

A THEORETICAL ESTIMATE OF THE TILT OF A SURFACE FOUNDATION DUE TO AN INCLINED ANCHOR LOAD APPLIED AT THE INTERIOR OF THE SOIL MEDIUM

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SUMMARY

This paper examines the problem of the interaction between a loaded rigid circular foundation located at the surface of an isotropic elastic halfspace and an inclined concentrated anchor load which is located at a finite depth along the axis of symmetry. Such inclined loads can be induced by, for example, anchor regions supporting earth retaining structures. The loaded rigid circular foundation resting in smooth contact with the elastic soil mass experiences a displacement and a tilt due to the action of the inclined anchor load. The magnitude of the rotational settlement is evaluated in exact closed form.

INTRODUCTION

Problems related to the interaction of foundations resting on a soil or rock mass and anchor regions which are situated at the interior of the soil or rock mass are of interest to several problems in geotechnical engineering. Interaction analyses of this nature can be utilized to examine problems such as anchorage efficiency and the desirable location of anchor regions used in *in-situ* testing techniques (Stagg and Zienkiewicz,²¹ Jaeger,⁶ Selvadurai¹⁶). In this connection, the interaction between a rigid circular foundation located at the surface of the soil medium and a Mindlin type axisymmetric internal load was examined by Selvadurai.¹⁷ The resulting axisymmetric displacement of the rigid foundation was evaluated in explicit form. When ground anchors are used to support earth retaining structures such as sheet pile walls and other bulkheads, the anchor loads are transmitted to the soil mass in arbitrary directions (Figure 1) (see e.g. Littlejohn,⁹ Trow,²² Kovacs *et al.*,⁷ Ostermayer,¹³ Kay and Qamar,⁸ Hanna,⁵ Banerjee¹ and Rowe and Booker¹⁵). The interaction of such inclined anchor loads and structural foundations located at the surface of the soil medium is therefore of interest in the assessment of the safe depth of location of the anchors (for example, Ostermayer¹³ arbitrarily suggests that in order to minimize soil displacements and damages to buildings the distance between a fixed anchor and an adjacent foundation should exceed 3 m). In this paper we consider a relatively simple idealized problem pertaining to the interaction of a rigid circular foundation resting on an isotropic elastic soil mass and an internal inclined anchor load which is located at some depth along the z -axis (Figure 2). The soil mass is idealized as an isotropic elastic halfspace and the anchor load is represented by a concentrated force Q (with components Q_h and Q_v in the horizontal and vertical directions respectively). It is also assumed that the rigid foundation is loaded by an axial force P and that the interface between the foundation and the soil mass is smooth. For most geotechnical problems of this nature it may be assumed that P is usually larger than Q . The axisymmetric interaction problem related to the internal anchor force Q_v and the

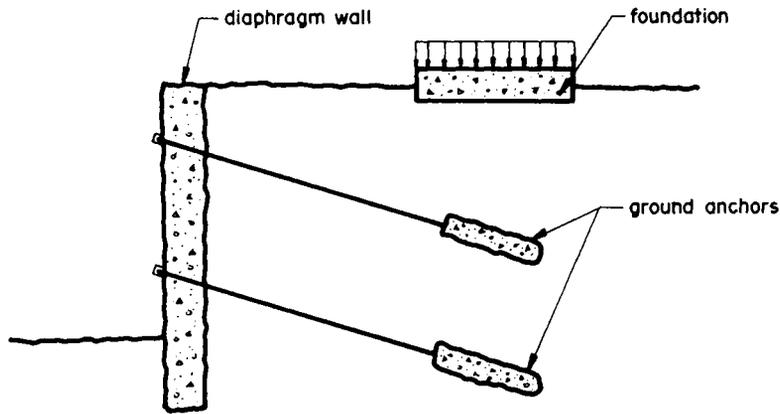


Figure 1. Anchoring systems for a diaphragm wall

loaded rigid circular foundation has been examined elsewhere (Selvadurai¹⁷). In this paper, therefore, we shall concentrate on the analysis of the asymmetric mixed boundary value problem corresponding to the horizontal component Q_h of the inclined anchor load and the loaded rigid circular foundation. By adopting an integral transform formulation, this mixed boundary value problem can be reduced to the solution of a set of dual integral equations. The solution of the dual system is facilitated by the generalized results given by Sneddon.^{19,20} The factor of primary geotechnical interest, namely, the resultant tilt experienced by the rigid foundation due to Q_h , is evaluated in exact closed form. The results for the horizontal anchor

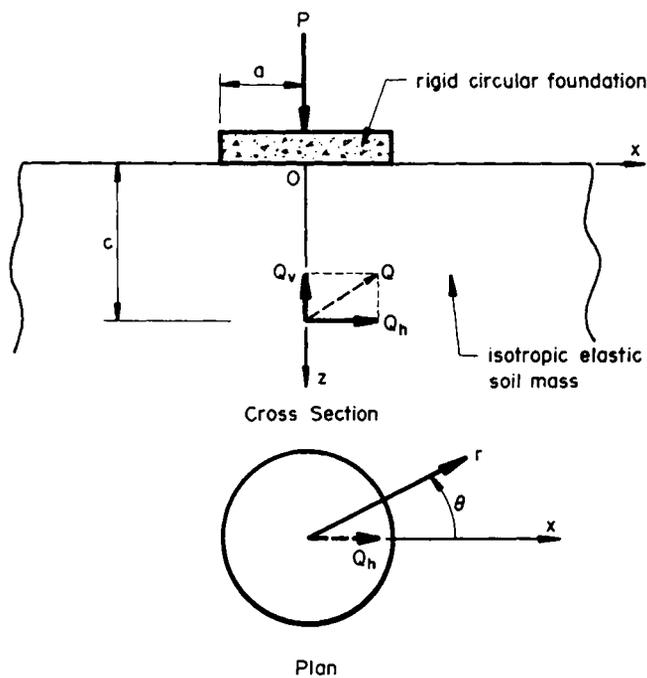


Figure 2. Geometry of the rigid foundation-internal anchor load system

load are combined with the existing solution for the vertical anchor load to develop explicit results for the settlement and tilt of a surface foundation due to an inclined anchor load.

GOVERNING EQUATIONS

The interaction problem related to the horizontally directed internal anchor load falls into the category of asymmetric boundary value problems in elasticity theory which can be analysed by employing the stress function techniques developed by Muki.¹² These stress functions are governed by the differential equations

$$\nabla^2 \nabla^2 \Phi(r, \theta, z) = 0; \quad \nabla^2 \chi(r, \theta, z) = 0 \quad (1)$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \quad (2)$$

is Laplace's operator referred to the (r, θ, z) system. The stresses and displacements in the elastic medium can be uniquely expressed in terms of these stress functions (see e.g. Muki¹² and Gurtin⁴). The expressions for the displacement and stress components in terms of $\Phi(r, \theta, z)$ and $\chi(r, \theta, z)$ take the forms

$$2Gu_r = -\frac{\partial^2 \Phi}{\partial r \partial z} + \frac{2}{r} \frac{\partial \chi}{\partial \theta} \quad (3a)$$

$$2Gu_\theta = -\frac{1}{r} \frac{\partial^2 \Phi}{\partial \theta \partial z} - 2 \frac{\partial \chi}{\partial r} \quad (3b)$$

$$2Gu_z = 2(1-\nu)\nabla^2 \Phi - \frac{\partial^2 \Phi}{\partial z^2} \quad (3c)$$

and

$$\sigma_{rr} = \frac{\partial}{\partial z} \left(\nu \nabla^2 - \frac{\partial^2}{\partial r^2} \right) \Phi + \frac{\partial}{\partial \theta} \left(\frac{2}{r} \frac{\partial}{\partial r} - \frac{2}{r^2} \right) \Psi \quad (4a)$$

$$\sigma_{\theta\theta} = \frac{\partial}{\partial z} \left(\nu \nabla^2 - \frac{1}{r} \frac{\partial}{\partial r} - \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right) \Phi - \frac{\partial}{\partial \theta} \left(\frac{2}{r} \frac{\partial}{\partial r} - \frac{1}{r^2} \right) \Psi \quad (4b)$$

$$\sigma_{zz} = \frac{\partial}{\partial z} \left[(2-\nu)\nabla^2 - \frac{\partial^2}{\partial z^2} \right] \Phi \quad (4c)$$

$$\sigma_{\theta z} = \frac{1}{r} \frac{\partial}{\partial \theta} \left[(1-\nu)\nabla^2 - \frac{\partial^2}{\partial z^2} \right] \Phi - \frac{\partial^2 \Psi}{\partial r \partial z} \quad (4d)$$

$$\sigma_{zr} = \frac{\partial}{\partial r} \left[(1-\nu)\nabla^2 - \frac{\partial^2}{\partial z^2} \right] \Phi + \frac{1}{r} \frac{\partial^2 \Psi}{\partial \theta \partial z} \quad (4e)$$

$$\sigma_{r\theta} = \frac{1}{r} \frac{\partial^2}{\partial \theta \partial z} \left(\frac{1}{r} - \frac{\partial}{\partial r} \right) \Phi - \left(2 \frac{\partial^2}{\partial r^2} - \frac{\partial^2}{\partial z^2} \right) \Psi \quad (4f)$$

where G and ν correspond to the linear elastic shear modulus and Poisson's ratio respectively.

Considering a Hankel integral transform development of the differential equations (1) it can be shown that the solutions of $\Phi(r, \theta, z)$ and $\Psi(r, \theta, z)$ appropriate for the halfspace region

$z > 0$ are

$$\Phi(r, \theta, z) = \frac{\cos \theta}{a^2} \int_0^\infty \xi [C(\xi) + D(\xi)z] e^{-\xi z/a} J_1(\xi r/a) d\xi \quad (5)$$

$$\Psi(r, \theta, z) = \frac{\sin \theta}{a^2} \int_0^\infty \xi F(\xi) e^{-\xi z/a} J_1(\xi r/a) d\xi \quad (6)$$

where $C(\xi)$, $D(\xi)$ and $F(\xi)$ are arbitrary functions.

INTERNAL LOADING OF THE HALFSPACE REGION

We first consider the problem of a halfspace region, with a traction free plane boundary and subjected to an internal concentrated force Q_h which is located at a depth c below the free surface and which acts along the x -direction. The solution to this problem was obtained by Mindlin and Cheng¹¹ in their treatment of nuclei of strain associated with a halfspace region. This exact solution is developed by considering a superposition of solutions for the Kelvin problem related to an infinite region and a distribution of tangential tractions based on Cerruti's solution for a halfspace region (see e.g. Love¹⁰). The result of primary interest to the analytical treatment of the anchor interaction problem corresponds to the surface displacement of the halfspace region. This displacement is given by

$$u_z(r, \theta, z) = \frac{Q_h r \cos \theta}{4\pi G} \left\{ -\frac{c}{(r^2 + c^2)^{3/2}} + \frac{(1 - 2\nu)}{(r^2 + c^2)^{1/2} [c + \sqrt{(r^2 + c^2)}]} \right\} = u_z^0(r) \cos \theta \quad (7)$$

It may be noted that this internal loading induces a state of asymmetric deformation about any horizontal plane $z = \text{const.}$

ANALYSIS OF THE INTERACTION PROBLEM

Attention is now focussed on the asymmetric interaction between the externally loaded rigid circular foundation of radius a ($a > 0$), and the horizontal anchor load which is applied at the interior of the halfspace region. The rigid foundation-elastic halfspace interface is assumed to be smooth. According to this assumption, the interface is incapable of sustaining tensile tractions. Therefore for the solution developed in the ensuing analysis to be physically admissible sufficient external loads (i.e. force P in Figure 2) should be present to prevent the development of tensile tractions. For convenience we have assumed that the rigid circular foundation is subjected to an axisymmetric load large enough to prevent the development of tensile tractions. This factor has to be verified after the completion of the analysis and the solutions developed will be valid only for those situations in which $\sigma_{zz}(r, \theta, 0)$ is everywhere compressive in the region $r < a$. Since the problem is formulated within the context of the linear theory of elasticity and since separation at the interface is suppressed we can examine the axisymmetric interaction problem related to the central force P and the asymmetric problem related to the internal anchor load Q_h separately (Figure 3). The former problem is, of course, Boussinesq's classical solution related to the indentation of an elastic halfspace by a rigid circular foundation (see e.g. Boussinesq,² Poulos and Davis,¹⁴ Selvadurai¹⁸). We shall concentrate on the analysis of the asymmetric problem and assume that owing to the action of the internal anchor load the rigid foundation experiences a rotation Ω .

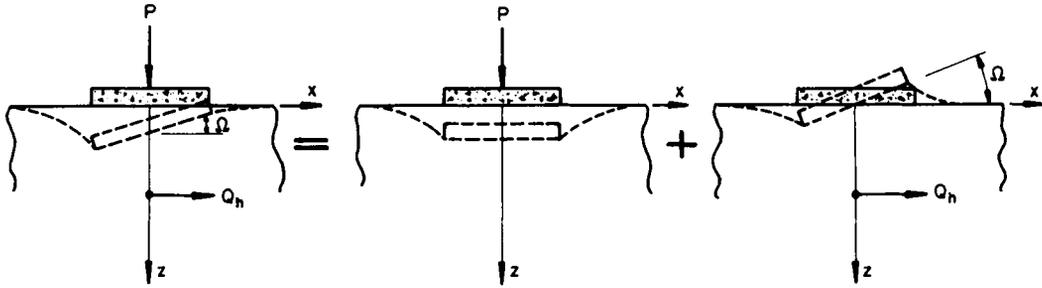


Figure 3. Combined interaction between a loaded rigid circular foundation and a horizontal anchor load

The mixed boundary conditions related to this problem can be summarized as follows:

(i) Since the contact is smooth we require

$$\sigma_{\theta z} = \sigma_{rz} = 0 \quad \text{on } z = 0 \quad \text{for all } r \geq 0 \tag{8}$$

(ii) The displacement boundary condition in the foundation region is

$$u_z(r, \theta, 0) = -\Omega r \cos \theta - u_z^0(r) \cos \theta; \quad 0 \leq r \leq a \tag{9}$$

(iii) The traction boundary condition exterior to the foundation is

$$\sigma_{zz}(r, \theta, 0) = 0; \quad a < r < \infty \tag{10}$$

Deriving the integral expressions for the stress components $\sigma_{\theta z}$ and σ_{rz} it becomes evident that, in order to satisfy the boundary conditions (8), we require

$$F(\xi) = 0; \quad C(\xi) = \frac{2\nu a}{\xi} D(\xi) \tag{11}$$

The remaining mixed boundary conditions (9) and (10) yield the following pair of dual integral equations for the unknown function $D(\xi)$:

$$\int_0^\infty \xi^2 D(\xi) J_1(\xi r/a) d\xi = f(r); \quad 0 \leq r \leq a \tag{12}$$

$$\int_0^\infty \xi^3 D(\xi) J_1(\xi r/a) d\xi = 0; \quad a < r < \infty$$

where

$$f(r) = \frac{Ga^3 \Omega r}{(1-\nu)} - \frac{Q_h a^3}{4\pi(1-\nu)} \left\{ \frac{rc}{(r^2+c^2)^{3/2}} - \frac{r(1-2\nu)}{(r^2+c^2)^{1/2}[c+\sqrt{(r^2+c^2)}]} \right\} \tag{13}$$

The solution of the dual system (12) can be easily achieved by making use of the generalized results given by Sneddon.¹⁹ Briefly, by making use of a substitution of the type $\xi^3 D(\xi) = \phi(\xi)$ and representing $\phi(\xi)$ in the form of the integral

$$\phi(\xi) = \int_0^1 \chi(t) \sin(\xi t) dt \tag{14}$$

it can be shown that the second equation of the dual system (12) is identically satisfied and the first equation yields an Abel-type integral equation for $\chi(\xi)$. The solution of this latter integral

equation completes the formal analysis of the problem (see e.g. Sneddon²⁰); i.e.

$$D(\xi) = \frac{2}{\pi\xi^3} \int_0^a \left\{ \frac{1}{t} \frac{d}{dt} \int_0^t \frac{r^2 f(r) dr}{\sqrt{(t^2 - r^2)}} \right\} \sin(\xi t) dt \quad (15)$$

Formal integral expressions can now be obtained for the displacement and stress fields in the medium ($\mathbf{u}(r, \theta, z)$ and $\boldsymbol{\sigma}(r, \theta, z)$) in terms of the function $D(\xi)$ defined by (15).

The result of primary interest to this paper concerns the evaluation of the resultant rotation experienced by the rigid circular foundation due to the internal horizontal anchor force. To develop this result we make use of the expression for the moment M_x exerted on the rigid circular foundation by the contact stresses $\sigma_{zz}(r, \theta, 0)$. Since the foundation is subjected to zero external moment we require

$$M_x = \int_0^a \int_{-\pi}^{\pi} \frac{1}{a^4} \left\{ \int_0^{\infty} \xi^3 D(\xi) J_1(\xi r/a) d\xi \right\} r^2 \cos^2 \theta dr d\theta = 0 \quad (16)$$

Evaluating (16) we obtain the following expression for the rotation of the rigid circular foundation which interacts with the internal horizontal anchor force Q_h ;

$$\Omega = \frac{3Q_h}{8\pi G a^2} \left[2(1-\nu) \frac{c}{a} \tan^{-1} \left(\frac{a}{c} \right) - \frac{C}{(a^2 + c^2)} - (1-2\nu) \right] \quad (17)$$

Limiting cases

It may be noted that as $c \rightarrow \infty$, the rotation $\Omega \rightarrow 0$ and the horizontal anchor force have no influence on the foundation located at the surface of the elastic soil medium. As $c \rightarrow 0$, $\Omega = -3(1-2\nu)Q_h/8\pi G a^2$; this result, in effect, represents the somewhat artificial situation in which the circular area corresponding to the foundation outline is enforced to undergo a rigid body settlement corresponding to $u_z(r, \theta, 0) = -\Omega r \cos \theta$ but allowing unrestricted displacement in the radial direction. This situation is, of course, physically unattainable since the interface is assumed to be frictionless. It is of interest to note that in the event of complete adhesion between the rigid circular foundation and the elastic medium, the equivalent tilt experienced by a rigid circular foundation due to a horizontal force acting at the interface centre is given by³

$$\Omega_a = -\frac{3(1-2\nu)Q_h}{4\pi G a^2 \{4 + \sigma^2\}} \quad (18a)$$

where

$$\sigma = \frac{\ln(3-4\nu)}{\pi^2} \quad (18b)$$

Owing to the assumption of frictionless contact at the interface, the result (17) is applicable for those values of c/a which give compressive contact stresses everywhere in the region $r < a$. The contact stresses σ_{zz} should be evaluated to ascertain such limits. It can be shown that the contact stress distribution in the foundation region due to the horizontal anchor load $\sigma_{zz}(r, \theta, 0) (= \sigma_{zz}^h)$ is given by the integral expression

$$\sigma_{zz}^h = \frac{\cos \theta}{a^4} \int_0^{\infty} \xi^3 D(\xi) J_1(\xi r/a) d\xi \quad (19)$$

Evaluating (19) we obtain

$$\sigma_{zz}^h = \frac{Q_h r \cos \theta H(r, c)}{2\pi^2 a^2 (1-\nu) \sqrt{(a^2 - r^2)}} \quad (20a)$$

where

$$H(r, c) = 3 \left\{ 2(1-\nu) \frac{c}{a} \tan^{-1} \left(\frac{a}{c} \right) - \frac{c^2}{(a^2 + c^2)} - (1-2\nu) \right\} \\ + \frac{a^2}{(r^2 + c^2)} \left\{ 2(1-2\nu) [1 + \beta \tan^{-1} \beta] - \frac{c^2}{(r^2 + c^2)} \left[2 \frac{(r^2 + c^2)}{(a^2 + c^2)} + 3 \tan^{-1} \beta \right] \right\} \quad (20b)$$

and

$$\beta = \sqrt{\left[\frac{a^2 - r^2}{(r^2 + c^2)} \right]} \quad (20c)$$

RESULTS FOR AN INCLINED ANCHOR LOAD

The results presented in the preceding section for the horizontal anchor load can be combined with the results already available for the vertical anchor load (Selvadurai¹⁷) to produce a generalized solution for the problem wherein the combined interaction between the externally loaded foundation and the elastic halfspace takes place in the presence of an inclined anchor load. We consider the problem in which a rigid circular foundation resting in smooth contact with an isotropic elastic halfspace is subjected to an external axisymmetric load P . The halfspace region is subjected to a concentrated anchor load Q which acts at a distance c from the plane boundary and is inclined at an angle ζ to the z -axis. The point of action of the anchor load is situated on the z -axis (see Figure 4). We assume that the magnitude of (c/a) is such that no tensile tractions are developed at the interface due to the combined interaction of P and Q . The settlements within the rigid foundation region $r \leq a$ are given by the expression

$$u_z(r, \theta, 0) = w_0 - \Omega r \cos \theta \quad (21)$$

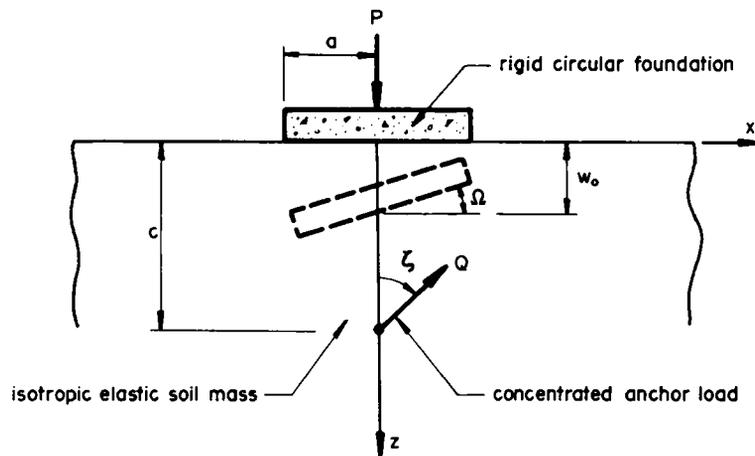


Figure 4. Combined interaction between a loaded rigid circular foundation and an inclined anchor load

where

$$w_0 = \frac{P(1-\nu)}{4aG} - \frac{Q(1-\nu) \cos \zeta}{4aG} \left[\frac{2 \tan^{-1} \left(\frac{a}{c} \right)}{\pi} + \frac{ac}{\pi(1-\nu)(a^2+c^2)} \right] \quad (22a)$$

$$\Omega = \frac{3Q \sin \zeta}{8\pi G a^2} \left[2(1-\nu) \frac{c \tan^{-1} \left(\frac{a}{c} \right)}{a} - \frac{c^2}{(a^2+c^2)} - (1-2\nu) \right] \quad (22b)$$

The results given here are applicable to the situation where a single anchor force acts at the interior of the soil mass. The analysis can, however, be easily extended to the problem of a series of individual anchors or a distribution of anchor loads located along the z -axis by a superposition of the solution (21). The normal contact stress at a foundation-elastic medium interface is given by

$$\sigma_{zz}(r, \theta, 0) = \frac{P}{2\pi a \sqrt{(a^2-r^2)}} + \frac{Q \cos \zeta V(r, c)}{2\pi a \sqrt{(a^2-r^2)}} + \frac{Qr \sin \zeta \cos \theta H(r, c)}{2\pi^2 a^2 \sqrt{(a^2-r^2)}} \quad (23)$$

where

$$V(r, c) = - \left\{ 2 \tan^{-1} \left(\frac{a}{c} \right) + \frac{ac}{(1-\nu)(a^2+c^2)} \right\} + \frac{ac}{(1-\nu)(r^2+c^2)^2} \\ \times \left\{ (1-2\nu)(r^2+c^2) + c^2 + \frac{(2a^2+c^2-r^2)}{(a^2+c^2)} + [(1-2\nu)(r^2+c^2) + 3c^2] \beta \tan^{-1} \beta \right\} \quad (24)$$

The expression (23) for the contact stress has to be evaluated for specific choices of P , Q , ζ , ν and (c/a) to ensure that the stress distribution everywhere in the region $r \leq a$ is comprehensive. In the particular instance when $P = Q (= Q_h)$ and $\zeta = \pi/2$ (i.e. a horizontal anchor load) it is found that the contact stresses remain compressive for all values of $(c/a) > 0.8$ for every choice of ν . This represents a situation in which the anchor force is at very close proximity to the foundation. Furthermore, since P is usually larger than Q we may assume that the development of tensile tractions at the interface is inhibited when the anchor force is located at a depth greater than the radius of the foundation.

CONCLUSIONS

The problem of interaction between a rigid circular foundation resting on the surface of a soil medium and an inclined anchor load acting within the soil mass has been investigated in the context of the linear theory of elasticity. This particular problem is of some interest to the study of soil and rock anchors used to support structures such as sheet piled walls and other earth retaining bulkheads. In particular the theoretical solutions provide some estimate of the settlements such anchor loads may induce on existing structures. The rotational tilt of the circular foundation can be evaluated in explicit form. Figure 5 illustrates the manner in which the rotational tilt of the circular foundation is influenced by the depth of location of the horizontal anchor force. It would appear that the rotational settlement becomes practically insignificant for $c/a > 10$.

An interpretation of the general results of this paper can be made by recourse to a numerical example. We consider a rigid foundation 8 m square (which is approximated by a circular rigid foundation of radius 4 m) which rests on a saturated soil medium with $G = 10^4$ kN/m²; $\nu = 1/2$. The rigid foundation is subjected to an axial load $P = 1000$ kN. Horizontal localized anchor loads $Q_h = 500$ kN act at depths 4 m, 8 m and 12 m from the foundation. The undrained elastic

settlement of the rigid foundation due to the axisymmetric load P is 3.125 mm. Owing to the internal anchor loads the foundation experiences a tilt; this tilt is such that the maximum and minimum settlements (which occur at $\theta = 0$ and $\theta = \pi$) are approximately 3.875 mm and 2.375 mm respectively. The analytical treatment of the rigid foundation-anchor load interaction makes several simplifying assumptions (such as the representation of the soil mass as an elastic halfspace; and the idealization of the anchor region as a localized load located along the axis) to render the problem mathematically tractable. The analytical procedure can be further developed to minimize these simplifying assumptions.

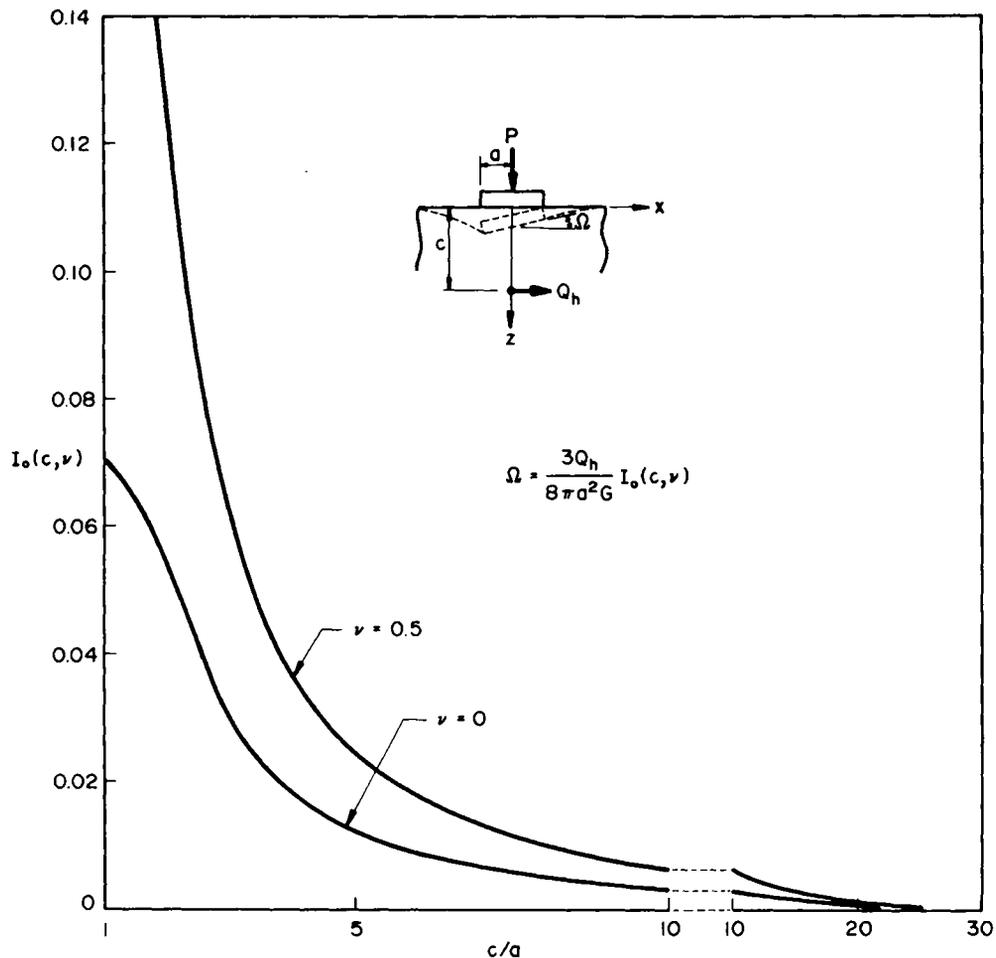


Figure 5. Influence factor for the rotation of the surface foundation due to a horizontal anchor load

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