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ORIGINAL ARTICLE

Rainfall infiltration-induced groundwater table rise in an unsaturated porous medium

L. Z. Wu[1] · **A. P. S. Selvadurai**[2]

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Abstract Increased rainfall associated with climate change is becoming a common occurrence and can lead to a variety of geoenvironmental hazards. Infiltration during rainfall and deformations of a soil mass are coupled, and the water table can easily experience a rise during these events. A study of the coupling process is important to increase our knowledge concerning the impact of a water level rise on the environment. Based on the theory of seepage and flow in porous media, the theory of elasticity and the soil–water characteristic curve, a two-dimensional hydro-mechanical model for an unsaturated porous medium is developed and incorporated in the COMSOL Multiphysics® software. The numerical procedure is capable of considering the influences of the soil–water characteristic curve. The effects of varying the boundary conditions on the coupled unsaturated infiltration and deformation equations are investigated. The examples demonstrate that the coupling effects significantly influence the position of the groundwater levels. The rate of change in the water table is closely related to the coupled infiltration and deformation in an unsaturated porous medium.

✉ L. Z. Wu
cewulz@gmail.com

A. P. S. Selvadurai
patrick.selvadurai@mcgill.ca

1 State Key Laboratory of Geohazard Prevention and Geoenvironment Protection, Chengdu University of Technology, Chengdu 610059, Sichuan, People's Republic of China

2 Department of Civil Engineering and Applied Mechanics, McGill University, Montreal, QC H3A 0C3, Canada

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Introduction

Rainfall-induced infiltration in unsaturated porous media is pervasive in nature. Heavy rainfall under extreme weather conditions, largely attributed to the effects of climate change, is expected to produce increased variations to the infiltration characteristics and the level of the water table in low lying areas such as valleys and slopes (Schnellmann et al. 2010; Tsai and Wang 2011). Infiltration during rainfall alters the water uptake giving rise to skeletal deformations; i.e., there are deformations that can be observed during water infiltration with a resulting water table change. A rise in the water table in response to a rainfall event is a complex process influenced by several factors including permeability, the initial soil–water conditions, the position of the water table, evapo-transpiration, land cover and use, rainfall intensity, etc. (Mansuco et al. 2012; van Gaalen et al. 2013).

Infiltration and flow in saturated and unsaturated heterogeneous porous media have been examined using analytical solutions. (Srivastava and Yeh 1991; Basha and Selvadurai 1998; Basha 1999, 2000; Chen et al. 2001; Zhan and Ng 2004; Selvadurai and Selvadurai 2010, 2014; Huang and Wu 2012; Wu et al. 2012a, b). Srivastava and Yeh (1991) derived analytical solutions to the *one-dimensional rainfall infiltration* toward the water table through a homogeneous, layered two-soil system. Based on the analytical solution presented by Srivastava and Yeh (1991), Zhan and Ng (2004) investigated the effect of hydraulic parameters and rainfall conditions on water infiltration into unsaturated ground. However, in reality, rainfall infiltration causes soil structure variations. During water infiltration into unsaturated porous medium, the porosity in the porous medium changes with the level of saturation. At the same time, stress modification in the unsaturated porous medium leads to porosity changes. Deformation in the unsaturated porous medium leads to variations in the porosity, which influences the water flow in the unsaturated porous medium. The deformation-seepage approach is of significance because it is helpful to better understand the natural process of water infiltration into an unsaturated porous medium. Analytical solutions to 1D and 2D coupled unsaturated infiltration and deformation problems during rainfall have received increased attention in recent years (Wu and Zhang 2009; Wu et al. 2012a, b, 2013). Analytical solutions to transient coupled infiltration problems have been obtained using Fourier integral and Laplace transform techniques (Wu and Zhang 2009; Wu et al. 2012a, b, 2013).

Numerical techniques are increasingly being used to examine coupled seepage and deformation in unsaturated porous media. Numerical procedures are necessary because real-life problems of rainfall infiltration involve complicated geometries, soil heterogeneity and complex initial conditions and boundary conditions (Kim 2000; Oka et al. 2010; Garcia et al. 2011). Solutions to such practical situations can rarely be obtained using analytical approaches (Garcia et al. 2011). Many numerical techniques that consider the hydro-mechanical processes involved in water infiltration into unsaturated porous media have been presented in the literature in soil science and geotechnique. Recent advances and references to historical findings are given by Mansuco et al. (2012). As an example, Griffiths and Lu (2005) examined the slope stability in an unsaturated elasto-plastic medium due to rainfall infiltration, where the Bishop's effective stress concept for an unsaturated porous medium and a one-dimensional suction theory were employed. Ehlers et al. (2004) employed a coupled seepage-deformation technique to examine the deformation and the localization of strains in an unsaturated porous medium due to infiltration. Kim (2000) presented a fully coupled numerical model for the water table fluctuation and land deformation in a partially saturated soil due to surface loading. Cho and Lee (2001) employed the *net stress concept* in a seepage-deformation coupled approach to examine the development of instability in unsaturated soil slopes. Using an unsaturated coupled hydro-mechanical model, Alonso et al. (2003) computed the deformations and the variation of the safety factor with time for an unstable slope in a profile of weathered over-consolidated clay. Multiphase coupled elasto-viscoplastic finite element analysis formulations have also been used to numerically investigate the generation of pore-water pressure and deformations during rainfall seepage into a one-dimensional soil column (Oka et al. 2010; Garcia et al. 2011).

The objective of this paper is to examine, using a computational approach, the problem of rainfall-induced water table advance in an unsaturated soil mass that also considers the coupled effects of unsaturated seepage and soil deformation. The van Genuchten model (1980), which fails to obtain the analytical solution to the coupled infiltration and deformation in unsaturated porous medium, is considered in the analysis. The effective stress and constitutive model in unsaturated soils proposed by Lu et al. (2010) is adapted in the modeling. The numerical model developed is based on poro-mechanical governing equations incorporating coupled groundwater flow and deformation of the soil mass. Using two case studies, we discuss the effect of boundary conditions on the coupling, and parametric results are presented to illustrate the influence of the processes of water infiltration considering coupling effects.

Governing equations for the hydro-mechanical processes involved in water table change

To effectively discuss the coupled infiltration and deformation of an unsaturated porous medium, we make the following assumptions: (1) the soil medium is homogeneous and elastic; (2) the soil fabric is deformable, the grains are non-deformable and water is incompressible; (3) hysteresis of the soil–water characteristic curve is not considered; (4) the pore-air pressure within the porous medium is assumed to be constant.

Static equilibrium equations

The equation of equilibrium for a soil mass can be written as:

$$\sigma_{ij,j} + f_i = 0 \tag{1}$$

where σ_{ij} is the total stress tensor, and f_i is the body force vector.

For an unsaturated soil, Lu et al. (2010) proposed an effective stress for all levels of saturations by modifying the saturation contribution:

$$\sigma' = (\sigma - u_\mathrm{a}) - \sigma^s \tag{2}$$

where σ' is the effective stress, σ is the total stress, u_a is the pore-air pressure, and σ^s is defined with a general functional form of

$$\sigma^s = -(u_\mathrm{a} - u_\mathrm{w}) \quad (u_\mathrm{a} - u_\mathrm{w}) \geq 0 \tag{3a}$$

$$\sigma^s = -S_\mathrm{e}(u_\mathrm{a} - u_\mathrm{w}) \quad (u_\mathrm{a} - u_\mathrm{w}) < 0 \tag{3b}$$

where u_w is the pore-water pressure, and $S_\mathrm{e} = 1/1[1 + (ah)^n]^m$ $[1 + (ah)^n]^m$ is the effective saturation.

The constitutive equation for the soil skeleton is as follows (Davis and Selvadurai 1996; Selvadurai 2000b):

$$\varepsilon_{ij} = \frac{\sigma'_{ij}}{2G} - \frac{\lambda\delta_{ij}\sigma'_{kk}}{2G(3\lambda + 2G)} \tag{4}$$

where ε_{ij} is the strain tensor; $G = \mu = E/2(1 + v)$; $\lambda = Ev/(1 + v)(1 - 2v)$ in which G is the shear modulus (modulus of rigidity), v is Poisson's ratio of the soil, and E is the elastic modulus of the soil. Also μ and λ are Lamé's constants.

The linearized strain–displacement relations are

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{5}$$

in which u_i are displacement components.

Porous media have complex mechanical properties (Schofield and Wroth 1968; Desai and Siriwardane 1984; Davis and Selvadurai 2002; Selvadurai 2007; Pietruszczak 2010; Ichikawa and Selvadurai 2012; Selvadurai and Suvorov 2012, 2014) and the elastic properties of a porous medium vary with the degree of saturation. The assumption (1) which relates to the elasticity of the porous medium simplifies the analysis of the coupled problem.

We restrict our attention to the class of problems where the deformation corresponds to a state of plane strain. The volumetric strain of the soil mass $\varepsilon_v = \varepsilon_{xx} + \varepsilon_{zz}$, where ε_{xx} and ε_{zz} are the normal strains in the x- and z-directions, respectively. From the assumption (4), the pore-air pressure is a constant and, therefore, the partial derivative of the pore-air pressure with respect to time is zero. The pore-water pressure $u_w = \gamma_w\ h$, γ_w is the unit weight of water and h is the depth to the phreatic surface or the pressure head. The total soil stress is assumed to remain constant with time. Substituting Eqs. (2) and (4) into Eq. (5), the partial derivative of the volumetric strain can be obtained as:

$$\frac{\partial\varepsilon_v}{\partial t} = \frac{-2}{C}\frac{\partial(S_e u_w)}{\partial t} \tag{6}$$

in which ε_v is the volumetric strain and $C = E/E(1 + v)(1 - 2v).(1 + v)(1 - 2v)$.

van Genuchten model

The van Genuchten model (1980) relates the volumetric water content (θ) to the hydraulic head (h) as follows:

$$\theta = \theta_r + \frac{\theta_s - \theta_r}{[1 + (ah)^n]^m} \tag{7}$$

where θ_r and θ_s are, respectively, the residual moisture content and the moisture content (θ) at saturation; a and n are both arbitrary parameters; $m = (n - 1)/n$.

Based on the relationship between the volumetric water content and the degree of saturation, i.e., $S_r = \theta(h)/\theta_s$, and according to Eq. (7), we obtain

$$S_r(h) = \frac{\theta}{\theta_s} = \theta_r/\theta_s + \frac{1 - \theta_r/\theta_E}{[1 + (ah)^n]^m} \tag{8}$$

The unsaturated hydraulic conductivity can be expressed in the form (Mualem 1976)

$$k = k_s S_e^{1/2}[1 - (1 - S_e^{1/m})^m]^2 \tag{9}$$

where k_s is the coefficient of infiltration at saturation. In the numerical examples, we consider only isotropic hydraulic conductivity. Therefore, the unsaturated hydraulic conductivities in both x- and z-directions (both k_{xx} and k_{zz}) are the same in this analysis.

Seepage equations

In this study, the two-dimensional Richards' equation is employed, which, when combined with Darcy's Law for a transversely isotropic medium, gives

$$\frac{\partial}{\partial x}\left(k_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial z}\left[k_{zz}\left(1 + \frac{\partial h}{\partial z}\right)\right] = \frac{\partial\theta}{\partial t} \tag{10}$$

where θ is the volumetric water content; k_{xx}, k_{zz} are the coefficients of hydraulic conductivity in the x- and z-direction, respectively, and t is the duration of rainfall. Implicit in Eq. (7) is the assumption of transverse isotropy of the porous medium, where the Cartesian coordinate directions coincide with the principal directions of permeability. The measurement of hydraulic conductivity properties of soils and rocks can employ both *steady state tests* (Selvadurai 2003, 2004, 2010, 2014; Selvadurai and Selvadurai 2007; Selvadurai and Głowacki 2008) and *transient tests* (Selvadurai and Carnaffan 1997; Selvadurai and Jenner 2012; Selvadurai and Najari 2013; Najari and Selvadurai 2014; Selvadurai et al. 2005, 2011).

At first, the specification of the hydraulic properties of the porous medium as transversely isotropic and the elastic deformability characteristics of the porous medium as isotropic may appear to be somewhat incongruous. The rationale for the treatment of the hydraulic properties as transversely isotropic is due to the dominant influence the hydraulic properties can have on the infiltration and groundwater movement and the lesser influence the elastic transverse isotropy can exert on the moisture movement.

The change in the volumetric water content in an unsaturated porous medium is related to the normal stress or strain and the soil matric suction. Dakshanamurthy et al. (1984) proposed that the constitutive equation for an unsaturated soil can be expressed as:

$$\theta = \beta\varepsilon_v + \omega(u_a - u_w) \tag{11}$$

in which $(u_a - u_w)$ is the matric suction and the coefficients β and ω are used to express the volume change modulus

$$\beta = \frac{E}{H(1-2v)} \tag{12a}$$

$$\omega = \frac{1}{R} - \frac{3\beta}{H} \tag{12b}$$

where H is the *elastic modulus* for the water phase with respect to the effective stress, R is the *elastic modulus* for the water phase with respect to matric suction $(u_a - u_w)$ (Dakshanamurthy et al. 1984), and $1/R$ is the envelope of the soil–water characteristic curve.

Assuming that H is a constant, and that R is a variable, we obtain from the van Genuchten model (1980).

$$\frac{1}{R} = \frac{\partial\theta}{\partial h}\frac{\partial h}{\partial\psi} = \left(-\frac{1}{\rho_w g}\right)(\theta_s - \theta_r)m\alpha n S_e^{1+1/m}(-\alpha h)^{n-1} \tag{13}$$

We now take the partial derivative of both sides in Eq. (8) with respect to time t, and substitute the result into Eq. (6), which yields the following expression:

$$\frac{\partial\theta}{\partial t} = -\rho_w g\left\{\frac{2\beta_0}{C}\left[S_r + h\frac{\partial S_r}{\partial h}\right] + \omega_0 + h\frac{\partial(1/R)}{\partial h}\right\}\frac{\partial h}{\partial t} \tag{14}$$

Therefore, the right hand side of Eq. (14) is a function of the pressure head.

Considering a Cartesian coordinate system, and substituting Eq. (14) into Eq. (9), we obtain the equation governing the transient hydro-mechanical process in an unsaturated region as follows:

$$\begin{aligned}\frac{\partial}{\partial x}\left(k_x\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial z}\left[k_z\left(1+\frac{\partial h}{\partial z}\right)\right] \\ = -\rho_w g\left\{\frac{2\beta_0}{C}\left[S_r + h\frac{\partial S_r}{\partial h}\right] + \omega_0 + h\frac{\partial(1/R)}{\partial h}\right\}\frac{\partial h}{\partial t}\end{aligned} \tag{15}$$

If the porous soil mass is considered to be rigid, Eq. (15) reduces to the unsaturated flow equation without coupling between infiltration and deformation in the porous medium.

Computational modeling

The region shown in Fig. 1 was discretized by 348 triangular finite elements using an adaptive mesh in COMSOL Multiphysics®. For purposes of the computational simulations, the model dimensions are set to 6 m in length by 1 m in height; the maximum mesh size of the model is 0.25 m long. The length for a thin soil layer is usually set 10 m compared with 1-m height, and the calculated result is very close to that achieved with a 6-m length. Hence, the 6-m length is considered to be reasonable. The initial water table is located at the base of the soil layer. The numerical procedure is presented in detail as follows: The COMSOL Model Wizard is selected for a 2D configuration; The time-dependent model is chosen to analyze the coupled PDEs; the geometry of a rectangle domain, the size of which is 600 × 100, is set in the examples; each equation in the coupled system of PDEs is represented by one Physics added to the coupled model. We already have one physics as coefficient form PDE. The initial condition is a function of height. Since the COMSOL™ mesh generation is adaptive, stabilized finite element methods are necessary to obtain numerical solutions to the two-dimensional coupled water infiltration and deformation problem for unsaturated porous media. If the solution changes when you refine the mesh, the solution is mesh dependent, hence the coupled model requires a finer mesh. The adaptive mesh refinement, which adds mesh elements based on an error criterion to resolve those areas where the error is large, can improve the solution accuracy. The mesh refinement is automatically adjusted when the groundwater level changes in the unsaturated porous medium. Comprehensive accounts of the use of this software and its limitations are given by Selvadurai and Selvadurai (2010, 2014), Selvadurai and Suvorov (2012) and Selvadurai et al. (2014).

Examples of water infiltration

In both the examples discussed in this section, the hydro-mechanical processes that take place during infiltration are examined using plausible boundary conditions to the domain. We apply this model to a glaciated geological setting where soil layers are comparatively thin and confined by extended bedrock. The material parameters used in the calculations are listed in Tables 1 and 2. Table 1 shows the material parameters used in the calculations of the unsaturated porous medium while Table 2 describes the basic soil properties.

Example 1

We consider the two-dimensional problem of water infiltration into a rectangular homogeneous unsaturated soil mass of width l and height D (Fig. 2). For the solution of the transient infiltration problem governed by the PDE (12) for the rectangular domain, it is necessary to specify one initial condition and four boundary conditions (Ozisik 1989; Selvadurai 2000a).

In Fig. 2, the initial condition for the deformation problem is assumed to be zero; the only free boundary is at the ground surface, and the other three boundaries are

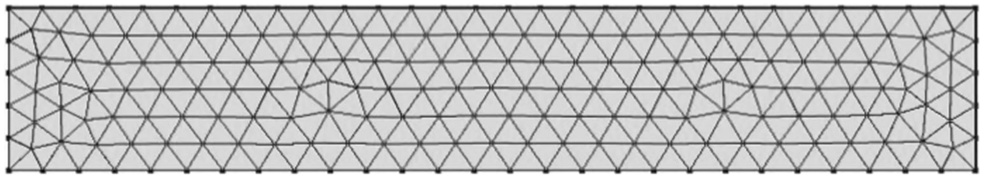

Fig. 1 Triangular finite element mesh of a rectangular domain for coupled infiltration and deformation in an unsaturated porous medium (maximum mesh dimension of the model is 0.25 m, and the initial water table is located at the *bottom* of the model)

Table 1 The material parameters used in the calculations involving an unsaturated porous medium

Saturated volumetric water content (θ_s)	Residual volumetric water content (θ_r)	Coefficient of permeability at saturation [k_s (m/s)]	Desaturation coefficient [α (m^{-1})]	Dimensionless rainfall intensity (q/k_s)
0.4	0.01	1×10^{-6}	0.01	0.9

Table 2 The basic soil properties of an unsaturated porous medium

Elastic modulus [E (MPa)]	Poisson's ratio (v)	Unit weight of soil mass [γ (kN/m^3)]	Unit weight of water [γ_ω (kN/m^3)]	Elastic modulus with respect to suction change [H (MPa)]
10	0.3	18.0	10.0	10

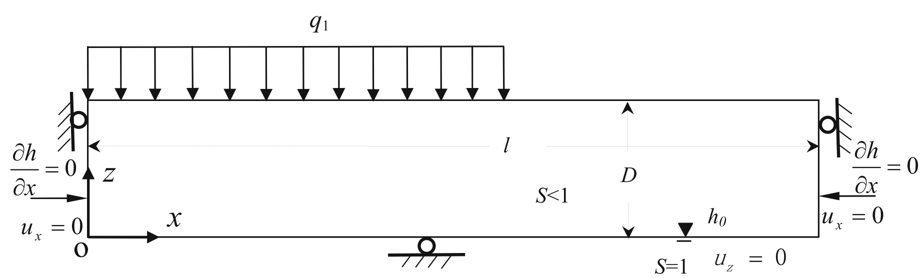

Fig. 2 Rainfall infiltration model in a two-dimensional finite region 6 m in width and 1 m in height

controlled by a fixed constraint. The initial pressure head distribution (i.e., the initial condition) is expressed as:

$$h(x, z, 0) = h_i \tag{16}$$

where h_i is the initial pressure head, which is a function of position. The water table is saturated at $x = 0$, i.e., $h(x, 0, 0) = 0$.

In this study, we consider only isotropic permeability. The boundary conditions consist of the lower and upper boundaries and the two sides of the domain, as shown in Fig. 2. At the lower boundary, which corresponds to the groundwater level, the hydraulic boundary condition is

$$\left[k(h)\frac{\partial h}{\partial z} + k(h)\right]_{z=0} = 0 \tag{17}$$

The lower boundary is usually located at the stationary groundwater table, where the pore-water pressure is 0, i.e., $h = 0$ at the start of infiltration.

In Fig. 2, the two side boundaries are subjected to a zero flux: i.e.

$$\left[k(h)\frac{\partial h}{\partial x}\right]_{x=0} = 0 \quad 0 < z < D \tag{18a}$$

$$\left[k(h)\frac{\partial h}{\partial x}\right]_{x=l} = 0 \quad 0 < z < D \tag{18b}$$

The upper boundary in Fig. 2 is subjected to a constant rainfall flux (q_1), which is specified as follows:

$$\left[k(h)\frac{\partial h}{\partial z} + k(h)\right]_{z=D} = q_1(x, t) \quad 0 < x < l/2 \tag{19}$$

Figure 3 illustrates the water table profiles with rainfall over time for both uncoupled and coupled conditions. In Fig. 3a, the coupling effect of deformations is ignored whereas the influence of the deformation on infiltration is accounted for in Fig. 3b. When dimensionless duration of rainfall t^* ($= t \times k_s/D$) is 5.364×10^{-3}, the water table under uncoupled conditions has reached the ground

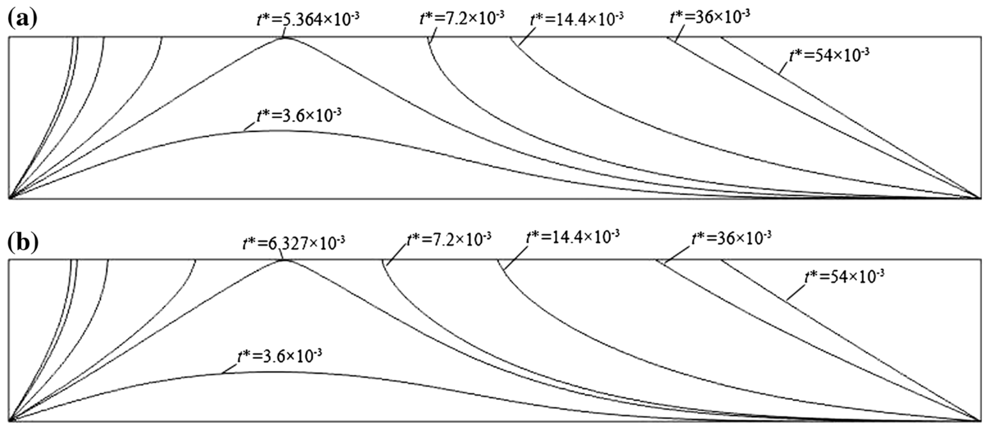

Fig. 3 The water table profile due to water infiltration in a finite region with time: **a** uncoupled conditions; **b** coupled conditions

surface in Fig. 3a. However, when coupling is accounted for, the water table reaches the ground surface at dimensionless duration of rainfall t^* ($=t \times k_s/D$) is 6.327×10^{-3} (Fig. 3b). From these results, we can conclude that the pressure head moves more slowly when the coupling effect is taken into consideration than for solutions where the coupling is ignored. In the latter stages of rainfall infiltration, the difference in the water table change between the uncoupled and coupled conditions is significant.

Figure 4 describes the change in the total deformation, which is achieved in the x- and z-directions with time when the hydro-mechanical coupling processes due to water infiltration are considered. The maximum displacement deformation occurs at the axis of symmetry of the porous domain. In Fig. 4, the deformation increases rapidly at the beginning of rainfall. After a prolonged duration, the deformation becomes stable, and little additional deformation is observed.

Example 2

We consider 2D infiltration through a homogeneous soil layer of thickness D. Figure 5 shows the problem where two-dimensional infiltration takes place through a soil layer of finite height D and finite extent b. In the computational simulation, the thickness D is taken as 1 m and b is 10 m to ensure $b/D = 10$ (assumed to approximately simulate a layer of infinite lateral extent.), and the width of the infiltration zone $2a$ is set to 2 m.

In Fig. 5, the initial condition for the deformation problem is zero, and the displacement boundary conditions are indicated, with zero vertical displacement prescribed at the base of the layer. The 2D transient infiltration problem involves a single initial condition and the infiltration boundary condition on the surface. The initial pore-water pressure profile in Example 2 is different from that in Example 1. The lower boundary is constrained by zero vertical displacement, where the pore-water pressure is $D/2$, i.e., $h(x, 0, 0) = D/2$. The water table is saturated at $z = 0$, i.e., $h(x, D/2, 0) = 0$.

Both sides of the unsaturated region shown in Fig. 5 extend to infinity, and the upper boundary is subjected to a rainfall flux (q_2) over a finite width when isotropic hydraulic conductivity is considered:

$$\left[k(h)\frac{\partial h}{\partial z} + k(h)\right]_{z=D} = q_2(x,t) \quad -a<x<a \tag{22a}$$

$$\left[k(h)\frac{\partial h}{\partial z} + k(h)\right]_{z=D} = 0 \quad x<-a \quad \text{or} \quad x>a \tag{22b}$$

In computational simulations, infinite domains can be modeled accurately provided the far-field behavior is accommodated through a suitable choice of infinite elements (see, e.g., Bettess 1977; Selvadurai and Gopal 1988). Boundary element modeling can also be used to model such far-field effects (Kangro and Nicolaides 2000; Yang and Hung 2001). In this study, the region is restricted to the finite domain, but the dimensions are selected to model an extended domain rather than an infinite domain. The extended domain is chosen with a region of width b in the horizontal direction and height D such that $b/D = 10$. The boundary conditions of the two vertical sides are controlled by zero flux.

Figure 6 shows changes in the water table over time in the infinite region for both uncoupled and coupled conditions. In the computational model, the height of the unsaturated region is $D = 1$ m, and the two sides are infinite in extent. The width in the horizontal direction of the rectangular domain is taken to be ten times greater than the height D, i.e., l (= 10 m), which can be regarded as an extended region, with D (=1 m). Since rainfall is concentrated in the central part of the upper surface of the domain (Fig. 5), the water table first rises along the center line of the z *axis*. The water table is always highest at the central axis (z *axis*) due to axial symmetry, i.e., the water table distribution is symmetric about the z *axis*. At

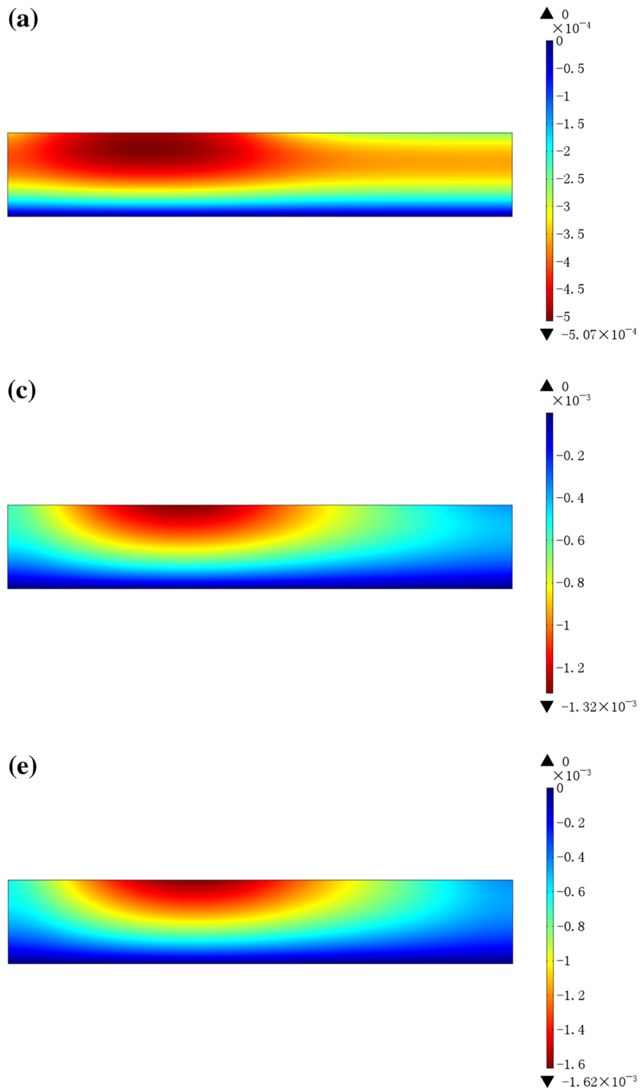

Fig. 4 The total displacement of the soil with time due to water infiltration in a finite region taking into account the hydro-mechanical processes: **a** $t^* = 3.6 \times 10^{-3}$; **b** 1.44×10^{-2}; **c** 2.88×10^{-2}; **d** 3.6×10^{-2}; **e** 5.4×10^{-2}

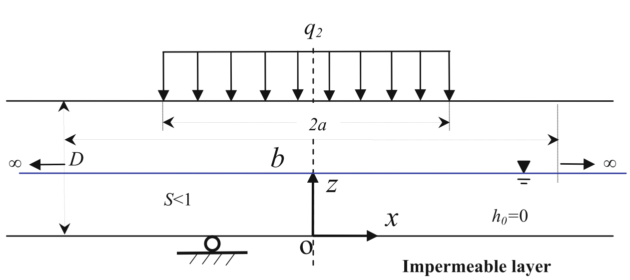

Fig. 5 Rainfall infiltration model in a two-dimensional infinite region

dimensionless duration of rainfall t^* ($t \times k_s/D$) $= 1.044 \times 10^{-3}$, the water table at the z *axis* under uncoupled conditions reaches the ground surface. However, when deformation and infiltration are coupled, the water table only reaches the ground surface along the z *axis* at dimensionless duration of rainfall $t^* = 1.38 \times 10^{-3}$. Whether the water table rises or not depends on the ratio of rainfall intensity to the saturated hydraulic conductivity, the duration of the rainfall, the initial conditions and the soil thickness. The coupling effect shows a smaller difference in Fig. 5 than in Fig. 3; this is related to the boundary conditions and region size. The two-dimensional area in Fig. 5 is bigger than that in Fig. 3. In comparison with unchanged water table at the bottom boundary in the 1D coupled problem (Wu and Zhang 2009), the 2D coupling effect shows a much smaller difference due to the finite extent of the rainfall region.

Figure 7 presents the profiles for the total soil displacement $u_z(x, z, t)$ with time due to water infiltration in an extended region where coupled water infiltration and deformation are considered and the porous medium is

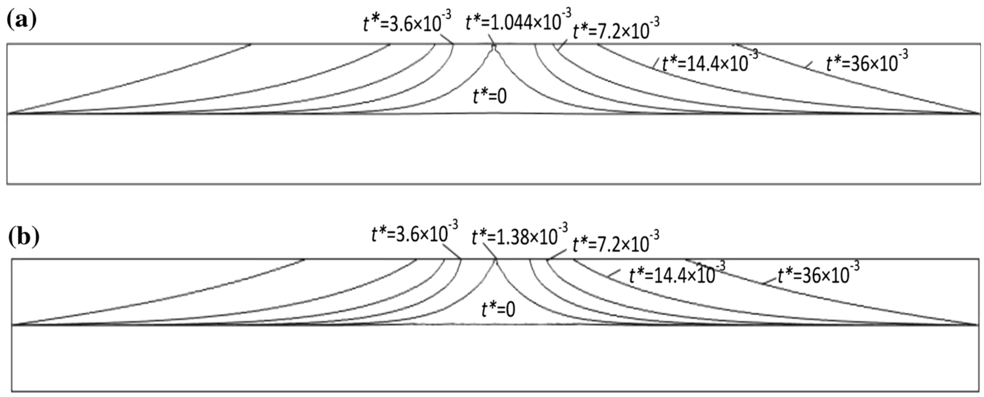

Fig. 6 The water table profile with time for an infinite region: **a** uncoupled conditions; **b** coupled conditions

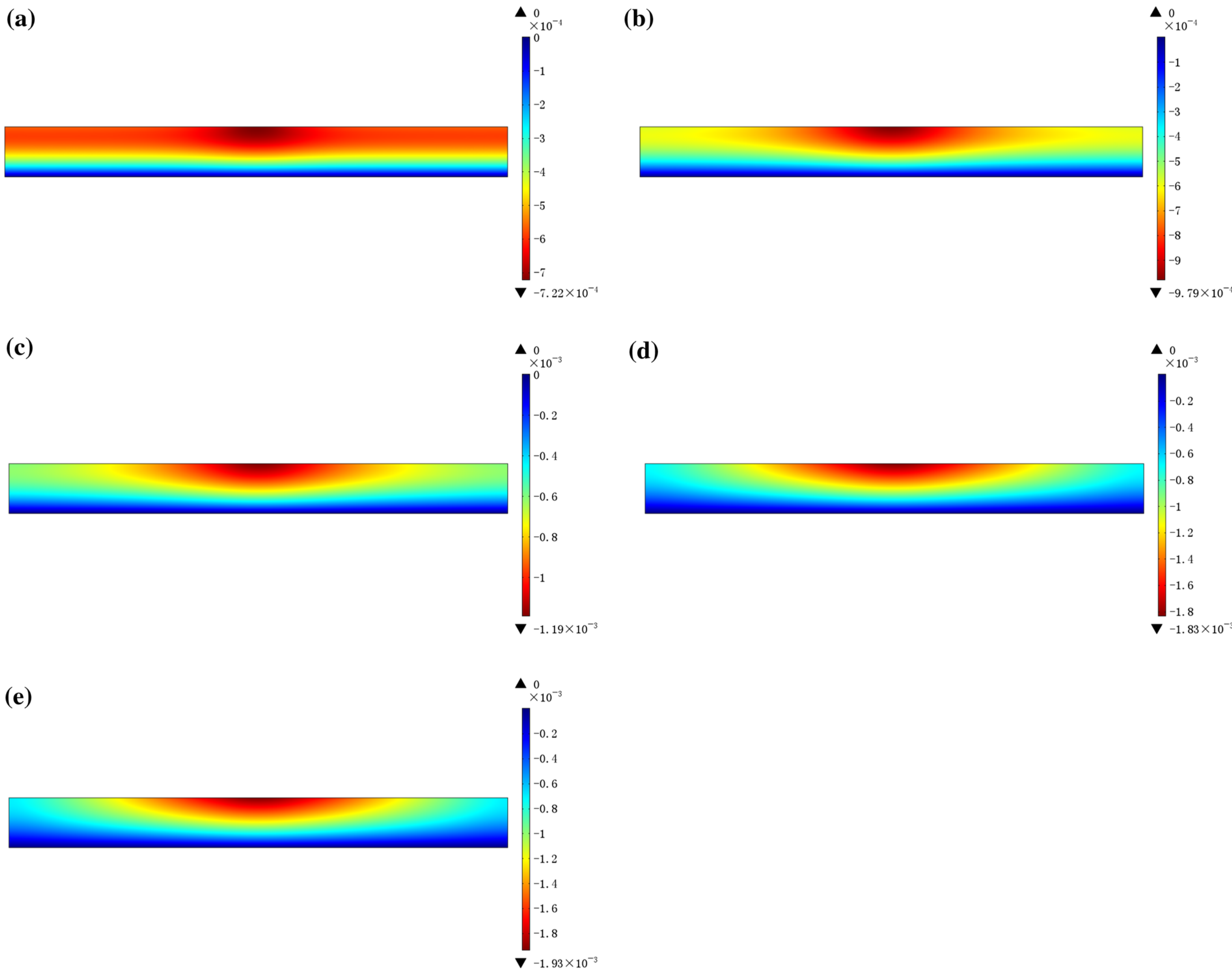

Fig. 7 The total displacement of the soil with time due to water infiltration in an infinite region taking into account the hydro-mechanical processes: **a** $t^* = 3.6 \times 10^{-3}$; **b** 1.8×10^{-2}; **c** 3.6×10^{-2}; **d** 1.8×10^{-1}; **e** 5.4×10^{-1}

unsaturated. At the early stage of rainfall, deformation occurs. For expansive soils, swelling occurs whereas collapse occurs for loess-type soils (Wu and Zhang 2009). The rate of the deformation gradually becomes slower with time and eventually stabilizes. Although rainfall continues, there is hardly any further change in the porosity of the soil domain. The maximum settlement occurs at the center of the ground surface. The maximum displacement $u_z(-x, z, t)$ increases with time, and becomes stable, as shown in Fig. 8.

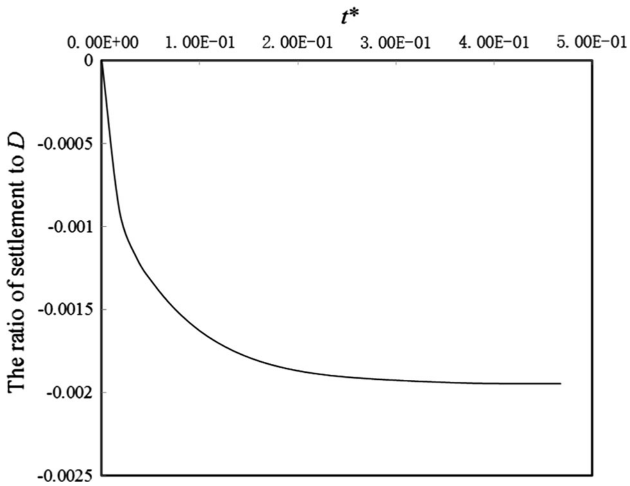

Fig. 8 Dimensionless displacement with time at the center point of the top surface of the infinite region ($z = 1$ m, $x = 0$)

Discussion and conclusions

Discussion

The 1D and 2D analytical solutions to coupled water infiltration and deformation in an unsaturated porous medium (Wu and Zhang 2009; Wu et al. 2013) usually employ exponential function approach proposed by Gardner (1958). However, the van Genuchten (1980) model that describes the soil–water characteristic curve (SWCC) is more suitable than the model proposed by Gardner (1958). The van Genuchten (1980) model, however, is difficult to apply to analytical methods that can be employed to solve coupled problems. In the numerical method, the van Genuchten (1980) model was employed to describe 1D coupled seepage and deformation in unsaturated soils based on poroelastic constitutive model (Kim 2000) and elasto-plastic one (Garcia et al. 2011). The numerical solution of 2D coupled seepage and deformation in unsaturated porous media are not encumbered by the form of the relationships describing SWCCs. The numerical solution can consider different rainfall-evaporation boundaries and initial conditions. The irreversible effects of soil plasticity that can contribute to permanent deformations and the influences of hysteresis arising from rainfall-evaporation cycles are not considered in the paper.

Conclusions

Based on the seepage theory for unsaturated porous medium, and the effective stress concept proposed by Lu et al. (2010) and the van Genuchten (1980) model of the soil–water characteristic curve, a two-dimensional coupled seepage and deformation model for an unsaturated porous medium are formulated to describe rainfall-induced water table changes. The effect of the type of boundary on the coupled unsaturated seepage and deformation equations is investigated. The examples presented in the paper demonstrate that the coupling between infiltration and soil deformation in an unsaturated porous medium significantly influences the position of the groundwater levels and water table alteration. The rate of rise in the water table is closely related to the coupled infiltration and deformation in an unsaturated porous medium. The coupling effect is related to boundary conditions and the extent of the domain. The 2D coupling effect shows a much smaller difference compared with the 1D coupled problem. By coupling the effects of deformation and water infiltration, we are able to gain a better understanding of rainfall-induced water table fluctuations and the attendant ground deformations.

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