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# The additional settlement of a rigid circular foundation on an isotropic elastic halfspace due to multiple distributed external loads

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The paper is concerned with the interaction between a rigid circular foundation resting on an elastic halfspace and a distributed external load of finite extent which acts at an exterior region of the halfspace. This problem serves as a model for the examination of the interaction between an existing structural foundation and a newly constructed adjacent foundation. The mathematical procedures which lead to the theoretical analysis of the title problem are summarized. It is shown that the additional elastic settlements experienced by the rigid circular foundation due to the action of a concentrated external force can be evaluated in exact closed form. This solution is used to generate results which are applicable to distributed loadings. From the results presented it is possible to determine the additional elastic settlement of a rigid circular foundation on an elastic halfspace due to multiple loads, of arbitrary shape and location, which act in its vicinity.

La communication traite de l'interaction entre une foundation rigide circulaire reposant sur un semi-espace et une charge extérieure répartie de surface finie qui agit sur une zone extérieure à la surface du semi-espace. Ce problème sert de modéle à l'étude de l'interaction d'une fondation structurale existante et d'une fondation contiguë nouvellement construite. Les procédés mathématiques qui ont conduit à l'analyse théorique du problème énoncé en titre sont résumés et il est montré que les tassements élastiques supplémentaires subis par la fondation rigide circulaire, dus à l'action d'une force externe concentrée, peuvent se calculer d'une manière exacte. Cette solution sert pour obtenir des résultats qui sont applicables à des charges réparties. A partir des données énoncées, il est possible de déterminer le tassement élastique supplémentaire que subira une fondation circulaire rigide sur un semi-espace élastique sous des charges multiples de forme et de localisation arbitraires, intervenant dans son voisinage.

## INTRODUCTION

The classical theoretical solutions in the area of soil–foundation interaction are primarily concerned with the examination of the performance of an isolated foundation. For example, Boussinesq's (1885) solution for the indentation of an isotropic elastic halfspace by a rigid circular foundation explicitly assumes that the region exterior to the foundation is free of loads or neighbouring foundations. However, the mutual interaction between a structural foundation and an externally placed surface load is important to the assessment of the settlement of an existing foundation due to surcharge loads applied in its vicinity.

This Paper examines the problem of the interaction between a loaded rigid circular foundation resting in smooth contact with an isotropic elastic halfspace and a distributed loading which acts at an exterior point on the surface of the halfspace. There is no analytical solution available for this linear elastostatic interaction problem. Owing to the rigid nature of the circular foundation it is not possible to use the influence charts developed by Newmark and others for the settlement of a halfspace region (see e.g. Poulos & Davis, 1974) to estimate settlements associated with the mutual interaction problem. For this reason an elasticity solution for the mutual interaction problem is developed.

The mathematical analysis of the basic problem is discussed. The problem related to the asymmetric deformation of the rigid circular foundation induced by the external concentrated force can be reduced to a mixed boundary value problem associated with a halfspace region. The dual integral equations associated with the mixed boundary problem are solved by using the standard techniques outlined by Sneddon (1977) and Sneddon & Lowengrub (1969). The Paper also shows the reciprocal relationship which exists between the displacement at an internal location of the rigid circular foundation due to the external concentrated force and the displacement of an external point due to a concentrated force applied at a point within the circular foundation. The applicability of such a reciprocal theorem to this class of interaction problem extends the use of classical results derived for the directly loaded foundation (see e.g. Poulos & Davis, 1974; Selvadurai, 1979) to the examination of mutual interaction effects.

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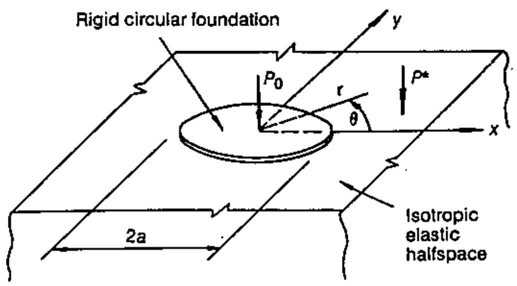


**Fig. 1. Geometry of the rigid circular foundation and the co-ordinate system**

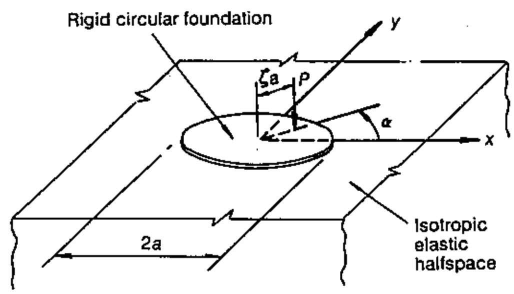


**Fig. 2. Eccentric direct loading of the rigid circular foundation**

The exact solution derived for the displacement of the rigid circular foundation due to a concentrated external load is used as a Green's function for the generation of equivalent results for the displacement of the rigid circular foundation due to distributed external loads. From the numerical results given it is possible to calculate the additional settlements induced in a rigid circular foundation on an elastic halfspace due to multiple loads of arbitrary extent and variable location.

## ANALYSIS

This Paper gives a brief account of the theoretical analysis of the interaction between an externally located surface force $P^*$ and a rigid circular foundation resting in smooth contact with an isotropic elastic halfspace. This interaction takes place in the presence of a direct load $P_0$ which acts at the centre of the rigid circular foundation (Fig. 1). The magnitude of $P_0$ is assumed to be such that the development of tensile contact stresses at the smooth interface is suppressed for all choices of magnitude and location of $P^*$. When this requirement is satisfied, the contact region at the rigid circular foundation–elastic halfspace interface always corresponds to a plane surface.

### *Results for directly loaded rigid circular foundation*

Some fundamental results relate to the directly loaded foundation. In particular, consider the problem of a rigid circular foundation resting in smooth contact with an isotropic elastic halfspace. The foundation (of radius $a$) is subjected to an eccentric vertical load $P$ which acts at the location $r = \zeta a$ and $\theta = \alpha$. The solution to this problem can be obtained by combining the separate solutions developed for the rigid circular foundation subjected to a central force (see e.g. Boussinesq, 1885; Sneddon, 1950) and a central moment (see e.g. Bycroft, 1956; Florence, 1961) about a horizontal axis. The analysis of these problems can be approached by using a Hankel transform technique. From Fig. 2, the settlements in the foundation region are given by

$$u_z(r,\theta,0) = \frac{P(1-\nu^2)}{2Ea}\left[1+\frac{3\zeta\rho}{2}\cos(\theta-\alpha)\right] \quad (1)$$

where $\rho = r/a$ and $0 \leqslant \rho \leqslant 1$.

Similarly, the surface displacements in the exterior region are given by

$$u_z(r,\theta,0) = \frac{P(1-\nu^2)}{2Ea}\left\{\frac{2}{\pi}\sin^{-1}\frac{1}{\rho}+\frac{3\zeta\rho}{2}\cos(\theta-\alpha) \times\left[1-\frac{2}{\pi}\tan^{-1}\sqrt{(\rho^2-1)}-\frac{2}{\pi}\frac{\sqrt{(\rho^2-1)}}{\rho^2}\right]\right\} \quad (2)$$

where $1 \leqslant \rho \leqslant \infty$. In equations (1) and (2) $E$ and $\nu$ denote Young's modulus and Poisson's ratio of the elastic material respectively.

### *Displacements of rigid circular foundation due to external load*

Consider now the interaction between a rigid circular foundation resting in smooth contact with an isotropic elastic halfspace and an external force $P^*$ which acts at the location $(\lambda a, 0, 0)$. It is assumed that perfect contact is maintained between the rigid foundation and the elastic halfspace by the application of a central force $P_0$. To facilitate the analysis of the problem it is assumed that the rigid circular foundation is subjected to fictitious additional force resultants $\bar{P}$ and $\bar{M}$ (as shown in Fig. 3) such that the foundation experiences zero vertical settlement in the $z$ direction within the foundation region $r \leqslant a$.

The surface of the halfspace region is now subject to the mixed boundary conditions

$$u_z(r,\theta,0) = 0, \qquad 0 \leqslant \theta \leqslant 2\pi, \quad 0 \leqslant r \leqslant a \quad (3)$$

$$\sigma_{zz}(r,\theta,0) = -p^*(r,\theta), \qquad 0 \leqslant \theta \leqslant 2\pi, \quad a < r < \infty \quad (4)$$

$$\sigma_{rz}(r,\theta,0) = 0, \qquad 0 \leqslant \theta \leqslant 2\pi, \quad 0 \leqslant r < \infty \quad (5)$$

where $p^*(r,\theta)$ is the external loading which can be

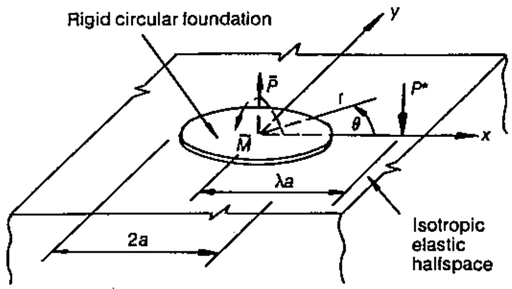


**Fig. 3. External loading of the rigid circular foundation and the associated corrective forces**

expressed as an even function of $\theta$. For the solution of this mixed boundary value problem a Hankel transform development of the equations of elastic equilibrium is used. Following Muki (1960) it can be shown that when the boundary condition (5) relating to the shear stress is satisfied, the appropriate expressions for $u_z$ and $\sigma_{zz}$ can be represented in the form

$$u_z(r,\theta,0) = 2(1-\nu)\sum_{m=0}^{\infty}\left[\int_0^{\infty}\Phi_m(\xi)\,\mathrm{J}_m(\xi r/a)\,\mathrm{d}\xi\right]\cos m\theta \quad (6)$$

$$\sigma_{zz}(r,\theta,0) = -\frac{E}{(1+\nu)}\sum_{m=0}^{\infty}\left[\int_0^{\infty}\xi\Phi_m(\xi)\,\mathrm{J}_m(\xi r/a)\,\mathrm{d}\xi\right]\cos m\theta \quad (7)$$

The unknown functions $\Phi_m(\xi)$ in equations (6) and (7) should be determined by satisfying the mixed boundary conditions (3) and (4). By assuming that $p^*(r,\theta)$ can be expressed in the form

$$p^*(r,\theta) = -\frac{E}{(1+\nu)}\sum_{m=0}^{\infty} g_m(r)\cos m\theta \quad (8)$$

the boundary conditions (3) and (4) can be reduced to a system of dual integral equations of the type

$$\int_0^{\infty}\Phi_m(\xi)\,\mathrm{J}_m(\xi r/a)\,\mathrm{d}\xi = 0, \quad 0 \leqslant r \leqslant a \quad (9)$$

$$\int_0^{\infty}\xi\Phi_m(\xi)\,\mathrm{J}_m(\xi r/a)\,\mathrm{d}\xi = g_m(r), \quad a < r < \infty \quad (10)$$

In equations (9) and (10) $\mathrm{J}_m$ is the $m$th order Bessel function of the first kind. This system of dual integral equations has been studied by Noble (1958), Sneddon & Lowengrub (1969) and others. For the concentrated loading

$$\begin{bmatrix} g_0(r) \\ g_m(r) \end{bmatrix} = \frac{P^*(1+\nu)\,a\Delta(r-l)}{4\pi^2 Er}\begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad (11)$$

where $\Delta(r-l)$ is the Dirac delta function. The solution of the systems of dual integral equations determines $\Phi_m(\xi)$. Using these results, formal expressions for the displacement and stress distribution in the elastic halfspace region can be obtained in integral series form.

In particular, the contact stress distribution induced beneath the rigid circular foundation subject to the boundary conditions (3)–(5) can be evaluated in explicit closed form, i.e.

$$\sigma_{zz}(r,\theta,0) = \frac{P^*\sqrt{(\lambda^2-1)}}{\pi^2 a^2\sqrt{(1-\rho^2)}\,[\lambda^2+\rho^2-2\lambda\rho\cos\theta]} \quad (12)$$

where $\lambda = l/a$ and $l$ is the distance between the external load and the centre of the rigid foundation.

This solution has been obtained by applying force resultants $\bar{P}$ and $\bar{M}$ within the foundation region $r \leqslant a$ to ensure $u_z(r,\theta,0) = 0$, in the region $r \leqslant a$. The force $\bar{P}$ required to achieve this condition is given by

$$\bar{P} = 2\int_0^a\int_0^{\pi}\sigma_{zz}(r,\theta,0)\,r\,\mathrm{d}r\,\mathrm{d}\theta = \frac{2P^*}{\pi}\sin^{-1}\frac{1}{\lambda} \quad (13)$$

Similarly, the moment $\bar{M}$ required to maintain zero displacement in $r \leqslant a$ is given by

$$\bar{M} = 2\int_0^a\int_0^{\pi}\sigma_{zz}(r,\theta,0)\,r^2\cos\theta\,\mathrm{d}r\,d\theta = P^*l\left[1-\frac{2}{\pi}\tan^{-1}\sqrt{(\lambda^2-1)}-\frac{2}{\pi}\frac{\sqrt{(\lambda^2-1)}}{\lambda^2}\right] \quad (14)$$

As the rigid circular foundation is not subjected to any loading other than the central force $P_0$ it is necessary to apply corrective force resultants $\bar{P}$ and $\bar{M}$ to the foundation, in the opposite sense. The displacements induced within the foundation area due to $-\bar{P}$ and $-\bar{M}$ can be obtained by using the preliminary results (1) and (2) developed for the directly loaded foundation. It can be shown that the settlement of the rigid circular foundation at an arbitrary location $(\rho a,\theta,0)$ due to the external load $P^*$ applied at $(\lambda a,0,0)$ is given by

$$u_z(r,\theta,0) = \frac{P^*(1-\nu^2)}{2aE}\left\{\frac{2}{\pi}\sin^{-1}\frac{1}{\lambda}+\frac{3\lambda\rho}{2}\cos\theta \times\left[1-\frac{2}{\pi}\tan^{-1}\sqrt{(\lambda^2-1)}-\frac{2}{\pi}\frac{\sqrt{(\lambda^2-1)}}{\lambda^2}\right]\right\}, \quad 0 \leqslant r \leqslant a \quad (15)$$

*Generalized results and reciprocal relationships*

The results derived for the displacement at an external point due to an internally loaded foundation and the displacement at a point within the foundation due to an externally placed load can be generalized to include arbitrary locations for the loaded and displaced points. This can be achieved by a straightforward change in the frame of reference. Consider a force $P_A$ applied at the location $(\rho a, \theta, 0)$ within the rigid circular foundation. The displacement at an external location B $(\lambda a, \phi, 0)$ is denoted by $u_z^{\mathrm{B}}$ (Fig. 4). Similarly $P_{\mathrm{B}}$ and $u_z^{\mathrm{A}}$ refer to the situation where the load and displacement locations are reversed. It can be shown that

$$\begin{Bmatrix} u_z^{\mathrm{A}} \\ u_z^{\mathrm{B}} \end{Bmatrix} = \frac{(1-\nu^2)H}{2aE} \begin{Bmatrix} P_{\mathrm{A}} \\ P_{\mathrm{B}} \end{Bmatrix} \quad (16)$$

where

$$H = \frac{2}{\pi}\sin^{-1}\frac{1}{\lambda} + \frac{3\lambda\rho}{2}\cos(\theta-\phi) \times \left[1 - \frac{2}{\pi}\tan\sqrt{(\lambda^2-1)} - \frac{2}{\pi}\frac{\sqrt{(\lambda^2-1)}}{\lambda^2}\right] \quad (17)$$

As a consequence of the result in equation (16) it is evident that the displacements $u_z^{\mathrm{A}}$ and $u_z^{\mathrm{B}}$ and the corresponding applied loads $P_{\mathrm{A}}$ and $P_{\mathrm{B}}$ satisfy Betti's reciprocal relationship

$$P_{\mathrm{A}} u_z^{\mathrm{B}} = P_{\mathrm{B}} u_z^{\mathrm{A}} \quad (18)$$

In general, the applicability of the reciprocal theorem to this class of soil–foundation interaction problem is valid only in situations where the foundation is in smooth or fully bonded contact with the soil medium. The reciprocal theorem does not extend to situations where dissipative phenomena such as Coulomb friction or finite friction effects are present at the interface. Nevertheless, the smooth and fully bonded cases provide useful bounds for practical purposes. In the case of undrained elastic behaviour of the soil medium ($\nu = 1/2$), the elastostatic solutions for the settlements for both the fully bonded and frictionless interface situations are identical. The applicability of the reciprocal theorem to this class of external load–foundation interaction problem extends the use of available elastostatic solutions for square and rectangular foundations to the examination of combined interaction problems.

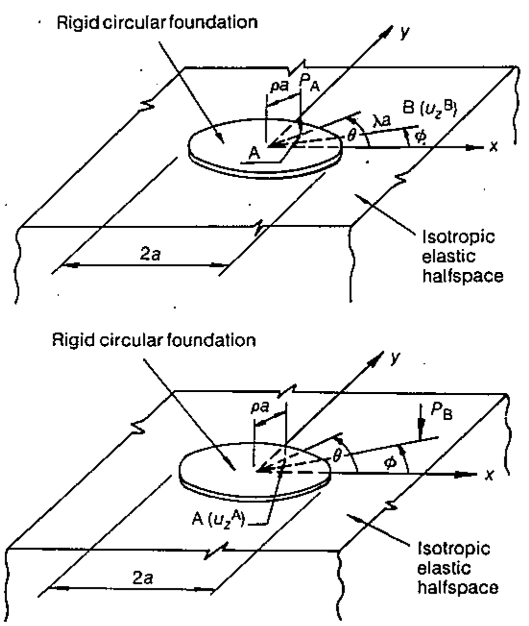


Fig. 4. Reciprocal states

## APPLICATIONS

The solution developed for the interaction between the rigid circular foundation and the external concentrated force can be generalized to include other forms of distributed external loading. This is achieved by a direct integration of the result in equation (15) within the limits of the loading region. Several external loading configurations are of interest to geotechnical application. The approach adopted here is to develop a solution for the interaction between the rigid circular foundation and a circular external loading of uniform intensity and arbitrary radius, which can then be used to examine the effects of distributed loadings of arbitrary shape and varying intensity.

Consider the problem of the interaction between a rigid circular foundation and a uniform circular external load of intensity $P_e/\pi\beta^2 a^2$ (where $P_e$ is the total external load acting within the circular area and $\beta a$ is the radius) which is at a distance $sa$ from the centre of the rigid circular foundation (Fig. 5). As $\beta$ becomes small the external uniform circular load resembles a localized or concentrated force. In Fig. 5 the frame of reference is altered to a specific direction N such that all angular measurements are referred to this direction. Considering the solution for the interaction between the external concentrated force and the rigid circular foundation it may be observed that

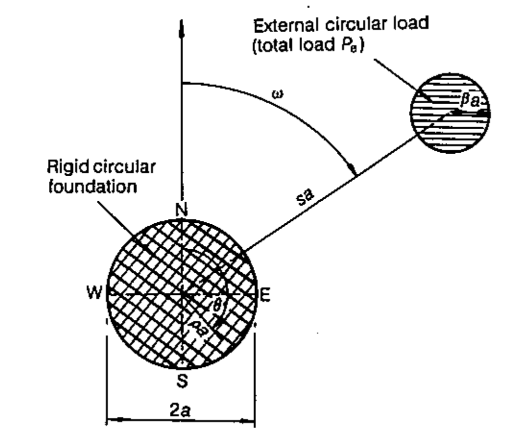


Fig. 5. Interaction between the rigid circular foundation and the external circular load

the settlement within the foundation region is given by

$$u_z(\rho a, \theta, 0) = \frac{(1-\nu^2)P_e}{2\pi a E} I_c\{s, \omega, \beta, \rho; \theta\} \quad (19)$$

where the influence function $I_c\{s, \omega, \beta, \rho, \theta\}$ is given by the multiple integral

$$I_c\{s, \omega, \beta, \rho, \theta\} = \frac{1}{\beta^2}\int_{\omega-\Omega}^{\omega+\Omega}\int_{\lambda_2(\phi)}^{\lambda_1(\phi)}\left\{\frac{2}{\pi}\sin^{-1}\frac{1}{\lambda}+\frac{3}{2}\lambda\rho\cos(\theta-\phi)\right.$$
$$\left.\times\left[1-\frac{2}{\pi}\tan^{-1}\sqrt{(\lambda^2-1)}-\frac{2}{\pi}\frac{\sqrt{(\lambda^2-1)}}{\lambda^2}\right]\right\}\lambda\,d\lambda\,d\phi \quad (20)$$

where

$$\left.\begin{aligned}\Omega &= \sin^{-1}(\beta/s)\\ \lambda_1(\phi) &= s\cos(\phi-\omega)+[\beta^2-s^2\sin^2(\phi-\omega)]^{\frac{1}{2}}\\ \lambda_2(\phi) &= s\cos(\phi-\omega)-[\beta^2-s^2\sin^2(\phi-\omega)]^{\frac{1}{2}}\end{aligned}\right\} \quad (21)$$

The influence function $I_c$ can be evaluated numerically (by using a numerical scheme based on Gaussian quadrature) to examine the effect of the distributed circular loading on the settlement of the rigid circular foundation. Fig. 6 shows the variation of $I_c^0(=I_c\{s, 0, \beta, 0, 0\})$, i.e. the settlement of the centre of the rigid circular foundation. The size of the distributed loading has an appreciable effect on the central settlement only when the external load approaches the rigid circular foundation. For values of $\beta \leqslant 1$ and $s > 3$ the central settlement of the rigid circular foundation due to the distributed loading is identical with that which is obtained for the concentrated external loading. The influence of the external circular loading on the settlement at specific locations of the rigid foundation is shown in Figs 7–9. The following convention is adopted

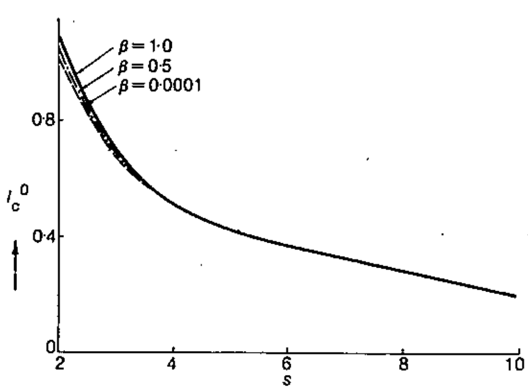


**Fig. 6. Variation of $I_c^0$ with size and location of external load $u_z(0,0,0) = [P_e(1-\nu^2)/2\pi aE]\, I_c^0$**

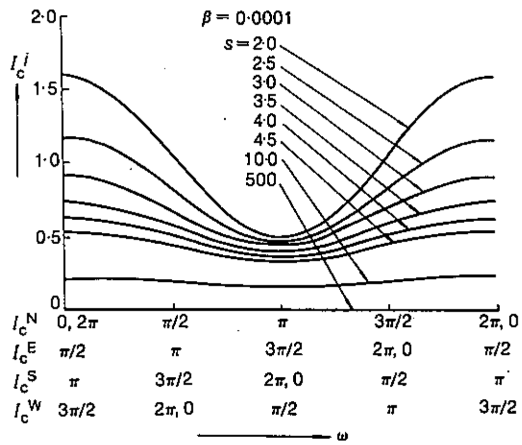


**Fig. 7. Influence coefficients $I_c^j(s, \omega, \beta)$**

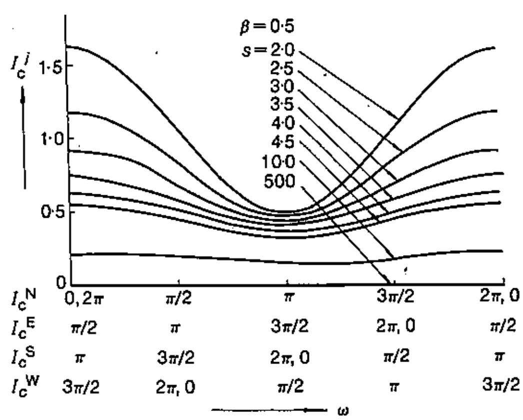


**Fig. 8. Influence coefficients $I_c^j(s, \omega, \beta)$**

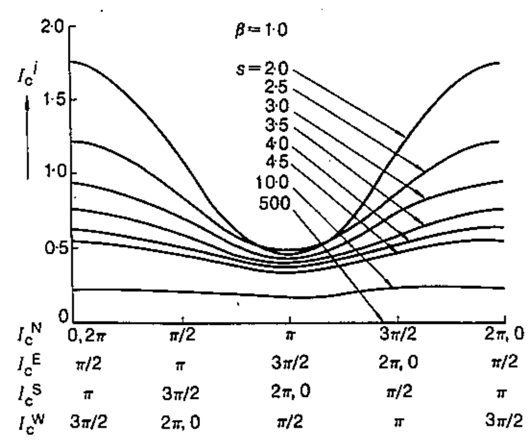


**Fig. 9. Influence coefficients $I_c^j(s, \omega, \beta)$**

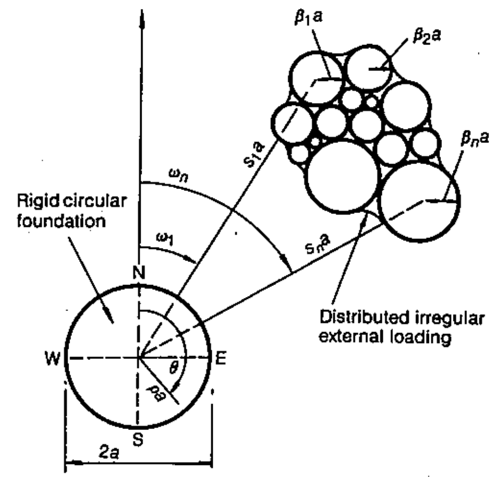


**Fig. 10. Solution scheme for the analysis of the interaction of rigid circular foundation with irregular external loading**

to describe the settlements at the edges N, E, S and W of the rigid foundation

$$\left.\begin{aligned} u_z(a,0,0) &= \frac{P_e^*(1-\nu^2)}{2\pi aE}[\bar{P}_e I_c^{\mathrm{N}}(s,\omega,\beta)] = u_z^{\mathrm{N}} \\ u_z\left(a,\frac{\pi}{2},0\right) &= \frac{P_e^*(1-\nu^2)}{2\pi aE}[\bar{P}_e I_c^{\mathrm{E}}(s,\omega,\beta)] = u_z^{\mathrm{E}} \\ u_z(a,\pi,0) &= \frac{P_e^*(1-\nu^2)}{2\pi aE}[\bar{P}_e I_c^{\mathrm{S}}(s,\omega,\beta)] = u_z^{\mathrm{S}} \\ u_z\left(a,\frac{3\pi}{2},0\right) &= \frac{P_e^*(1-\nu^2)}{2\pi aE}[\bar{P}_e I_c^{\mathrm{W}}(s,\omega,\beta)] = u_z^{\mathrm{W}} \end{aligned}\right\} \quad (22)$$

where $P_e^*$ is a normalizing load and $\bar{P}_e = P_e/P_e^*$. Again, it is evident that the effects of the variable size of the external circular loads become noticeable only when the load approaches the boundary of the rigid circular foundation.

From these findings it is possible to represent a distributed loading of arbitrary shape as a collection of circular loading regions of uniform extent. The overall effect of the irregular loading on the additional settlement of the rigid circular foundation can be obtained by superposition, as shown in Fig. 10.

Considering the influence of the single circular load it can be shown that the total displacement (i.e. the displacement due to the central load $P_0$ and the additional settlement due to the external load $P_e$) of the rigid circular foundation is given by

$$\begin{aligned} u_z(\rho a,\theta,0) = {} & \frac{P_0(1-\nu^2)}{2aE}+\frac{P_e(1-\nu^2)}{2\pi aE} \\ & \times\left[\frac{1}{4}\sum_{j=\mathrm{N,S,W,E}}\{I_c^j\}+\frac{\rho\cos\theta}{2}\right. \\ & \left.\times\{I_c^{\mathrm{N}}-I_c^{\mathrm{S}}\}+\frac{\rho\sin\theta}{2}\{I_c^{\mathrm{E}}-I_c^{\mathrm{W}}\}\right] \end{aligned} \quad (23)$$

where

$$I_c^{()} = I_c^{()}\{s,\omega,\beta\}$$

$$4I_c^0 = \sum_{j=\mathrm{N,S,W,E}} I_c^j$$

$$I_c^{\mathrm{N}}+I_c^{\mathrm{S}} = I_c^{\mathrm{W}}+I_c^{\mathrm{E}}$$

It can be verified easily that when $\rho = 1$ and $\theta = 0, \pi/2, \pi$ or $3\pi/2$, equation (23) yields the values for $(u_z^j+u_z^0)$ ($j = \mathrm{N},\ldots,\mathrm{E}$; $u_z^0 = P(1-\nu^2)/2aE$) in the presence of both $P_0$ and $P_e$. Generalizing equation (23) to a series of $n$ circular loaded areas as shown in Fig. 10 gives

$$\begin{aligned} u_z(\rho a,\theta,0) = {} & \frac{P_0(1-\nu^2)}{2aE}+\frac{P_e^*(1-\nu^2)}{8\pi aE} \\ & \times\left\{\sum_{i=1}^{n}\left[\sum_{j=\mathrm{N}}^{\mathrm{E}}\bar{P}_{ei}I_{ci}^j\right]\right. \\ & +2\rho\cos\theta\left[\sum_{i=1}^{n}\bar{P}_{ei}(I_{ci}^{\mathrm{N}}-I_{ci}^{\mathrm{S}})\right] \\ & \left.+2\rho\cos\theta\left[\sum_{i=1}^{n}\bar{P}_{ei}(I_{ci}^{\mathrm{E}}-I_{ci}^{\mathrm{W}})\right]\right\} \end{aligned} \quad (24)$$

where

$$\bar{P}_{ei} = P_{ei}/P_e^*$$

$$I_{ci}^{()} = I_c^{()}\{s_i,\omega_i,\beta_i\}$$

This result can be used to develop values for the settlement of the rigid circular foundation due to an external load of arbitrary shape and/or varying intensity.

EXAMPLE

As an example, the procedure is applied to the group interaction of a series of silos. Fig. 11 shows the plan configuration of a series of silo foundations to be constructed in the vicinity of an existing

**Table 1**

| Load region | $s_i$ | $\omega_i$ | $\beta_i$ | $\bar{P}_{ei}$ | $\bar{P}_{ei}I_{ci}^{\mathrm{N}}$ | $\bar{P}_{ei}I_{ci}^{\mathrm{S}}$ | $\bar{P}_{ei}I_{ci}^{\mathrm{W}}$ | $\bar{P}_{ei}I_{ci}^{\mathrm{E}}$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 4·00 | $\pi/4$ | 1·0 | 1·0 | 0·60 | 0·42 | 0·42 | 0·60 |
| 2 | 4·00 | $\pi/2$ | 1·0 | 1·0 | 0·50 | 0·50 | 0·37 | 0·62 |
| 3 | 4·00 | $3\pi/4$ | 1·0 | 1·0 | 0·42 | 0·60 | 0·42 | 0·60 |

rigid circular foundation. The proposed silo foundations each carry a load of $P_s$. The radii of the additional silo foundations and the location of their centres from the existing rigid circular foundation are shown in Fig. 11. The calculations for estimating the elastic settlements that may occur at the locations N, S, W and E of the rigid circular foundation are outlined in Table 1. The influence coefficients $I_c^j$ (applicable for $\beta_l = 1$) are given in Fig. 9. The value of $P_c^*$ is set equal to $P_s$.

Hence, the additional elastic settlements induced in the rigid circular foundation at the locations N, S, W and E due to the proposed silo group are given by

$$[u_z^{N}: u_z^{S}: u_z^{W}: u_z^{E}] = \frac{P_S(1-\nu^2)}{2\pi a E}[1{\cdot}52: 1{\cdot}52: 1{\cdot}21: 1{\cdot}82]$$

The average settlement at the centre of the rigid circular foundation is given by

$$u_z^{0} = \frac{P_S(1-\nu^2)}{2\pi a E}\left[\frac{1}{4}\sum_{j=N}^{E} u_z^{j}\right] \simeq \frac{1{\cdot}52 P_S(1-\nu^2)}{2\pi a E}$$

This is in agreement with the value that can be calculated from the results given in Fig. 6.

## CONCLUSIONS

The Paper presents an analysis of the problem of the interaction between a rigid circular foundation and an external loading which is formulated within the framework of the classical theory of elasticity. It is shown that the settlement of the rigid circular foundation due to an externally placed concentrated force can be evaluated in exact closed form. This result can be used as an influence of Green's function to examine the effects of other forms of distributed loading. Numerical results are presented for the settlement of a rigid circular foundation which is subjected to a central concentrated force and a uniform circular external load. This solution can be used to compute the settlements experienced by the rigid circular foundation due to multiple external loads or external loads of arbitrary configuration. Theoretical and numerical results are presented.

The Paper also illustrates reciprocal relationships that exist between the displacement induced at an exterior point due to a directly loaded foundation and the displacement in the rigid circular foundation due to an externally located force. The reciprocity property extends to any form of surface foundation which rests in smooth or fully bonded contact with the elastic medium, i.e. a halfspace or layer region. This reciprocity property can be used to great advantage in the examination of interaction between rigid rectangular foundations and externally placed loads. The results presented can be used to produce influence charts which will facilitate the computation of settlements of foundation groups.

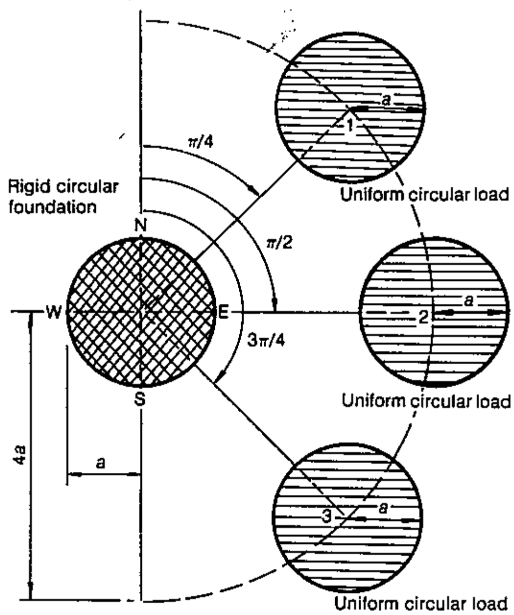


**Fig. 11. Interaction between the rigid circular foundation and multiple external loads**

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