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FUNDAMENTAL RESULTS CONCERNING THE SETTLEMENT OF A RIGID FOUNDATION ON AN ELASTIC MEDIUM DUE TO AN ADJACENT SURFACE LOAD

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SUMMARY

This paper examines the interaction between a rigid circular foundation resting in smooth contact with an isotropic elastic halfspace and a concentrated surface load which acts at a finite distance from the foundation. Owing to the action of the external load the rigid foundation experiences an extra settlement and a tilt. The expressions for the extra settlement and the tilt are evaluated in exact closed form. It is also shown that these deformations due to the external load satisfy Betti's reciprocal theorem. The auxiliary solution required for the application of the reciprocal theorem is derived from the analysis of the problem of a rigid circular foundation resting in smooth contact with an elastic medium and subjected to an eccentric load. The results developed for the interaction between the rigid circular foundation and the external concentrated load are utilized to generate, among others, solutions for the settlement and tilt induced at a rigid foundation due to its interaction with uniformly or non-uniformly distributed loads with circular and square plan shapes.

INTRODUCTION

Much of the geotechnical literature on soil-foundation interaction is concerned with the examination of the performance of isolated foundations.[1–5] Classical theoretical solutions in this area, such as the interaction between a rigid circular foundation and a soil medium which exhibits elastic,[5–7] ideally plastic[8,9] or viscoelastic[10] characteristics, explicitly neglect the effect of neighbouring foundations or the influence of additional exterior surface loads. In many situations in geotechnical engineering practice it becomes important to assess the behaviour of a foundation due to its interaction with another structural foundation or some form of external loading. The interaction between individual piles or anchors which constitute the group is a very obvious example of this type of problem (see e.g. Reference 11). The analysis of interaction between foundations and internally located anchor loads has also received some recent attention.[12–15] The interference between closely spaced footings, silo foundations, rafts, etc., constitutes a similar class of problem. The mutual interaction between a structural foundation and an external load is of some importance to the assessment of settlement of existing foundations due to surcharge loads applied in its vicinity.

This paper examines the problem of the interaction between a loaded rigid circular foundation resting in smooth contact with an isotropic elastic halfspace and a concentrated force which acts at an exterior point (Figure 1). The analysis of the problem can be approached by

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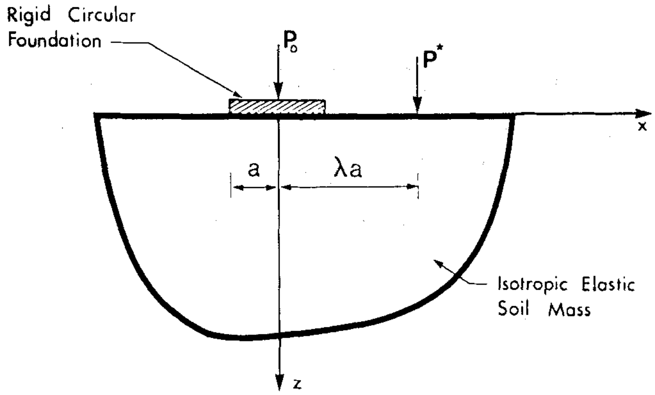

Figure 1. Geometry of the interaction problem

employing an integral transform formulation of the governing equations. The asymmetric boundary value problem yields a system of dual integral equations which can be solved by making use of standard procedures.[16,17] It is found that (i) the displacements of the rigid circular foundation due to the externally applied load, and (ii) the external displacements of the surface of the halfspace due to an eccentrically loaded foundation, constitute two force-displacement fields which satisfy the reciprocal theory of Betti.[18] The applicability of such a reciprocal theorem to this class of interaction problems extends the use of classical results derived for directly loaded foundations[2,5] to the analysis of mutual interaction.

The exact closed form results derived for the settlements of the rigid circular foundation due to the action of an external concentrated force are used to generate solutions for other forms of distributed external loadings. Specific numerical results are given for the central displacements and rotations induced in the rigid circular foundation due to action of external loads with square and circular plan shapes.

ANALYSIS

The category of problems which deals with the analysis of settlement of foundations resting on an elastic soil mass has received considerable attention. Complete accounts of these investigations are given by Galin,[19] Uflyand,[20] Poulos and Davis,[2] de Pater and Kalker,[21] Selvadurai[5] and Gladwell.[22] Basic investigations pertaining to the directly loaded foundation examine the settlement of rigid foundations with circular and rectangular plan shapes which exhibit varying degrees of interface phenomena and which are subjected to symmetric and non-symmetric external loads. The soil medium is usually approximated as a linear elastic halfspace or a layer of finite depth which may possess non-homogeneous or anisotropic material properties. Classical elasticity, despite its limitations, serves as a useful first approximation for the representation of the time independent mechanical behaviour of cohesive soils at working stress levels. Solutions derived from such analyses have found useful application in geotechnical engineering. This paper first examines the problem of the interaction between a loaded rigid circular foundation resting in smooth contact with an elastic halfspace and a concentrated normal surface load (P^*) which acts in the vicinity of the foundation. In the ensuing treatment it is assumed that the load P_0 applied at the centre of the foundation is sufficiently large to prevent the development of tensile contact stresses at the smooth soil–

foundation interface. When this condition is satisfied the contact region at the soil–foundation interface always corresponds to a plane surface. The analysis is thus restricted to the examination of the additional settlements induced in the rigid circular foundation due to the adjacent load P^*. It is shown that the foundation displacements due to P^* can be evaluated in exact closed form.

Surface settlements due to the direct loading of the rigid circular foundation

Before examining the solution to the title problem, it is convenient to record here certain results related to the settlement of a rigid circular foundation subjected to an eccentric load (Figure 2). Consider the problem of a rigid circular foundation resting in smooth contact with

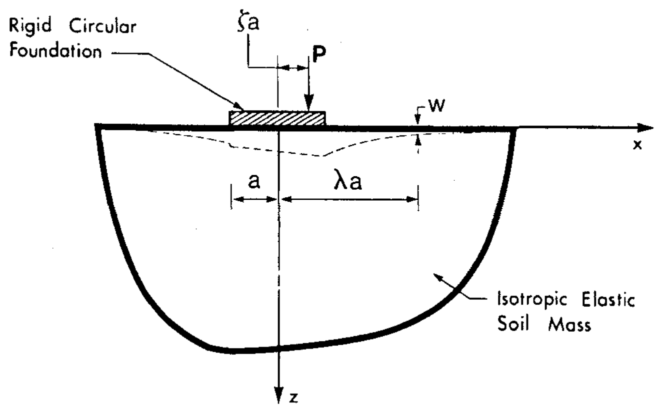

Figure 2. Eccentric loading of the rigid circular foundation

an isotropic elastic halfspace. The foundation (of radius a) is subjected to an eccentric load P at the location $r=\zeta a$, $\theta=0$. The solution to this problem can be obtained by combining the separate solutions developed for the rigid circular foundation subjected to a central force,[6,7] and a central moment about a horizontal axis.[23,24] Briefly, the analysis of the problem can be reduced to the solution of two separate sets of dual integral equations of the type

$$\begin{aligned} &\int_0^\infty A_0(\xi)J_0(\xi r/a)\,\mathrm{d}\xi = w_0; \qquad 0\le r\le a \\ &\int_0^\infty \xi A_0(\xi)J_0(\xi r/a)\,\mathrm{d}\xi = 0; \qquad a<r<\infty \end{aligned} \tag{1}$$

and

$$\begin{aligned} &\int_0^\infty A_1(\xi)J_1(\xi r/a)\,\mathrm{d}\xi = \varphi_0 r; \qquad 0\le r\le a \\ &\int_0^\infty \xi A_1(\xi)J_1(\xi r/a)\,\mathrm{d}\xi = 0; \qquad a<r<\infty \end{aligned} \tag{2}$$

for the unknown functions $A_i(\xi)(i=0,1)$. In (1) and (2) w_0 and φ_0 are the rigid settlements and rotation of the rigid circular foundation. The solution of the dual systems (1) and (2) can be readily obtained from the generalized results given by Sneddon.[16,25] The result of particular interest to this paper concerns the vertical surface settlement $u_z(r, \theta, 0)$ of the isotropic elastic soil medium due to the eccentrically loaded function. The displacement within the foundation region is given by

$$u_z(r, \theta, 0) = \frac{P(1-\nu^2)}{2Ea}\left[1 + \frac{3\zeta\rho}{2}\cos\theta\right]; \qquad 0 \leq \rho \leq 1 \tag{3}$$

Similarly in the exterior to the circular foundation

$$u_z(r, \theta, 0) = \frac{P(1-\nu^2)}{2Ea}\left[\frac{2}{\pi}\sin^{-1}\left(\frac{1}{\rho}\right) + \frac{3\zeta\rho}{2}\cos\theta\left\{1 - \frac{2}{\pi}\tan^{-1}\sqrt{(\rho^2-1)} - \frac{2}{\pi}\frac{\sqrt{(\rho^2-1)}}{\rho^2}\right\}\right]; \qquad 1 \leq \rho \leq \infty \tag{4}$$

Interaction between the rigid circular foundation and an external force

Attention is now directed to the problem of the interaction between the rigid circular foundation resting in smooth contact with an isotropic elastic halfspace and an external load P^* acting normal to the surface at the location $(\lambda a, 0, 0)$. Furthermore, it is assumed that the rigid circular foundation is subjected to a central force P_0 sufficiently large to prevent the development of tensile contact stresses at the smooth interface for all choices of λ and P^*. The settlements associated with P_0 can be obtained from (3) and (4) and attention is restricted to the additional settlements induced at the circular foundation region due to P^*. To examine this problem, the rigid foundation is further subjected to a central force $\bar{P}$ and a moment $\bar{M}$ about the y-axis. The magnitudes of $\bar{P}$ and $\bar{M}$ are such that the circular foundation experiences zero vertical displacement in the z-direction (Figure 3). The contact region is now subjected to the boundary conditions

$$u_z(r, \theta, 0) = 0; \qquad 0 \leq \theta \leq 2\pi; \qquad 0 \leq r \leq a \tag{5}$$

$$\sigma_{zz}(r, \theta, 0) = -p^*(r, \theta); \qquad 0 \leq \theta \leq 2\pi; \qquad a < r < \infty \tag{6}$$

$$\sigma_{rz}(r, \theta, 0) = 0; \qquad 0 \leq \theta \leq 2\pi; \qquad 0 \leq r < \infty \tag{7}$$

where $p^*(r, \theta)$ is an even function of θ. Following the Hankel transform development of the equations of elastic equilibrium[26] it can be shown that when the boundary condition (7) relating

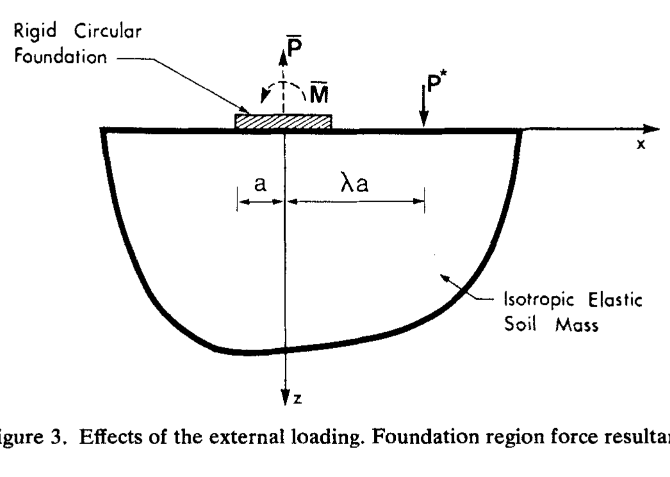

Figure 3. Effects of the external loading. Foundation region force resultants

to the shear stress is satisfied, the appropriate expressions for u_z and σ_{zz} reduce to the following on the plane $z=0$:

$$u_z(r,\theta,0)=2(1-\nu)\sum_{m=0}^{\infty}\left\{\int_0^{\infty}\Phi_m(\xi)J_m(\xi r/a)\,\mathrm{d}\xi\right\}\cos m\theta \tag{8}$$

$$\sigma_{zz}(r,\theta,0)=\frac{-E}{(1+\nu)}\sum_{m=0}^{\infty}\left\{\int_0^{\infty}\xi\Phi_m(\xi)J_m(\xi r/a)\,\mathrm{d}\xi\right\}\cos m\theta \tag{9}$$

In (8) and (9) $\Phi_m(\xi)$ are unknown functions which should be determined by satisfying the mixed boundary conditions (5) and (6). Assuming that the external loading $p^*(r,\theta)$ can be expressed in the form

$$p^*(r,\theta)=-\frac{E}{(1+\nu)}\sum_{m=0}^{\infty}g_m(r)\cos m\theta \tag{10}$$

the boundary conditions (5) and (6) can be reduced to the systems of dual integral equations

$$\int_0^{\infty}\Phi_m(\xi)J_m(\xi r/a)\,\mathrm{d}\xi=0;\qquad 0\leq r\leq a \tag{11}$$

$$\int_0^{\infty}\xi\Phi_m(\xi)J_m(\xi r/a)\,\mathrm{d}\xi=g_m(r);\qquad a<r<\infty \tag{12}$$

The analysis of the system of dual integral equations (11) and (12) has been examined by several authors including Noble,[27] Sneddon and Lowengrub[17] and others and the details of the method will not be pursued here. It is sufficient to note that for the concentrated loading

$$\begin{Bmatrix} g_0(r) \\ g_m(r) \end{Bmatrix}=\frac{P^*(1+\nu)a\Delta(r-l)}{4\pi^2Er}\begin{Bmatrix} 1 \\ 2 \end{Bmatrix} \tag{13}$$

where $\Delta(r-l)$ is the Dirac delta function. The solution of the dual systems determines $\Phi_m(\xi)$. The contact stress distribution induced beneath the rigid circular foundation subject to the boundary conditions (5)–(7) is given by

$$\sigma_{zz}(r,\theta,0)=\frac{P^*\sqrt{(\lambda^2-1)}}{\pi^2a^2\sqrt{(1-\rho^2)}(\lambda^2+\rho^2-2\lambda\rho\cos\theta)} \tag{14}$$

where $\rho=r/a$, $\lambda=l/a$. Also the displacement and stress fields corresponding to the problem posed above reduce to zero as $r, z\to\infty$. It is of interest to note that this distribution of contact stress beneath the foundation (equation (14)) has a form identical (except for a multiplicative constant) to that of the distribution of charge density on a thin conducting disc of radius a due to a point charge located at a distance λa (see e.g. Reference 28). This observation extends the long established analogy between the free distribution of charge on a thin conducting disc and the distribution of contact stress beneath a centrally loaded rigid circular foundation indenting an isotropic (or transversely isotropic) homogeneous elastic halfspace.[25,29]

The force and moment resultants ($\bar{P}$ and $\bar{M}$) required to maintain zero axial displacement in the foundation region are given by

$$\bar{P}=2\int_0^a\int_0^{\pi}\sigma_{zz}(r,\theta,0)r\,\mathrm{d}r\,\mathrm{d}\theta=\frac{2P^*}{\pi}\sin^{-1}\left(\frac{1}{\lambda}\right) \tag{15}$$

$$\bar{M}=2\int_0^a\int_0^{\pi}\sigma_{zz}(r,\theta,0)r^2\cos\theta\,\mathrm{d}r\,\mathrm{d}\theta=P^*l\left[1-\frac{2}{\pi}\tan^{-1}\sqrt{(\lambda^2-1)}-\frac{2}{\pi}\frac{\sqrt{(\lambda^2-1)}}{\lambda^2}\right] \tag{16}$$

As indicated previously, the rigid circular foundation is only subjected to the external central force P_0. Therefore the foundation should be subjected to further force and moment resultants, equal in magnitude but opposite in sense to $\bar{P}$ and $\bar{M}$ to achieve this requirement. The application of such force resultants will induce settlements in both the foundation and exterior regions. By using the relationships (3), (4), (15) and (16) it can be shown that the settlement of the rigid circular foundation at an arbitrary location $(\rho a, \theta, 0)$ (within the foundation area) is given by

$$u_z(r,\theta,0)=\frac{P^*(1-\nu^2)}{2aE}\left[\frac{2}{\pi}\sin^{-1}\left(\frac{1}{\lambda}\right)+\frac{3\lambda\rho\cos\theta}{2}\left\{1-\frac{2}{\pi}\tan^{-1}\sqrt{(\lambda^2-1)}-\frac{2}{\pi}\frac{\sqrt{(\lambda^2-1)}}{\lambda^2}\right\}\right]$$

$$0\leq\rho\leq 1 \qquad (17)$$

Reciprocal relationships and the effects of distributed loadings

Consider the displacement w^* of the rigid circular foundation at the location $(\zeta a, 0, 0)$ due to the externally placed load P^* applied at $(\lambda a, 0, 0)$ (Figure 4). The result (17) gives

$$w^*=\frac{P^*(1-\nu^2)}{2aE}\left[\frac{2}{\pi}\sin^{-1}\left(\frac{1}{\lambda}\right)+\frac{3\lambda\zeta}{2}\left\{1-\frac{2}{\pi}\tan^{-1}\sqrt{(\lambda^2-1)}-\frac{2}{\pi}\frac{\sqrt{(\lambda^2-1)}}{\lambda^2}\right\}\right] \qquad (18)$$

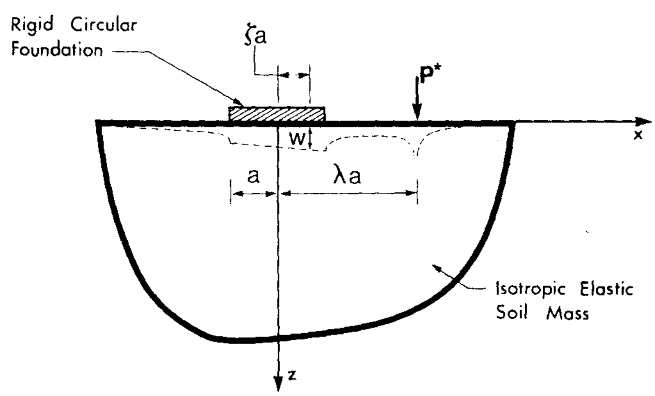

Figure 4. Displacements of the foundation due to externally placed load

Similarly, the surface displacement of the elastic halfspace at an exterior point $(\lambda a, 0, 0)$ due to the internal loading of the rigid circular foundation by a concentrated force P applied at the location $(\zeta a, 0, 0)$ is denoted by w (Figure 2). The value of w can be obtained from the result (4). By comparing this result with (18) it is evident that the two states satisfy Betti's reciprocal relation

$$Pw^*=P^*w \qquad (19)$$

The applicability of Betti's reciprocal relationship to the study of the interaction between the rigid foundation and the elastic halfspace can be used to considerable advantage in the analysis of foundation settlements induced by externally placed loads. The auxiliary solution (i.e. the solution to the problem related to a directly loaded foundation) can be examined, relatively conveniently, by using several analytical procedures such as dual integral equation formulations,[25] complex potential function techniques[30] and numerical procedures such as finite element, finite difference and other discretization procedures.[2,5] The reciprocal relationship proposed above can be generalized to include arbitrary locations for the point of application of the loads and for the position at which the displacements are evaluated. This amounts to a straightforward change in the frame of reference. Consider a rigid circular foundation in smooth contact with an isotropic elastic halfspace and subjected to an internal concentrated force P at the location $A(\rho a, \theta, 0)$. The displacement at the location $B(\lambda a, \varphi, 0)$ is denoted by $\bar{w}$ (Figure 5). Similarly, the displacement of the rigid circular foundation at $A^*(\rho a, \theta, 0)$

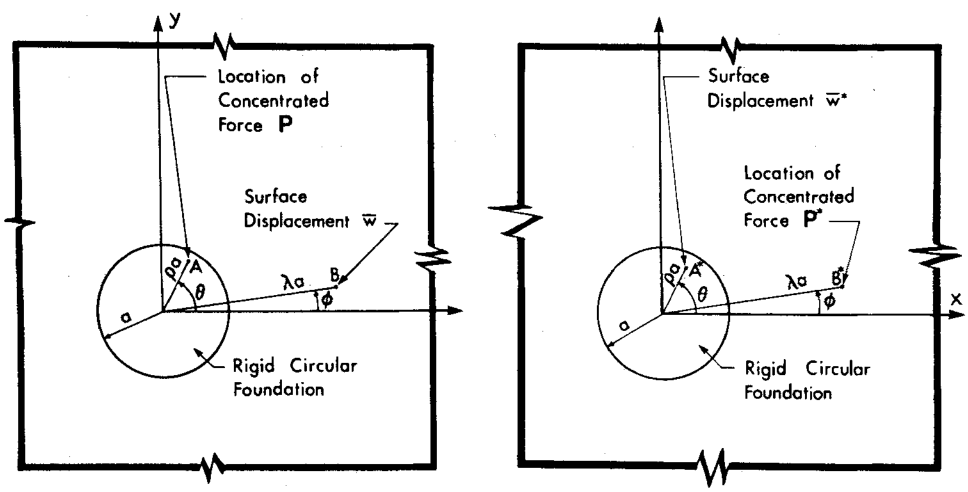

Figure 5. Reciprocal states for the loading of the rigid circular foundation—plan view

due to the external load P^* applied at $B(\lambda a, \varphi, 0)$ is denoted by $\bar{w}^*$. Betti's reciprocal relationship yields

$$\frac{\bar{w}}{P}=\frac{\bar{w}^*}{P^*}=\frac{(1-\nu^2)}{2aE}\left[\frac{2}{\pi}\sin^{-1}\left(\frac{1}{\lambda}\right)+\frac{3}{2}\lambda\rho\cos(\theta-\varphi)\left\{1-\frac{2}{\pi}\tan^{-1}\sqrt{(\lambda^2-1)}-\frac{2}{\pi}\frac{\sqrt{(\lambda^2-1)}}{\lambda^2}\right\}\right] \tag{20}$$

Interaction between the rigid circular foundation and distributed external loads

The result (20) provides the generalized influence function which may be used to compute the additional settlement of the rigid circular foundation induced by a distributed external load. There are, of course, a large number of specific forms of distributed loadings which are of interest to geotechnical engineering. In this section three specific forms of distributed external loading will be considered. The central displacement and tilt experienced by the rigid circular foundation due to the action of the distributed loadings are evaluated.

Uniform circular load

Consider the problem of a uniform circular load of radius βa and uniform stress intensity $q_e(=Q_e/\pi\beta^2 a^2$: where Q_e is the total load) located at a distance sa (between centres) from the rigid circular foundation (Figure 6). The central displacement (w_{0c}) of the rigid circular

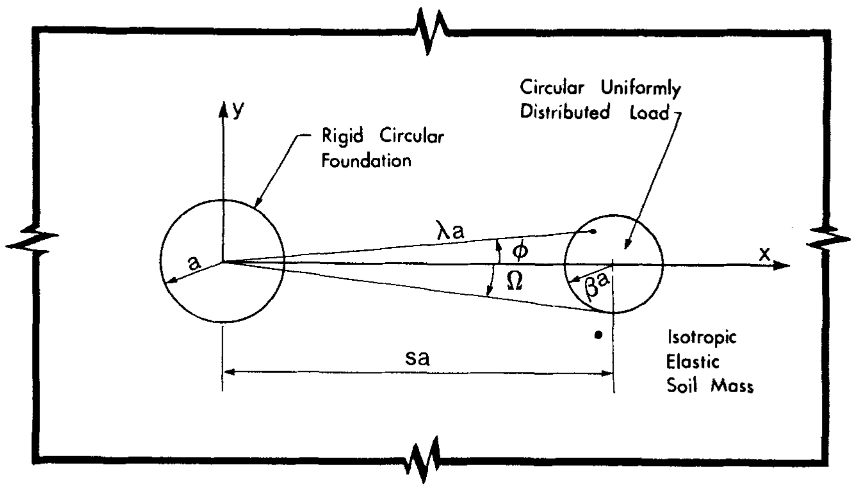

Figure 6. Interaction between the rigid circular foundation and the circular, uniformly distributed external load

foundation due to the external distributed load can be obtained in the integral form

$$\frac{w_{0c}}{Q_e(1-\nu^2)/\pi Ea}=I_{0c}(s,\beta) \tag{21}$$

where

$$I_{0c}(s,\beta)=\frac{1}{\beta^2}\int_0^{\Omega}\int_{\lambda_1(\varphi)}^{\lambda_2(\varphi)}\left\{\frac{2}{\pi}\lambda\sin^{-1}\left(\frac{1}{\lambda}\right)\right\}d\lambda\, d\varphi \tag{22}$$

and

$$\Omega=\sin^{-1}\left(\frac{\beta}{s}\right)$$
$$\lambda_1(\varphi)=s\cos\varphi-\sqrt{(\beta^2-s^2\sin\varphi)} \tag{23}$$
$$\lambda_2(\varphi)=s\cos\varphi+\sqrt{(\beta^2-s^2\sin\varphi)}$$

The rotation of the rigid circular foundation due to the external distributed load is defined by $\tan\chi_c=(w_{ac}-w_{0c})/a$, (where w_{ac} is the settlement at the periphery of the foundation). The appropriate integral expression for the evaluation of $\tan\chi_c$ can be reduced to the form

$$\frac{\tan\chi_c}{3Q_e(1-\nu^2)/2\pi Ea^2}=R_c(s,\beta) \tag{24}$$

where

$$R_c(s,\beta)=\frac{1}{\beta^2}\int_0^{\Omega}\int_{\lambda_1(\varphi)}^{\lambda_2(\varphi)}\lambda^2\cos\varphi\left\{1-\frac{2}{\pi}\tan^{-1}\sqrt{(\lambda^2-1)}-\frac{2}{\pi}\frac{\sqrt{(\lambda^2-1)}}{\lambda^2}\right\}d\lambda\, d\varphi \tag{25}$$

Non-uniform circular load

The type of non-uniformity considered resembles the singular contact stress distribution which occurs beneath a rigid circular foundation resting on an isotropic elastic halfspace (i.e.

a stress distribution of the type $\sigma_{zz}(r, 0) = P_0/2\pi a\sqrt{a^2 - r^2}$. This, of course, does *not* mean that the exterior circular loading acts as a rigid circular foundation; none the less it is useful in illustrating the effect of non-uniformity of the external loading on the settlements of the rigid circular foundation. The central settlement at the foundation is given by

$$\frac{w_{0n}}{Q_e(1-\nu^2)/\pi Ea} = I_{0n}(s, \beta) \tag{26}$$

where

$$I_{0n}(s, \beta) = \frac{1}{\pi\beta^2} \int_0^{\Omega} \int_{\lambda_1(\varphi)}^{\lambda_2(\varphi)} \frac{\lambda \sin^{-1}(1/\lambda)\, d\lambda\, d\varphi}{[\beta^2 - \{\lambda^2 + s^2 - 2\lambda s \cos\varphi\}]^{1/2}} \tag{27}$$

The rotation of the rigid circular foundation due to the non-uniform external load is given by

$$\frac{\tan \chi_n}{3Q_e(1-\nu^2)/2\pi Ea^2} = R_n(s, \beta) \tag{28}$$

where

$$R_n(s, \beta) = \frac{1}{2\pi\beta} \int_0^{\Omega} \int_{\lambda_1(\varphi)}^{\lambda_2(\varphi)} \frac{\{\pi\lambda^2 - 2\lambda^2 \tan^{-1}\sqrt{(\lambda^2-1)} - 2\sqrt{(\lambda^2-1)}\}}{\{\beta^2 - \lambda^2 - s^2 + 2s\lambda \cos\varphi\}^{1/2}} \cos\varphi\, d\lambda\, d\varphi \tag{29}$$

Uniform square load

In order to evaluate the central displacement of the rigid circular foundation due to an external uniformly distributed load with a square plan shape (Figure 7), it is convenient to employ a Cartesian co-ordinate formulation. The integral expression for the central settlement (w_{0s}) of the rigid circular foundation due to a square distributed load (total load Q_e; and side dimension δa, placed symmetrically about the x-axis) reduces to

$$\frac{w_{0s}}{Q_e(1-\nu^2)/\pi Ea} = I_s(s, \beta) \tag{30}$$

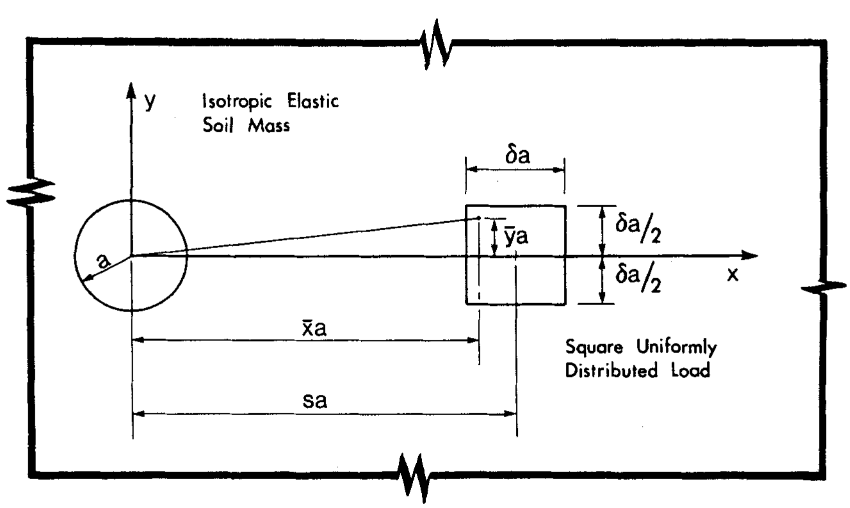

Figure 7. Interaction between the rigid circular foundation and the square, uniformly distributed external load

where

$$I_s(s,\beta)=\frac{2}{\delta^2}\int_{s-\delta/2}^{a+\delta/2}\int_0^{\delta/s}\sin^{-1}\left\{\frac{1}{\sqrt{(\bar{x}^2+\bar{y}^2)}}\right\}\,d\bar{x}\,d\bar{y} \tag{31}$$

Also, the rotation of the rigid circular foundation due to the externally placed uniform square load is given by

$$\frac{\tan\chi_s}{3Q_e(1-\nu^2)/2\pi Ea^2}=R_s(s,\beta) \tag{32}$$

where

$$R_s(s,\beta)=\frac{\pi}{\delta^2}\int_{s-\delta/2}^{s+\delta/2}\int_0^{\delta/2}\left\{1-\frac{2}{\pi}\tan^{-1}\sqrt{(\bar{x}^2+\bar{y}^2-1)}-\frac{2}{\pi}\frac{\sqrt{(\bar{x}^2+\bar{y}^2-1)}}{(\bar{x}^2+\bar{y}^2)}\right\}\,d\bar{x}\,d\bar{y} \tag{33}$$

Finite line loads

The additional settlement at the centre of the rigid circular foundation due to an aligned finite line load (of total load T and length ca where $ca<\infty$; see e.g. Figure 8(a)) is given by

$$w_0^{LR}=\frac{T(1-\nu^2)}{\pi aE}\left[\left(\frac{s+c}{c}\right)\sin^{-1}\left(\frac{1}{s+c}\right)-\frac{s}{c}\sin^{-1}\left(\frac{1}{s}\right)+\frac{1}{c}\ln\left\{\left|\frac{(s+c)+\sqrt{[(s+c)^2-1]}}{s+\sqrt{(s^2-1)}}\right|\right\}\right] \tag{34}$$

Similarly when the finite line load acts perpendicular to the x-axis (of total load T and length $2ta<\infty$; see e.g. Figure 8(b)) the additional central displacement of the rigid circular foundation can be evaluated in the following exact closed form:

$$w_0^{LT}=\frac{T(1-\nu^2)}{\pi aE}\left[\sin^{-1}\left(\frac{1}{\sqrt{(t^2+s^2)}}\right)+\frac{1}{2t}\ln\left\{\frac{\sqrt{(t^2+s^2-1)}+t}{\sqrt{(t^2+s^2-1)}-t}\right\}+\frac{s}{2t}\ln\left\{\left|\frac{s^2\sqrt{(t^2+s^2-1)}+t}{s^2\sqrt{(t^2+s^2-1)}-t}\right|\right\}\right] \tag{35}$$

It may be verified that as $c, t\to 0$, (34) and (35) reduce to the result obtained for the additional settlement induced at the centre of the rigid circular foundation due to a concentrated force. The result for the rotation experienced by the rigid circular foundation due to the external line loads can be evaluated in integral form by employing the procedures outlined earlier.

Concentrated moment

The solution (17) developed for the additional settlement of the rigid circular foundation due to an externally placed concentrated force can also be utilized to generate, for example, the solution for the settlement of the rigid foundation due to an externally placed concentrated couple. Consider the action of a concentrated couple of magnitude M, at the location $(\lambda a, 0, 0)$ in the anti-clockwise sense (Figure 9). The additional settlement induced at the centre of the rigid circular foundation due to the concentrated couple is given by

$$w_0^{M}=\frac{M(1-\nu^2)}{\pi a^2E}\,\frac{1}{\lambda\sqrt{(\lambda^2-1)}} \tag{36}$$

Again, an expression (similar to (24) or (28)) can be derived for the rotation experienced by the rigid circular foundation due to the action of the externally placed couple.

The general technique outlined here can be readily extended to cover multiple external loading regions of arbitrary shape and varying stress intensity. The integral expressions (22),

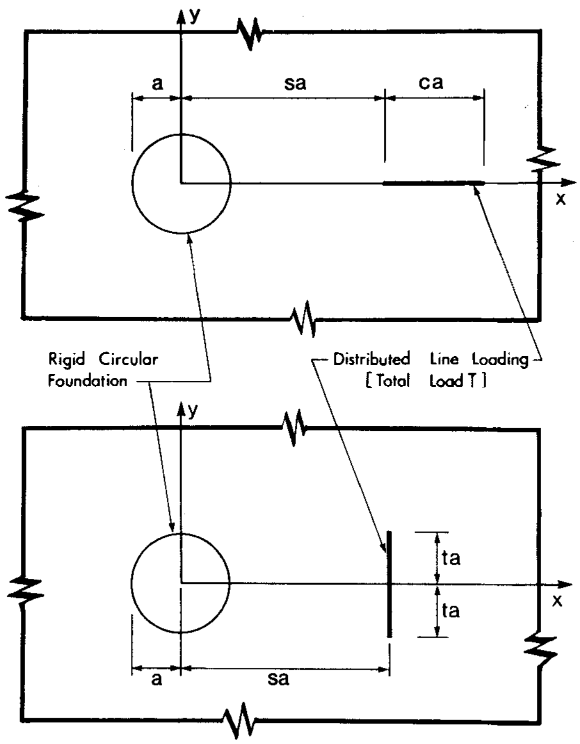

Figure 8. Interaction between the rigid circular foundation and line loadings

(25), (27) etc., which represent the influence factors for the central settlement and rotation of the rigid circular foundation induced by the various forms of distributed loading, do not appear to reduce to any simple closed forms. These integrals can, however, be evaluated by employing a numerical scheme based on Gaussian quadrature. The mathematical solutions for the non-dimensional central displacements and rotations (induced by the distributed external loadings) are presented in a manner which enables the examination of the following

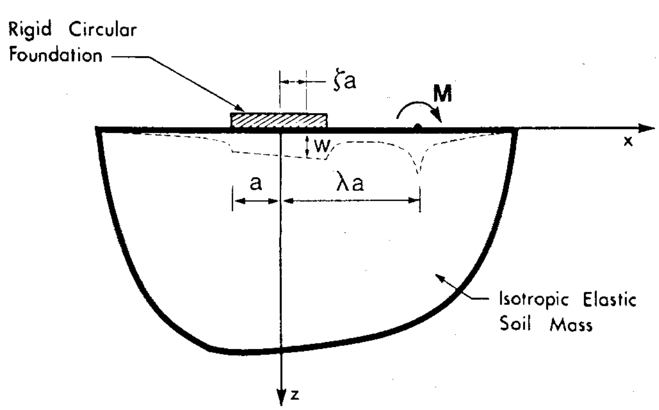

Figure 9. Displacements of the foundation due to external couple

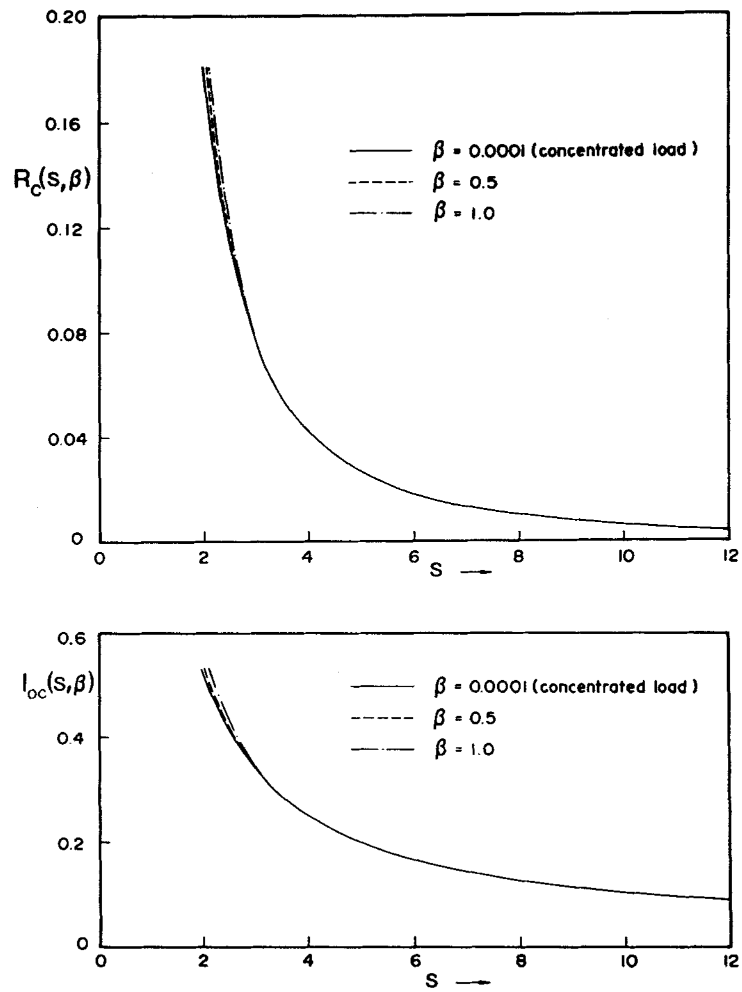

Figure 10. Influence coefficients for the central settlements and rotation of a rigid circular foundation due to an external uniform circular load

effects: (i) the spacing between the rigid circular foundation and the external load (s), (ii) the dimensions of the external load (β, δ), and (iii) the shape and distribution of the external load. The Figures 10–12 illustrate the influence of these factors on the central settlement and the rotation.

The contact stress distribution

The contact stress distribution at the rigid foundation–elastic halfspace interface due to the combined action of the central force P_0 and the external force Q_e (which acts at the location $(\lambda a, \varphi, 0)$) is given by

$$\sigma_{zz}(r, \theta, 0) = \frac{P_0}{\pi a^2\sqrt{(1-\rho^2)}}\left[1 - \frac{Q_e}{\pi P_0}\left\{\frac{(\lambda^2-1)}{[\lambda^2+\rho^2-2\lambda\rho\cos(\theta-\varphi)} - 2\sin^{-1}\left(\frac{1}{\lambda}\right) - \frac{3\rho\cos\theta}{2\lambda}(\pi\lambda^2 - 2\lambda^2\tan^{-1}\sqrt{(\lambda^2-1)} - 2\sqrt{(\lambda^2-1)})\right\}\right] \tag{37}$$

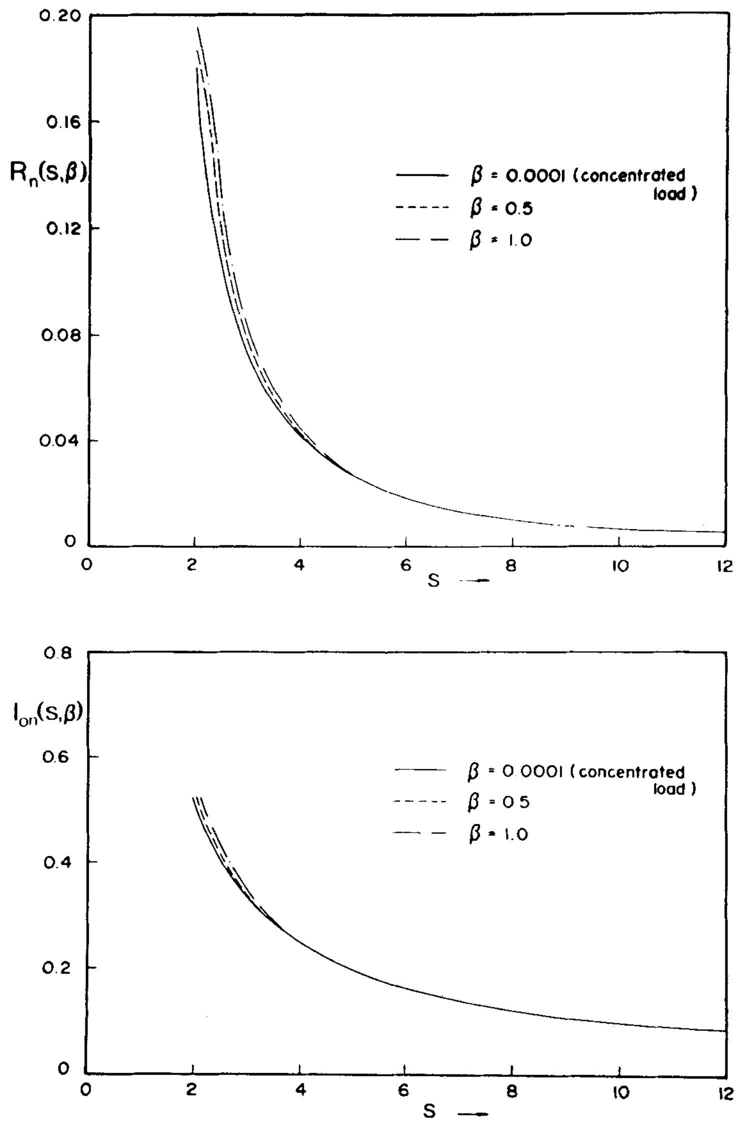

Figure 11. Influence coefficients for the central settlement and rotation of a rigid circular foundation due to an external non-uniform circular load

The contact stresses induced at the interface due to the combined action of the central force and the distributed external loadings can be obtained by an integration of the result (37) within appropriate limits. As an approximation, the development of tensile contact stresses at the smooth interface can be examined by directly using the solution (37) for the contact stresses induced by P_0 and Q_e. For example, when $P_0 = Q_e$ the contact stresses become tensile when $\lambda \simeq 1{\cdot}40$. Similar bounds can be established for other forms of distributed external loadings. For the present purposes it is sufficient to note that P_0 is sufficiently large to prevent the development of tensile stresses at the smooth interface.

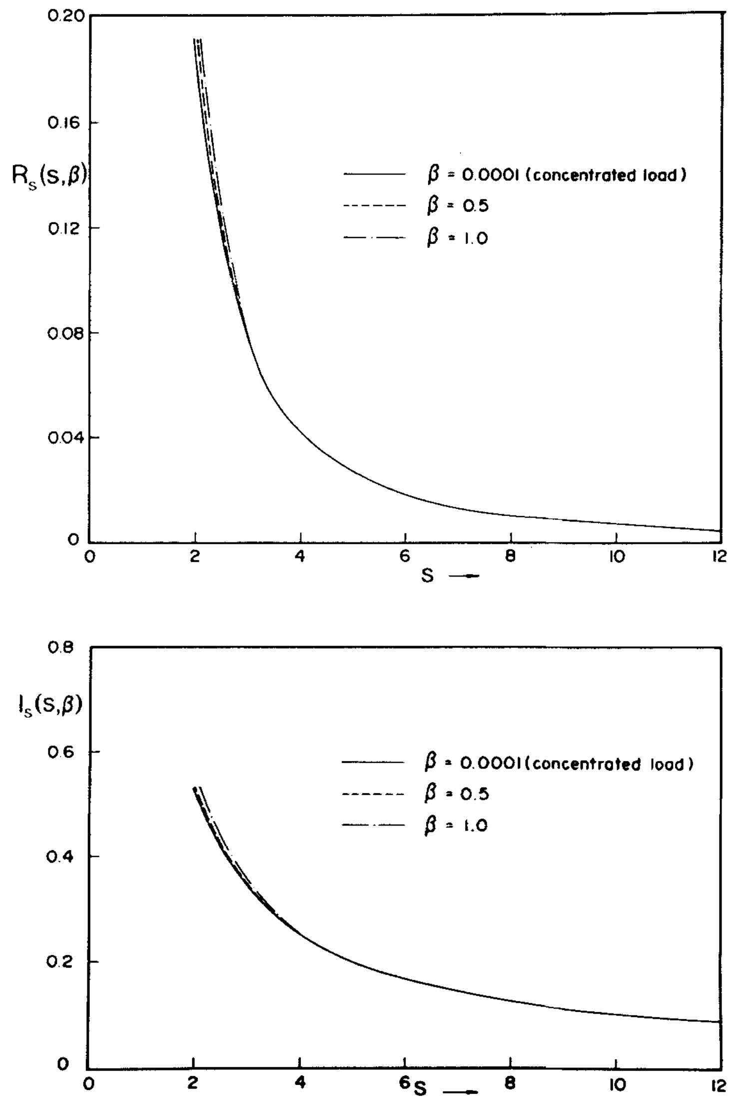

Figure 12. Influence coefficients for the central settlement and rotation of a rigid circular foundation due to an external uniform square load

CONCLUSIONS

The problem of the interaction between a rigid circular foundation resting in smooth contact with an isotropic elastic halfspace and an external distributed load is examined in the context of the linear theory of elasticity. It is shown that the results for the settlement of the foundation due to the external loading can be obtained by appeal to Betti's reciprocal theorem. The auxiliary solution required for the application of the reciprocal theorem can be obtained from an analysis of the problem related to a directly loaded rigid foundation. The various techniques available for the analysis of the directly loaded foundation are well documented in the literature on soil–foundation interaction. The central settlement and the rotation of the rigid circular foundation induced by three types of distributed loading are evaluated in explicit form. The numerical results indicate that the adjacent surface loads have a significant influence on the settlement of the rigid foundation. As expected, this influence diminishes as the spacing between the foundation and the external loading increases. Furthermore, at large spacing, the nature of the external load distribution has little influence on the settlements of the rigid circular foundation. This observation is in agreement with St. Venant's classical hypothesis concerning the effects of equipollent load distributions. The numerical results presented in the paper are of interest to the computation of time-independent settlements induced in silo foundation groups resting on cohesive soil media due to their mutual interaction.

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