Plane Strain Expansion of a Cylindrical Cavity in Clay. A Simple Interpretation of the Pressuremeter Test," *Geotechnique*, Vol. 22, 1972, pp. 451–457.
17. Palmer, A. C., and Mitchell, R. J., "Plane Strain Expansion of a Cylindrical Cavity in Clay," *Stress-Strain Behaviour of Soils*, R. H. G. Parry, ed., Foulis, Henley-on-Thames, 1972, pp. 588–599.
18. Prevost, J.-H., and Hoeg, K., "Analysis of Pressuremeter in Strain-Softening Soil," *Journal of the Geotechnical Engineering Division*, ASCE, Vol. 101, 1975, pp. 717–732.
19. Rivlin, R. S., "Large Elastic Deformation of Isotropic Materials VI.—Further Results in the Theory of Torsion, Shear and Flexure," *Philosophical Transactions of the Royal Society*, Series A, Vol. 242, 1949, pp. 173–195.
20. Rivlin, R. S., and Ericksen, J. L., "Stress-Deformation Relations for Isotropic Materials," *Journal for Rational Mechanics and Analysis*, Vol. 4, 1955, pp. 323–425.
21. Rivlin, R. S., "Some Topics in Finite Elasticity," *Structural Mechanics, Proceedings of the 2nd Symposium on Naval Structural Mechanics*, J. N. Goodier and N. J. Hoff, eds., Pergamon Press, Oxford, England, 1960, pp. 169–189.
22. Selvadurai, A. P. S., and Spencer, A. J. M., "Second-Order Elasticity with

Axial Symmetry; I.—General Theory," *International Journal of Engineering Science*, Vol. 10, 1972, pp. 97–114.
23. Smith, I. M., "Incremental Numerical Analysis of a Simple Deformation Problem in Soil Mechanics," *Geotechnique*, Vol. 20, 1970, pp. 357–372.
24. Smith, I. M., and Kay, S., "Stress Analysis of Contractive or Dilative Soil," *Journal of the Soil Mechanics and Foundations Division*, ASCE, Vol. 97, 1971, pp. 981–997.
25. Spencer, A. J. M., "The Static Theory of Finite Elasticity," *Journal of the Institute of Mathematics and Its Applications*, Vol. 6, 1970, pp. 164–200.
26. Truesdell, C., and Noll, W., "The Non-linear Field Theories of Mechanics," *Encyclopaedia of Physics*, Vol. 3/3, Springer-Verlag, Berlin, 1965.
27. Vesic, A. S., "Expansion of Cavities in Infinite Soil Mass," *Journal of the Soil Mechanics and Foundations Division*, ASCE, Vol. 98, 1972, pp. 265–290.
28. Wroth, C. P., "In Situ Measurement of Initial Stresses and Deformation Characteristics," *Proceedings of the ASCE Specialty Conference on In Situ Measurement of Soil Properties*, Raleigh, N.C., Vol. 2, 1975, pp. 181–231.
29. Wroth, C. P., and Windle, D., "Analysis of the Pressuremeter Test Allowing for Volume Change," *Geotechnique*, Vol. 25, 1975, pp. 598–604.

Appendix II.—Notation

The following symbols are used in this paper:

a = radius of cylindrical cavity;
$\mathbf{B}$ = Cauchy-Green strain matrix;
$\mathbf{B}^{-1}$ = inverse of $\mathbf{B}$;
C_R = correction factor for large strain and dilatancy effects;
$\mathbf{F}$ = deformation gradients;
$F(\mathbf{B})$ = constitutive functional;
I = invariant;
$\mathbf{I}$ = identity matrix;
I_1, I_2, I_3 = invariants of $\mathbf{B}$;
$\underset{\sim}{n}$ = direction cosines of unit normal to surface in deformed configuration;
p = scalar pressure;
R, Θ, Z = cylindrical polar coordinates of particle in reference configuration;
r, θ, z = cylindrical polar coordinates of particle in deformed configuration;
$\mathbf{T}$ = traction vector in deformed configuration;
V = volume of element in deformed configuration;
V_0 = volume of element in undeformed configuration;
X_1, X_2, X_3 = rectangular Cartesian coordinates of particle in reference configuration;
x_1, x_2, x_3 = rectangular Cartesian coordinates of particle in deformed configuration;
$\Gamma(I)$ = constitutive function;
ΔV = dilatant volume change;
$\boldsymbol{\epsilon}$ = Lagrangian strain matrix;
ηa = displacement at boundary of cylindrical cavity;
λ_1, λ_2 = positive constants characterizing kinematic constraints;
ν = dilatancy angle;
ρ = R/a: nondimensional radial coordinate;
$\boldsymbol{\sigma}$ = Cauchy stress matrix in deformed configuration;

σ_h	=	in situ horizontal stress state;
$\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}$	=	components of $\boldsymbol{\sigma}$ refer to cylindrical polar coordinate system;
Φ^{i*}	=	constitutive function;
$\Phi^{i}(\eta)$	=	shear stress-strain response for incompressible soil;
$\Phi^{d}(\eta)$	=	shear stress-strain response for dilatant soil;
$\Phi^{d}_{s}, \Phi^{d}_{w}$	=	shear stress-strain responses;
$\chi_0, \chi_1, \chi_{-1}$	=	scalar functions which depend on I_1, I_2, and I_3;
$\Psi(\eta)$	=	pressure applied at boundary of cylindrical cavity; and
Ω, μ	=	substitution parameters.