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A Statistical Correlation Between Permeability, Porosity, Tortuosity and Conductance

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Abstract A statistical evaluation of 13,000 numerical simulations of random porous structures is used to establish a correlation between permeability, porosity, tortuosity and conductance. The random structures are generated with variable porosities, and parameters such as the permeability and the tortuosity are determined directly from the structures. It is shown that the prevalent definition of tortuosity, as the ratio of length of the real flow path to the projected path in the overall flow direction, does not correlate with permeability in the general case. Also, the correlation between the conductance of the medium, as an indicator of the accessible cross section of a flow path and permeability is no more reliable than the permeability–porosity correlation. However, if the definition of tortuosity is corrected using the cross-sectional variations, the resulting parameter (i.e., the minimum-corrected tortuosity) has a reliable correlation with permeability and can be used to estimate permeability with an acceptable error for most of the simulations of the random porous structures. The feasibility of extending the conclusions from 2-dimensional to 3-dimensional configurations and the numerical percolation thresholds for random structures are also discussed.

Keywords Permeability · Porosity · Tortuosity · Medium conductance · Statistical analysis

1 Introduction

Permeability is a key parameter for examining transport phenomena in porous media encountered in various scientific and engineering applications. Permeability (and diffusivity) is normally considered to be a function of porosity (Nield and Bejan 2006; Yazdchi et al.

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2011), while other parameters, tortuosity in particular (Hunt and Ewing 2009; Ghanbarian et al. 2013; Pisani 2016), are linked to the correlations to improve the level of accuracy (Koponen et al. 1997; Tamayol and Bahrami 2011; Zamel et al. 2010; Nam and Kaviany 2003; Das et al. 2010; Webb and Pruess 2003). The permeability–porosity correlation has been studied extensively in the literature (e.g., Adler and Jacquin 1987; Adler 1988; Lemaitre and Adler 1990) and the efficiency of using porosity to estimate permeability is often questioned (Adler et al. 1990). The tortuosity factor was first introduced by Carman (1937) for tortuous capillary tubes to correct the tortuosity effects on the driving force (e.g., pressure) and the velocity (Bear 1972; Bear and Cheng 2010; Ichikawa and Selvadurai 2012). Bear (1972) concluded that permeability, as an isotropic measure of fluid transport, depends on three properties: porosity, average medium conductance, and tortuosity. The medium conductance was related to the cross sections of the channels through which the flow takes place. Bear (1972) assumed that medium conductance and tortuosity parameters are uncorrelated. His analysis is in fact the basis for subsequent studies to clarify the tortuosity concept and for improving correlations with permeability. Despite the numerous research conducted, the permeability–porosity–tortuosity correlations are not supported by experiments (Ghanbarian et al. 2013). There is no consensus in relation to the definition of tortuosity in the literature. Tortuosity is interpreted as a value not measured directly from the porous structures but rather, back-calculated from the measured permeability values. These concerns have relegated tortuosity from a physical concept to an adjustable parameter in the permeability correlations (Ghanbarian et al. 2013). It is still not clear in the literature whether tortuosity is an intrinsic (geometric) parameter of the porous structure or whether it is dependent on the transport process itself. In other words, is it possible to characterize the flow in a specific porous structure by a process-independent measure of tortuosity (with or without including porosity)?

The above questions and concerns are addressed in this paper. A new approach for estimating the permeability of a general porous structure was recently presented (Rezaei Niya and Selvadurai 2017). In this approach, the pressure drop in a 2D passage in a general case was analytically estimated. The porous structure was then analyzed and its connection tree determined to show the various passages in the structure. Employing the analytical results, the ratio of pressure drop to flow rate for each passage was calculated. The flow rate distribution was computed such that mass conservation was preserved and the pressure drop in parallel passages became equal; as a result, fluid energy dissipation from input to output was independent of the fluid path. The permeability value was calculated accordingly. The procedures were successfully applied to the study of fluid flow in wormhole features created by CO_2 acidized fluid flow through carbonate rocks (Selvadurai et al. 2017).

Using this approach, the permeability of 13,000 numerical realizations of random structures are evaluated. The permeability–porosity–tortuosity correlations for different tortuosity definitions are examined and discussed. The conductance parameter presented by Bear (1972) is also examined in a statistical context. Since this parameter was defined merely as a mathematical parameter in Bear's analysis, different geometrical interpretations are presented and discussed here. Bear (1972) defined his parameter to indicate the cross sections of the passages; therefore, non-dimensionalized overall average of cross sections of the structure, average of the minimum cross sections of the rows, average of the maximum cross sections of the rows, and average of the hydraulic average of the cross sections of the rows are considered and studied. It is shown that the tortuosity value does not correlate with the permeability of the porous medium. Also, the conductance parameter cannot predict the permeability values any more accurately than the permeability–porosity correlations. However, if the cross-sectional vari-

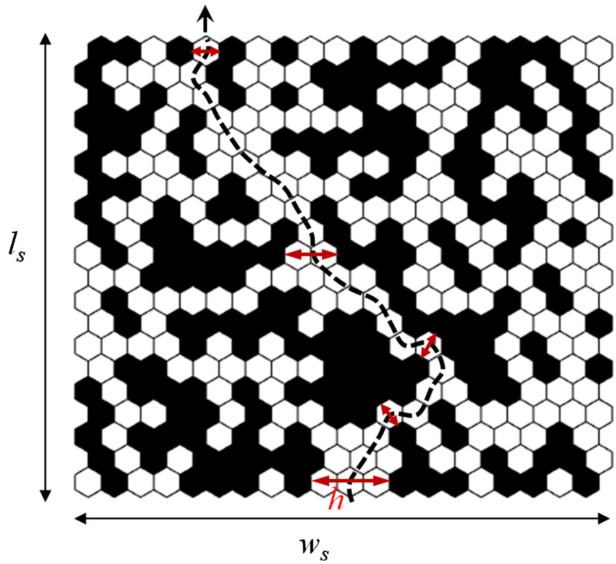

Fig. 1 A pictorial illustration of a random porous structure used in this study. The darker regions are the solid materials, and the lighter regions are the accessible pore spaces. The dashed line shows a typical flow path and the width change along the path

ations are included in the tortuosity definition, there is a positive correlation with permeability.

2 Numerical Analysis

In this study, 13 batches of random porous structures were constructed. A pictorial view of a typical random porous structure is shown in Fig. 1. Each batch contained 1000 random structures with a pre-determined average porosity. The porosity is defined by $\varepsilon = A_p/A_t$ where A_p and A_t are the pore space and total area, respectively. The average porosities for the batches ranged from $0.35, 0.4, \ldots, 0.95$. Each random structure is made up of a 20×20 hexagonal mesh. The physically inadmissible (blocked) samples were specified and replaced with new admissible random structures. The random structures were obtained using the MATLAB random generator algorithm. The details of the mesh and permeability estimation process, the accuracy of the method, and the verification of the results have been presented in a previous study (Rezaei Niya and Selvadurai 2017). The random structures and the solution process are 2-dimensional. In reality, the solid material should form a continuous fabric; in the case of 3-dimensional configurations, this is easily achieved since accessible flow paths can exist even if any simple cross section indicates the absence of a continuous 2-dimensional flow path. An example is the hexagonal packing of a set of uniform spheres. In this case, any two-dimensional section will indicate the absence of a continuous flow path, whereas in three dimensions the porous medium will contain accessible flow paths. As a result, each 2D structure has to be considered as equivalent (and not simply the cross section) of a realistic 3D structure. This equivalency and the reliability of extending the conclusions to 3-dimensional structures are also discussed in this paper. Porosities lower than 0.3 were not

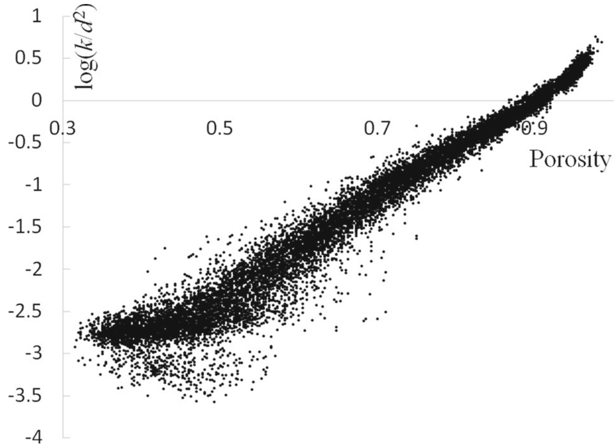

Fig. 2 The calculated permeability values for the random structures with different porosities

considered here as they reach the percolation threshold for this specific mesh size. This issue is discussed in subsequent sections. Since the connections tree for the porous structure needs to be determined as a part of the permeability estimation process (Rezaei Niya and Selvadurai 2017), tortuosity of the samples can be obtained directly from the porous structure.

3 Results and Discussions

The permeability estimation is based on a recently developed algorithm (Rezaei Niya and Selvadurai 2017). Using the Stokes' equation, it is shown that the pressure drop in a passage can be estimated to within an accuracy of 10%, from the average length of the streamlines in the passage, which is determined from the dimensions of the surrounding boundaries. The pressure drop in different passages is calculated as a function of the flow rate in the passage. Next, the flow rates are evaluated such that the overall pressure drop in the structure from all passages is equal. Using this approach, the permeability of a structure can be estimated with great computational efficiency compared to conventional methods (Bear and Cheng 2010) with errors less than 10% (Rezaei Niya and Selvadurai 2017).

Figure 2 shows the permeability–porosity correlation calculated for the random samples. The figure shows that for the same porosity, the permeability can vary up to two orders of magnitude. If the deviations of the permeability values from the average value at each porosity traced, it can be seen that these deviations are more significant in the lower porosity ranges and diminish at higher values. The results show that 19.1% of the samples lie outside of the $\log(k/d^2)\big|_{avg.} \pm 0.2$ span (37 to 58% deviation from the average value). In other words, permeability can at best be estimated from the porosity value with errors less than 58% with a confidence coefficient[1] of 19.1%. If the span is narrowed to $\log(k/d^2)\big|_{avg.} \pm 0.1$ (21 to 26% deviation from the average value), the confidence coefficient changes to 42.8%. (i.e.,

[1] Strictly speaking, the confidence coefficient definition has a subtle difference than the concept used here (Montgomery 2009). The confidence coefficient α for a parameter β is defined such that if, in repeated random samplings, a large number of confidence intervals are constructed, α percent of these intervals contain the β value. Here, the confidence coefficient is defined as the probability that a random selected sample lies inside the

Table 1 The confidence coefficients for the permeability–porosity correlation in different porosity ranges for the random structures

Porosity range	37–58% error $(\log(k/d^2)\big\|_{avg.} \pm 0.2)$	21–26% error $(\log(k/d^2)\big\|_{avg.} \pm 0.1)$
$\varepsilon \geq 0.9$	0.19 %	10.2 %
$0.8 \leq \varepsilon \leq 0.9$	1.49 %	17.3 %
$0.7 \leq \varepsilon \leq 0.8$	9.18 %	36.9 %
$0.6 \leq \varepsilon \leq 0.7$	24.8 %	56.4 %
$0.5 \leq \varepsilon \leq 0.6$	43.8 %	70.7 %
$\varepsilon \leq 0.5$	25 %	50.4 %

42.8% of the samples will have permeability values that are more than 21% different from the average value).

The calculated confidence coefficients for the results in different porosity ranges are presented in Table 1. The table shows that the error of the permeability–porosity correlation for porosities greater than 0.8 is less than 58% with a confidence coefficient of less than 1.5% (i.e., for more than 98.5% of the samples, the permeability values are less than 58% different from the average permeability for that porosity). Alternatively, the error is less than 26% with a confidence coefficient of less than 18% (i.e., for more than 82% of the samples). In conclusion, it can be stated that the permeability of a porous structure with porosities *greater than* 0.8 can be estimated with a *high* level of confidence with errors less than 58%. The table also shows that any permeability–porosity correlation for porosities *less than* 0.7 is unreliable and the estimation errors are more than 37% for at least 25% of the samples and more than 21% for at least 50% of the samples.

To study the permeability–tortuosity correlation, the term tortuosity needs to be defined in a quantitative way. Tortuosity is defined in the literature as (the square of) the ratio of average (minimum) length of the real flow paths (e.g., the length of the dashed line in Fig. 1, assuming that the flow enters from the bottom and exits from the top) to the length of the projected path in the overall flow direction (l_s in Fig. 1) (Ghanbarian et al. 2013). Here, all the possible paths from the *input* to the *output* in the random porous structures are determined and then the tortuosity is calculated based on the length of the minimal path ($\tau_{\min}$), the average length of all possible paths (τ_{avg}), and the length of the maximal path ($\tau_{\max}$). Figure 3 shows the correlation between permeability and average tortuosity (τ_{avg}) (the permeability correlation with minimal path and maximal path tortuosities are shown in the Supplementary Materials, Section SM.1). Figure 3 shows that there is no meaningful correlation between permeability and average tortuosity. It can also be shown (Supplementary Materials, Figure SM.2) that the simulations considered as outliers in the permeability–porosity correlation are distributed in a disorderly manner for all the different tortuosity values and, as a result, tortuosity cannot be used to improve the estimation of permeability. In other words, if tortuosity is defined in this way, the permeability–porosity–tortuosity correlations are not necessarily more accurate than the conventional permeability–porosity correlations. Also, tortuosity is not correlated with porosity (Supplementary Materials, Figure SM.3).

It can be claimed that the length of minimal path, the average length of all possible paths, and the length of the maximal path are not qualified representatives of the flow path length

Footnote 1 continued
confidence interval. This slight change in definition, however, does not affect the discussions and conclusions presented.

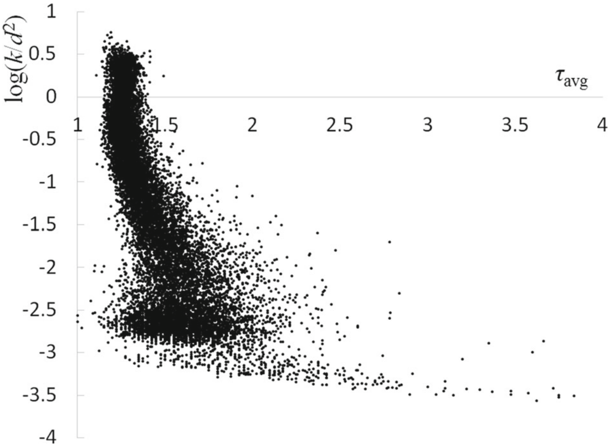

Fig. 3 The correlation between permeability and average tortuosity for the random structures

used in the tortuosity definition in the literature and the permeability–tortuosity correlation can be considerably improved if a proper definition is employed as the average flow path length. While the significance of the correlation has to be determined separately for each specific characteristic length employed in the tortuosity definition, the limitations of using such characteristic lengths (which are defined only based on the flow path length) to improve the permeability predictions can be explained at least for specific groups of porous structures. As an example, while the permeability of a structure drastically increases because of a wormhole creation (Selvadurai et al. 2017), the flow path length does not generally change significantly. Even in the very basic modeling of a porous structure as a bundle of capillary tubes, if same-length parallel tubes are replaced by an equivalent tube having the same total cross-sectional area, the porosity and tortuosity parameters remain unchanged, while the permeability of the structure changes considerably.

Bear (1972) predicted that the medium conductance, as an indicator of the cross sections of a channel, correlates with the permeability values. It was assumed that the viscous force per unit volume of fluid at a point inside a channel is proportional to the mass-averaged velocity of the particle at that point (Eq. (4.7.5) in Bear 1972). In Bear's approach, the conductance of the channel (with dimensions of L^2) was defined by this ratio. The conductance for a point at a distance r from the axis of a circular channel of radius r_0 was calculated as an example and was shown to be $(r_0^2 - r^2)/4$. The equation of conservation of linear momentum of the fluid system was averaged over the Representative Elementary Volume (REV) of the porous medium and the average equation of motion for an inhomogeneous fluid in laminar flow through a porous medium was determined. This equation included the average medium conductance.

In this study, various geometrical parameters are defined and calculated to model and analyze the medium conductance concept. Figure 4 shows the permeability–conductance correlation. In this figure, average conductance (B_{avg}) is employed, which is defined as the non-dimensionalized overall average of the cross-sections of the structure. Other defined parameters and their correlations can be found in the supplementary materials (Section SM.2). While Fig. 4 suggests that the permeability–conductance correlation can be significant for conductance values greater than 0.3, further analysis (Section SM.2) confirms that the accuracy and the confidence coefficient resulting from the conductance is not an improvement

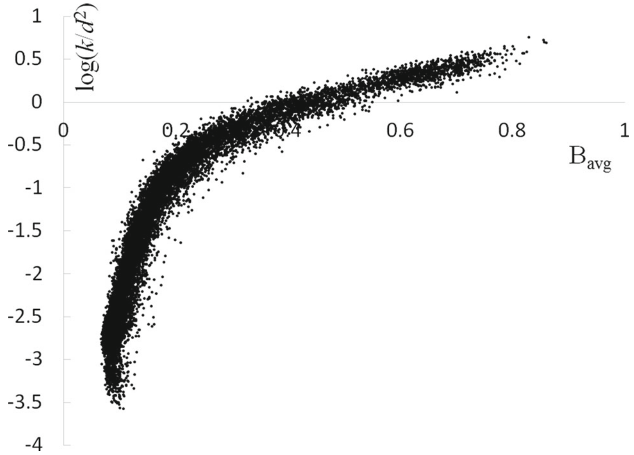

Fig. 4 The correlation between permeability and average conductance for the random structures

compared to the results previously extracted from the porosity analysis. In other words, the conductance cannot improve the permeability prediction correlations.

Bear (1972) also assumed that conductance and tortuosity are uncorrelated. If this assumption is not employed, then the cross-section effects have to be included in the definition of tortuosity. In this case, considering the correlation between the flow rate and the cross section in a 3D circular pipe and a 2D parallel channel (Selvadurai 2000; Rezaei Niya and Selvadurai 2017), the tortuosity definition could be modified as

$$\tau_{c,\mathrm{3D}} = \frac{1}{l_s} \int_{s=0}^{s=s_{\max}} \left(\frac{w_s}{D}\right)^4 \mathrm{d}s \tag{1}$$

$$\tau_{c,\mathrm{2D}} = \frac{1}{l_s} \int_{s=0}^{s=s_{\max}} \left(\frac{w_s}{h}\right)^3 \mathrm{d}s \tag{2}$$

where l_s, D, h, and $s_{\max}$ are the sample thickness (length of projected flow path), the diameter of the circular path in 3D, the width of the channel in 2D, and the length of the real path, respectively (Fig. 1). Here, w_s is a length scale of the sample used for non-dimensionalization and, in 2D, can be considered as the width of the sample (Fig. 1) and in 3D, as the square root of the cross-sectional area of the sample. Integration is performed along the length of each path (s). Considering the correlations between pressure drop and permeability, it can be shown (Rezaei Niya and Selvadurai 2017) that this definition results in an accurate permeability–tortuosity correlation, *if the input and the output of the porous structure are connected through a single path*. It can also result in an accurate correlation in a complex network if the integration is weighted in different sections of the path based on the ratio of the flow rates of the sections. Clearly, the flow rate is not equal in different sections of the path because of the interconnectivity of the structure.

In a complex structure, when the flow rate distribution is not known, this corrected tortuosity can be calculated for every path connecting the input to the output. Here, the minimum-corrected tortuosity is considered for further studies (the results for average, maximum, and parallel-paths-assumption corrected tortuosities are presented in the Supplementary Materials, Section SM.3). It is in fact equivalent to the assumption that all the fluid flows through the path of least resistance. Figure 5 shows the correlation between minimum-

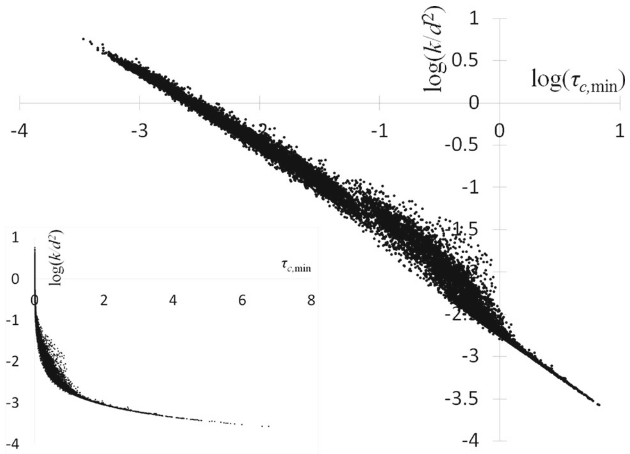

Fig. 5 The correlation between permeability and minimum-corrected tortuosity for the random structures (log–log graph). The semi-log graph is shown in the left-hand corner

corrected tortuosity and permeability and indicates that the correlation has an acceptable accuracy in the entire range of permeabilities; i.e., using minimum-corrected tortuosity, permeability can be estimated with an error margin of 58% with the confidence coefficient of 5.12% (compared to 19.1% for the porosity correlation). Alternatively, $\log(k/d^2)\big|_{avg.} \pm 0.1$ (21 to 26% error in the permeability value) has a confidence coefficient of 19.9% (compared to 42.8% for the porosity correlation). The correlation is particularly reliable in high and low ranges of the tortuosity value. For minimum-corrected tortuosities greater than 1 ($\log(\tau_{c,\min}) > 0$), the error margin of 11–12% ($\log(k/d^2)\big|_{avg.} \pm 0.05$) has a confidence coefficient of 4.63%. In other words, permeability can be estimated from minimum-corrected tortuosity with errors less than 12% for more than 95% of the samples in this range. On the other hand, for $\log(\tau_{c,\min}) < -2$, an error margin of 21–26% ($\log(k/d^2)\big|_{avg.} \pm 0.1$) has the confidence coefficient of 2.36%. The correlation between permeability and minimum-corrected tortuosity can be expressed as

$$\log(k/d^2)_{2D} = \begin{cases} -0.864\log(\tau_{c,\min}) - 2.21\pm 0.1 \text{ (for 93.2\% of the samples)} & \tau_{c,\min} < -1.5 \\ 0.615(\log(\tau_{c,\min}))^4 + 1.66(\log(\tau_{c,\min}))^3 + 0.878(\log(\tau_{c,\min}))^2 - 1.53\log(\tau_{c,\min}) & \\ -2.72 \pm 0.2 \quad \text{(for 90.8\% of the samples)} & -1.5 < \tau_{c,\min} < 0 \\ -1.05\log(\tau_{c,\min}) - 2.72 \pm 0.05 \text{ (for 95.4\% of the samples)} & \tau_{c,\min} > 0 \end{cases} \tag{3}$$

The R-squared of the fitted graph to the average permeability values for the above correlations are 0.9992, 0.9986, and 0.9988, respectively. The outliers (Fig. 5) are traced in a porosity–permeability graph to investigate whether the above correlations could be improved using the porosity values. As no meaningful trends were captured in this study (Supplementary Materials, Section SM.4), it was concluded that this correlation cannot be improved and the permeability–tortuosity–porosity correlation is not necessarily more accurate than the permeability–tortuosity correlation.

Several items should be discussed and emphasized here. First, the solution process and the samples used are 2 dimensional, whereas real porous materials are mostly 3D structures. However, the connections tree of each structure was determined during the solution process (Rezaei Niya and Selvadurai 2017). The connections trees are in fact dimensionless. In other words, when a connections tree is obtained (in which the loss of each branch is determined), it is not possible to distinguish whether this connections tree is derived from a 2D or a 3D structure. In fact, each 2D structure corresponds to a 3D structure.[2] As a result, it is expected that the discussions and the conclusions resulting from 2D studies are applicable to 3D structures, specifically when the statistical analysis of the random structures are involved. However, 3D structures are expected to be topologically more complex and diverse than 2D structures; therefore, it is prudent to analyze more complex 2D structures when the conclusions can be extended to 3D structures. Based on this hypothesis, if the number of paths from input to the output in each structure is determined (Figure SM.8) and only structures with more than 10,000 paths are considered, the correlation between permeability and corrected tortuosity becomes considerably more accurate as the confidence coefficients for $\log(k/d^2) = f(\log(\tau_{c,\min})) \pm 0.2$ and $\log(k/d^2) = f(\log(\tau_{c,\min})) \pm 0.1$ estimations improve to 0.73% and 8.2%, respectively (compared to 5.12 and 19.9% for the case when all the samples were analyzed). In this case, the confidence coefficients for the permeability–porosity correlation are also improved to 4.0 and 23.4% (compared to 19.1 and 42.8%). However, the accuracy level of this extension of 2D permeability–porosity correlation to 3D structures can be affected by another issue that is not covered here: While each 2D structure can correspond to a 3D structure through the connections tree, there are no assurances that these structures have the same porosities. In other words, a 2D structure can correspond to a 3D structure with a different porosity value and as a result, the expansion of the porosity-related conclusions of 2D studies to 3D structures requires further attention.

As discussed before and is seen in Fig. 2, this study covers random structures with porosities greater than 0.3. While the percolation thresholds for 2D (20 × 20 mesh) and 3D (20 × 20 × 20 mesh) structures are 0.05 and 0.0025, the random structures are mostly inaccessible for porosities less than 0.4 and 0.23 for 2D and 3D structures, respectively. Figure 6 shows the ratio of the percentage of the admissible structures (admissibility percentage) with different porosities for both 2D and 3D cases. In each case, the ratio is calculated through the consideration of 10,000 random structures. The results suggest that natural porous media with porosities lower than these numerical percolation thresholds (i.e., 0.23 for 3D structures, e.g., sandstone or limestone (Ghanbarian et al. 2013) cannot be studied as 'random structures'. Figure 6 also suggests that the permeability percentage increment is considerably sharper for the 3D case, which again indicates that 3D structures can be more diverse than their 2D counterparts and the more complex the 2D structures, the better they represent 3D structures.

It should also be mentioned that while minimum-corrected tortuosity has a reliable correlation with permeability and can be used to estimate the permeability of a structure, there is no direct method to measure or estimate this parameter [even when the network of the structure is fully determined (Supplementary Materials, Section SM.5)] and further studies are required in this area. However, the permeability–tortuosity correlation can lead to the following conclusions:

[2] More precisely, each group of 2D structures having the same connections tree corresponds to a group of 3D structures with the same connections tree.

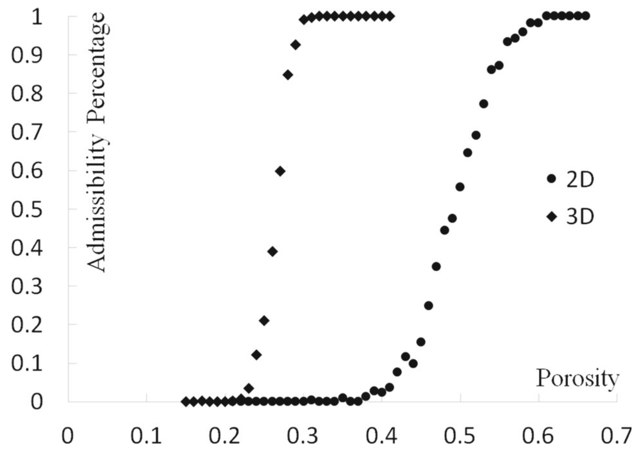

Fig. 6 The admissibility percentage in different porosities for random 2D and 3D structures

- There is an intrinsic, geometric, and process-independent parameter (minimum-corrected tortuosity) that can be used to characterize the flow process in a specific porous structure with an acceptable level of confidence.
- If a passage can be specified in a porous structure that has the highest overall cross-sectional area, permeability can be estimated with errors less than 58 for 95% of the cases by assuming all the fluid flows through that single passage. If the structure is complex enough, the error would be less than 26% for 92% of the cases.
- Even if this dominant passage cannot be specified for a porous structure, the closest passage can be employed. However, the accuracy level in this case has to be modified accordingly.
- If the porous structure is reshaped or new passages (connected defects) are created, while the dominant passage is not affected, the permeability value remains within the 37–58% error span (regardless how much the porosity is changed).

Finally, it must be emphasized that the above conclusions are based on the assumptions of Stokes' flow and fully saturated (or constant saturation level) conditions (Rezaei Niya and Selvadurai 2017). If the saturation level of a porous medium changes, the network involved in the transport process will change and the 'active porous structure' needs to be redefined and studied. Also, the results might be affected by the 20×20 mesh size and bigger mesh sizes need to be studied to verify these conclusions.

4 Conclusions

The correlation between permeability and porosity, tortuosity and medium conductance was studied using 13,000 numerical simulations of flow processes in random structures. The outcomes of this statistical investigation can be summarized as follows:

- The permeability can be estimated reliably from the porosity only for porosities greater than 0.8. The permeability–porosity correlation becomes unreliable for porosities lower than 0.7. This specifically highlights the fact that correlations such as the Kozeny–Carman equation are unreliable for general cases, as mentioned previously in the literature (Nield and Bejan 2006; Hunt and Ewing 2009; Ichikawa and Selvadurai 2012)

- The prevalent tortuosity parameter (as the ratio of lengths of the real flow path to the projected flow path) does not have a meaningful correlation with permeability and cannot be employed to improve the accuracy of the permeability–porosity correlations.
- The average medium conductance, used as an indicator of the cross sections of the porous structures, correlates with permeability. However, permeability and porosity are correlated better than permeability and medium conductance.
- If the tortuosity definition is corrected and the cross-section variations are included, a new parameter (i.e., minimum-corrected tortuosity) can be obtained, which has a reliable correlation with permeability. This permeability–tortuosity correlation is considerably more reliable at high values (greater than 1) and low values (lower than 0.01) of the minimum-corrected tortuosity. This correlation cannot be improved by incorporating the porosity parameter.
- There is a single, geometrical, and process-independent parameter (i.e., minimum-corrected tortuosity) that can be used to reliably characterize the flow process in porous structures in the Stokes' flow regime.
- Assuming that the fluid is only flowing along a path of least-resistance (i.e., minimum energy dissipation) in the porous structures, the permeability can be estimated with errors less than 58% for more than 95% of the cases for the random structures in all porosities.

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References

Adler, P.M.: Fractal porous media III: transversal Stokes flow through random and Sierpinski carpets. Transp. Porous Med. **3**, 185–198 (1988)

Adler, P.M., Jacquin, C.G.: Fractal porous media I: longitudinal Stokes flow in random carpets. Transp. Porous Med. **2**, 553–569 (1987)

Adler, P.M., Jacquin, C.G., Quiblier, J.A.: Flow in simulated porous media. Int. J. Multiphase Flow **16**, 691–712 (1990)

Bear, J.: Dynamics of Fluids in Porous Media. Dover Publications, New York (1972)

Bear, J., Cheng, A.H.-D.: Modeling Groundwater Flow and Contaminant Transport. Springer, Dordrecht (2010)

Carman, P.C.: Fluid flow through granular beds. Trans. Inst. Chem. Eng. **15**, 150–166 (1937)

Das, P.K., Li, X., Liu, Z.S.: Effective transport coefficients in PEM fuel cell catalyst gas diffusion layers: beyond Bruggeman approximation. Appl. Energy **87**, 2785–2796 (2010)

Ghanbarian, B., Hunt, A.G., Ewing, R.P., Sahimi, M.: Tortuosity in porous media: a critical review. Soil Sci. Soc. Am. J. **77**, 1461–1477 (2013)

Hunt, A., Ewing, R.: Percolation theory for flow in porous media, Lect. Notes Phys. 771. Springer, Berlin (2009)

Ichikawa, Y., Selvadurai, A.P.S.: Transport Phenomena in Porous Media: Aspects of Micro/Macro Behaviour. Springer, London (2012)

Koponen, A., Kataja, M., Timonen, J.: Permeability and effective porosity of porous media. Phys. Rev. E **56**, 3319–3325 (1997)

Lemaitre, R., Adler, P.M.: Transport in fractals three-dimensional Stokes flow through random media and regular fractals. Transp. Porous Med. **5**, 325–340 (1990)

Montgomery, D.C.: Design and Analysis of Experiments, 7th edn. Wiley, New York (2009)

Nam, J.H., Kaviany, M.: Effective diffusivity and water-saturation distribution in single- and two-layer PEMFC diffusion medium. Int. J. Heat Mass Transf. **46**, 4595–4611 (2003)

Nield, D.A., Bejan, A.: Convection in Porous Media, 3rd edn. Springer, New York (2006)

Pisani, L.: A geometrical study of the tortuosity of anisotropic porous media. Transp. Porous Med. **114**, 201–211 (2016)

Rezaei Niya, S.M., Selvadurai, A.P.S.: The estimation of permeability of a porous medium with a generalized pore structure by geometry identification. Phys. Fluids **29**, 037101 (2017)

Selvadurai, A.P.S.: Partial Differential Equations in Mechanics, Vol. 2, The Biharmonic Equation Poisson's Equation. Springer, Berlin (2000)

Selvadurai, A.P.S., Couture, C.-B., Rezaei Niya, S.M.: Permeability of wormholes created by CO_2-acidized water flow through stressed carbonate rocks. Phys. Fluids **29**, 096604 (2017)

Tamayol, A., Bahrami, M.: Transverse permeability of fibrous porous media. Phys. Rev. E **83**, 046314 (2011)

Webb, S.W., Pruess, K.: The use of Fick's law for modeling trace gas diffusion in porous media. Transp. Porous Med. **51**, 327–341 (2003)

Yazdchi, K., Srivastava, S., Luding, S.: Microstructural effects on the permeability of periodic fibrous porous media. Int. J. Multiphase Flow **37**, 956–966 (2011)

Zamel, N., Astrath, N.G.C., Li, X., Shen, J., Zhou, J., Astrath, F.B.G., Wang, H., Liu, Z.-S.: Experimental measurements of effective diffusion coefficient of oxygen-nitrogen mixture in PEM fuel cell diffusion media. Chem. Eng. Sci. **65**, 931–937 (2010)