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THE ECCENTRIC LOADING OF A RIGID CIRCULAR FOUNDATION EMBEDDED IN AN ISOTROPIC ELASTIC MEDIUM

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SUMMARY

This paper examines the problem of the eccentric loading of a rigid circular disc-shaped foundation embedded in bonded contact with an isotropic elastic medium of infinite extent. The solution of this problem is achieved by using a Hankel integral transform technique which reduces the problem to two sets of dual integral equations. These systems of dual integral equations represent the equations which govern the axisymmetric and asymmetric states of deformation induced by the loaded foundation. Explicit results are derived both for the displacement and rotation of the circular foundation and for the contact stress at the interface.

INTRODUCTION

The class of problems which investigate the behaviour of foundations embedded in soil media is of considerable interest to geotechnical engineering. In particular, these results find useful application in the geotechnical study of ground and rock anchors and in the *in-situ* testing of soils.[1–4] In these investigations pertaining to embedded foundations it is usually assumed that the loads are applied to the foundation in a symmetric fashion. In this paper we examine the problem of a rigid circular disc-shaped foundation deeply embedded in bonded contact with a homogeneous isotropic elastic medium of infinite extent and subjected to an eccentric load (Figure 1). The analytical treatment of this soil–foundation interaction problem is facilitated by treating the circular rigid foundation as a thin disc-shaped inclusion. The depth of location of the foundation is such that the presence of external boundaries does not influence its mechanical behaviour. It is further assumed that the soil medium remains adhered to the circular foundation, thereby preventing any separation or slippage at the soil–foundation interface. This assumption would seem consistent with the mechanical response of deeply embedded foundations. In the present investigation the soil medium is represented as a linearly deformable isotropic elastic solid; the analytical techniques could, however, be extended to include effects of transverse isotropy. The idealizations outlined above suggest that the results developed for the eccentrically loaded circular foundation problem would be of particular interest in examining the behaviour of embedded foundations supporting unevenly distributed or inclined anchor loads.[5]

The analytical study of disc-shaped rigid inclusions embedded in elastic media has received some degree of attention. The solution to the problem of a thin rigid circular penny-shaped inclusion embedded in an infinite isotropic elastic solid and subjected to a constant displacement normal to its plane was examined by Collins.[6] Keer[7] has considered a similar problem in which the bonded disc is displaced in its own plane. Kassir and Sih[8] subsequently extended these

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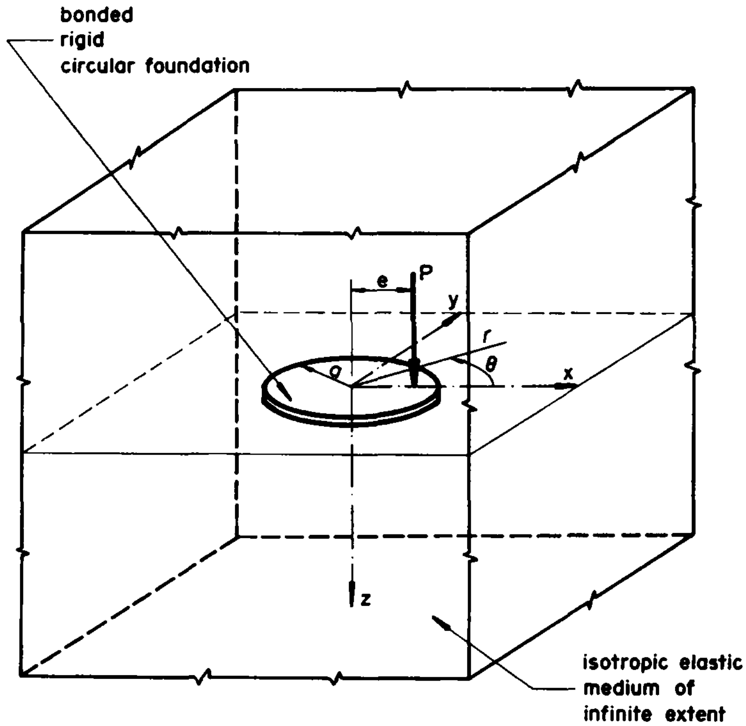

Figure 1. Geometry of the embedded disc foundation

investigations to problems relating to embedded elliptical disc inclusions. In all of the above investigations it is explicitly assumed that the inclusion remains bonded to the elastic medium at all times. The class of problems in which debonding, separation or slippage occurs at one or both of the inclusion faces is examined by Hunter and Gamblen[9] and Keer.[10] The solutions to loaded disc inclusion problems also occur as limiting cases for results developed for ellipsoidal and spheroidal rigid inclusion problems (Kanwal and Sharma,[11] Selvadurai[3]).

To examine the behaviour of the embedded rigid circular disc foundation subjected to an eccentric load, we make use of the antisymmetry of the problem which exists about the plane $z = 0$ (Figure 1). The bonded embedded disc foundation problem can thus be reduced to a mixed boundary value problem associated with a halfspace region. Using a Hankel transform formulation this mixed boundary value problem is reduced to two sets of dual integral equations. One set corresponds to the state of axisymmetric deformation of the embedded foundation induced by the applied load P. The second set corresponds to the state of asymmetry induced by the applied moment $M_0 = Pe$ (e = eccentricity of the load). The solution of these sets of dual integral equations is readily obtained from the generalized results given by Titchmarsh[12] and Sneddon.[13,14] Results of primary interest to geotechnical engineering, namely, the displacement and rotation of the eccentrically loaded circular foundation, are evaluated in exact closed form.

GOVERNING EQUATIONS

The problem related to the eccentrically loaded circular foundation can be represented as the superposition of solutions due to the effects of a central load P and a central moment $M_0(=Pe)$.

These problems constitute, respectively, axisymmetric and asymmetric states of deformation about the z-axis. The solution for the axisymmetric loading of the embedded foundation is already available in the literature cited earlier. These investigations employ analytical techniques based on complete potential function techniques,[6] singularity methods[11] and direct spheroidal harmonic function techniques.[3] In this paper we present a further method which employs a dual integral equation formulation. The solution to the problem of a disc foundation subjected to a central moment is obtained in an analogous manner.

In connection with the solution of both the axisymmetric and asymmetric problems outlined above it is convenient to employ a formulation based on the combination of the strain potential approach of Love[15] and its extension to asymmetric problems proposed by Muki.[16] It can be shown that these representations are specific reductions of the general method of analysis of the equations of classical elasticity in terms of the Boussinesq–Somigliana–Galerkin solution (see, e.g. Gurtin[17]). Proofs of the completeness of these representations are given by Truesdell[18] and Gurtin.[17] Also, the uniqueness of solutions derived from these potentials can be established by appeal to Kirchhoff's uniqueness theorem.[17]

Briefly, the solution of the displacement equations of equilibrium, in the absence of body forces, is representable in terms of biharmonic function $\Phi(r, \theta, z)$ and a harmonic function $\Psi(r, \theta, z)$; i.e.

$$\nabla^2\nabla^2\Phi(r, \theta, z) = 0 \qquad \nabla^2\Psi(r, \theta, z) = 0 \tag{1}$$

where $\nabla^4 = \nabla^2\nabla^2$ and

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \tag{2}$$

is Laplace's operator referred to the cylindrical polar coordinate system. The components of the displacement vector $\mathbf{u}$ and the Cauchy stress tensor $\boldsymbol{\sigma}$ referred to the cylindrical polar coordinate system (r, θ, z) can be expressed in terms of the derivatives of Φ and Ψ. We have

$$2Gu_r = -\frac{\partial^2\Phi}{\partial r\,\partial z} + \frac{2}{r}\frac{\partial\Psi}{\partial\theta} \tag{3a}$$

$$2Gu_\theta = -\frac{1}{r}\frac{\partial^2\Phi}{\partial\theta\,\partial z} - 2\frac{\partial\Psi}{\partial r} \tag{3b}$$

$$2Gu_z = 2(1-\nu)\nabla^2\Phi - \frac{\partial^2\Phi}{\partial z^2} \tag{3c}$$

where G and ν are the linear elastic shear modulus and Poisson's ratio of the material respectively. Similarly, the components of the stress tensor are given by

$$\sigma_{rr} = \frac{\partial}{\partial z}\left(\nu\nabla^2 - \frac{\partial^2}{\partial r^2}\right)\Phi + \frac{\partial}{\partial\theta}\left(\frac{2}{r}\frac{\partial}{\partial r} - \frac{2}{r^2}\right)\Psi \tag{4a}$$

$$\sigma_{\theta\theta} = \frac{\partial}{\partial z}\left(\nu\nabla^2 - \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right)\Phi - \frac{\partial}{\partial\theta}\left(\frac{2}{r}\frac{\partial}{\partial r} - \frac{2}{r^2}\right)\Psi \tag{4b}$$

$$\sigma_{zz} = \frac{\partial}{\partial z}\left[(2-\nu)\nabla^2 - \frac{\partial^2}{\partial z^2}\right]\Phi \tag{4c}$$

$$\sigma_{\theta z} = \frac{1}{r}\frac{\partial}{\partial\theta}\left[(1-\nu)\nabla^2 - \frac{\partial^2}{\partial z^2}\right]\Phi - \frac{\partial^2\Psi}{\partial r\,\partial z} \tag{4d}$$

$$\sigma_{rz} = \frac{\partial}{\partial r}\left[(1-\nu)\nabla^2 - \frac{\partial^2}{\partial z^2}\right]\Phi + \frac{1}{r}\frac{\partial^2 \Psi}{\partial \theta\, \partial z} \tag{4e}$$

$$\sigma_{r\theta} = \frac{1}{r}\frac{\partial^2}{\partial \theta\, \partial z}\left(\frac{1}{r} - \frac{\partial}{\partial r}\right)\Phi - \left(2\frac{\partial^2}{\partial r^2} - \frac{\partial^2}{\partial z^2}\right)\Psi \tag{4f}$$

It may be noted that for axial symmetry $\Phi = \Phi(r, z)$ and $\Psi = 0$; thus the results (3) and (4) for the displacements and stresses reduce to the representation in terms of Love's strain potential.

THE EMBEDDED FOUNDATION PROBLEM

We consider the embedded rigid circular foundation, of radius a, which is subjected to an eccentric load P acting at the location $(e, 0, 0)$ (Figure 1). Owing to the action of this eccentric load the disc foundation experiences a rigid body translation δ in the positive z-direction and a rigid body rotation Ω about the axis $\theta = \pi/2$. It is evident that the embedded foundation thus formulated is antisymmetric in the normal stress σ_{zz} and in the displacements u_r and u_θ, about the plane $z = 0$. Therefore we may restrict the analysis to a single halfspace region of the infinite elastic medium, in which the plane $z = 0$ is subjected to the following mixed boundary conditions:

(i) for the axisymmetric problem:

$$u_r(r, 0) = 0; \qquad r \geq 0 \tag{5a}$$

$$u_z(r, 0) = \delta; \qquad 0 \leq r \leq a \tag{5b}$$

$$\sigma_{zz}(r, 0) = 0; \qquad a < r < \infty \tag{5c}$$

(ii) for the asymmetric problem

$$u_r(r, 0) = u_\theta(r, 0) = 0; \qquad r \geq 0 \tag{6a}$$

$$u_z(r, 0) = \Omega r \cos\theta; \qquad 0 \leq r \leq a \tag{6b}$$

$$\sigma_{zz}(r, 0) = 0; \qquad a < r < \infty \tag{6c}$$

For the solution of the axisymmetric and asymmetric problems we introduce the nth-order Hankel transform of the function $\phi(r, \theta, z)$ as follows:

$$\bar{\phi}^n(\xi, \theta, z) = H_n\{\phi, (r, \theta, z); \xi\} = \int_0^\infty r\phi(r, \theta, z) J_n(\xi r/a)\, d\xi \tag{7a}$$

The appropriate Hankel inversion theorem is

$$\phi(r, \theta, z) = H_n^{-1}\{\bar{\phi}^n(\xi, \theta, z); r\} = \frac{1}{a^2}\int_0^\infty \xi\bar{\phi}^n(\xi, \theta, z) J_n(\xi r/a)\, d\xi \tag{7b}$$

THE AXISYMMETRIC PROBLEM

We restrict our attention to the halfspace region $z \geq 0$. operating on the first equation of (1) with the zero-order Hankel transform we obtain a fourth-order ordinary differential equation for the transform of the non-zero Love strain potential function $\Phi^0(\xi, z)$. Since the displacement and stress fields derived from this function should tend to zero as $r, z \to \infty$, the solution for $\Phi(r, z)$

appropriate for the region $z \geqslant 0$ takes the form

$$\Phi(r, z) = \frac{1}{a^2} \int_0^\infty \xi[A(\xi) + B(\xi)z] \exp(-\xi z/a) J_0(\xi r/a)\, d\xi \tag{8}$$

where $A(\xi)$ and $B(\xi)$ are arbitrary functions. (For axial symmetry $\Psi \equiv 0$). From (3a) and (8) it is evident that, in order to satisfy the boundary condition (5a), we require

$$B(\xi) = \frac{\xi}{a} A(\xi) \tag{9}$$

By making use of the above result and with the aid of the expressions for $u_z(r, z)$ and $\sigma_{zz}(r, z)$ given by (3c) and (4c) respectively, it can be shown that the boundary conditions (5b) and (5c) are equivalent to

$$\begin{aligned} -\frac{(3-4\nu)}{a^4} \int_0^\infty \xi^3 A(\xi) J_0(\xi r/a)\, d\xi &= 2G\delta; \qquad 0 \leqslant r \leqslant a \\ 2(1-\nu) \int_0^\infty \xi^4 A(\xi) J_0(\xi r/a)\, d\xi &= 0; \qquad a < r < \infty \end{aligned} \tag{10}$$

Introducing the substitutions

$$-\frac{2G\delta a^4}{(3-4\nu)} = \delta_0; \qquad \rho = \frac{r}{a}; \qquad \xi^3 A(\xi) = C(\xi) \tag{11}$$

the equations (10) can be reduced to the following pair of dual integral equations:

$$\begin{aligned} H_0[\xi^{-1} C(\xi); \rho] &= \delta_0; \qquad 0 \leqslant \rho \leqslant 1 \\ H_0[C(\xi); \rho] &= 0; \qquad 1 < \rho < \infty \end{aligned} \tag{12}$$

The solution of this system of dual integral equations is given by Sneddon[13,14] and the details of the method of solution will not be pursued here. The general solution of (12) is valid for the situation in which the prescribed displacement is an arbitrary function of r (say $\omega(\rho)$). This general solution takes the form

$$C(\xi) = \frac{2}{\pi} \left[\cos(\xi) \int_0^1 \frac{s\omega(s)\, ds}{\sqrt{(1-s^2)}} + \int_0^1 \frac{s\, ds}{\sqrt{(1-s^2)}} \int_0^1 \omega(st) \xi t \sin(\xi t)\, dt \right] \tag{13}$$

From this general result we obtain

$$C(\xi) = \frac{2\delta_0 \sin \xi}{\pi \xi} = \xi^3 A(\xi) \tag{14}$$

Formal integral expressions for the displacement and stress fields in the halfspace region $z \geqslant 0$ can be determined by making use of the equations (3), (4), (8) and (14).

The development presented here is primarily concerned with the evaluation of the load–displacement relationship for the embedded circular foundation. To this end we shall evaluate the normal stress distribution at the bonded interfaces $z = 0^+$ and $z = 0^-$. (The superscripts $(^+)$ and $(^-)$ refer to the location of the interface normal stresses with respect to the separate halfspace regions $z \geqslant 0$ and $z \leqslant 0$, respectively.) In view of the fact that the axisymmetric problem formulated above is antisymmetric in σ_{zz} about $z = 0$, we have $\sigma_{zz}(r, 0^+) = -\sigma_{zz}(r, 0^-)$ in $r \leqslant a$.

Considering the integral expression for σ_{zz} we have

$$\sigma_{zz}(r, 0^+) = -\frac{8G\delta(1-\nu)}{\pi(3-4\nu)a}\int_0^\infty \sin\xi J_0(\xi r/a)\,d\xi \tag{15}$$

From Erdelyi[19] we note that the integral expression (15) is equivalent to

$$\begin{aligned} \sigma_{zz}(r, 0^+) &= -\frac{8G\delta(1-\nu)}{\pi(3-4\nu)\sqrt{(a^2-r^2)}}; \qquad 0<r<a \\ \sigma_{zz}(r, 0^+) &= 0; \qquad a<r<\infty \end{aligned} \tag{16}$$

The total force exerted by the elastic medium on the embedded circular foundation is given by

$$P = 2\pi\int_0^a r[\sigma_{zz}(r, 0^+) - \sigma_{zz}(r, 0^-)]\,dr \tag{17}$$

Evaluating (17) we obtain the load-displacement relationship for the axisymmetrically loaded embedded rigid circular foundation. With the understanding that the displacement δ occurs in the direction of the applied force we obtain

$$P = \frac{32\delta Ga(1-\nu)}{(3-4\nu)} \tag{18}$$

This expression for the 'translational stiffness' of the embedded foundation is in agreement with the results obtained by Collins,[6] Kanwal and Sharma[11] and Selvadurai[3] by employing different analytical methods. It is of interest to note that when the elastic medium is incompressible ($\nu = \frac{1}{2}$), the translational stiffness of the disc foundation (i.e. $P/\delta Ga = 16$) is twice that obtained for the stiffness of an axially loaded rigid circular foundation resting on an isotropic incompressible elastic halfspace (i.e. Boussinesq's solution[20] gives $P/\delta Ga = 8$). This result is analogous to the similarity that exists between Kelvin's solutions for a concentrated force acting in an infinite space and Boussinesq's solution for a concentrated force acting at the surface of a halfspace, in the particular limit of material incompressibility. Also we note that the expression for the shear stress on $z = 0$

$$\sigma_{rz}(r, 0) = -\frac{(1-2\nu)}{a^4}\frac{\partial}{\partial r}\left[\int_0^\infty \xi^3 A(\xi) J_0(\xi r/a)\,d\xi\right] \tag{19}$$

vanishes for all r, when $\nu = \frac{1}{2}$. Therefore it may be concluded that *in the limit of material incompressibility*, the solution to the problem of an axisymmetrically loaded foundation (with a shape symmetric about $z = 0$) embedded in bonded contact with an isotropic elastic medium can be directly recovered from the solution of the appropriate problem related to an elastic halfspace (and *vice versa*). This result is of particular importance in geotechnical engineering since the initial elastic behaviour of saturated cohesive soils loaded in an undrained manner can be described by an incompressible elastic model.

THE ASYMMETRIC PROBLEM

We now examine the asymmetric problem related to an embedded disc foundation which is subjected to a central moment $M_0(=Pe)$. The resulting rigid rotation of the foundation about the axis $\theta = \pi/2$ is Ω. Since the problem is antisymmetric about $z = 0$ we shall restrict our attention to the region $z \geq 0$. Considering the form of the boundary conditions (6) we seek solutions of the differential equations (1) which admit representations

$$\Phi(r,\theta,z)=M(r,z)\cos\theta;\qquad \Psi(r,\theta,z)=N(r,z)\sin\theta \tag{20}$$

Using (16) in (1) we obtain two partial differential equations for the functions $M(r, z)$ and $N(r, z)$. By operating on these equations with the first-order Hankel transform we obtain a pair of ordinary differential equations for $\bar{M}^1(\xi, z)$ and $\bar{N}(\xi, z)$. The solutions of $M(r, z)$ and $N(r, z)$ appropriate for the region $z \geq 0$ are

$$M(r,z)=\frac{1}{a^2}\int_0^\infty \xi[D(\xi)+zE(\xi)]\exp(-\xi z/a)J_1(\xi r/a)\,d\xi \tag{21}$$

$$N(r,z)=\frac{1}{a^2}\int_0^\infty \xi F(\xi)\exp(-\xi z/a)J_1(\xi r/a)\,d\xi \tag{22}$$

where $D(\xi)$, $E(\xi)$ and $F(\xi)$ are arbitrary functions. The displacement components u_r and u_θ derived from these functions and (20) are given by

$$2Gu_r(r,0)=\frac{\cos\theta}{a^2}\left[\int_0^\infty \frac{\xi^2}{a}\left\{\frac{\xi}{a}D(\xi)-E(\xi)\right\}J_0(\xi r/a)\,d\xi - \int_0^\infty \xi\left\{\frac{\xi}{a}D(\xi)-E(\xi)-2F(\xi)\right\}\frac{J_1(\xi r/a)}{r}\,d\xi\right] \tag{23}$$

$$2Gu_\theta(r,0)=\frac{\sin\theta}{a^2}\left[-\int_0^\infty \frac{2\xi^2}{a}F(\xi)J_0(\xi r/a)\,d\xi-\int_0^\infty \xi\left\{\frac{\xi}{a}D(\xi)-E(\xi)-2F(\xi)\right\}\frac{J_1(\xi r/a)}{r}\,d\xi\right] \tag{24}$$

From (23) and (24) it is evident that the boundary conditions (6a) will be satisfied for all r and θ if we set

$$E(\xi)=\frac{\xi}{a}D(\xi);\qquad F(\xi)=0 \tag{25}$$

Using the above results and the expressions for u_z and σ_{zz} given by (3c) and (4c) respectively, it can be shown that the boundary conditions (6b) and (6c) are equivalent to

$$\begin{aligned}&\left\{-\frac{(3-4\nu)}{a^4}\int_0^\infty \xi^3 D(\xi)J_1(\xi r/a)\,d\xi\right\}\cos\theta=2G\Omega r\cos\theta; && 0\leq r\leq a\\ &\left\{\frac{2(1-\nu)}{a^5}\int_0^\infty \xi^4 D(\xi)J_1(\xi r/a)\,d\xi\right\}\cos\theta=0; && a<r<\infty\end{aligned} \tag{26}$$

Introducing the substitutions

$$-\frac{2G\Omega a^5}{(3-4\nu)}=\Omega_0;\qquad \rho=\frac{r}{a};\qquad \xi^3D(\xi)=L(\xi) \tag{27}$$

the equations (26) can be reduced to the following pair of dual integral equations:

$$\begin{aligned}&H_1[\xi^{-1}L(\xi);\rho]=\Omega_0\rho; && 0\leq\rho\leq 1\\ &H_1[L(\xi);\rho]=0; && 1<\rho<\infty\end{aligned} \tag{28}$$

The solution of this system of dual integral equations is given by Titchmarsh[12] and also by Copson.[21] The final form of the solution given by Titchmarsh occurs in a rather complicated form. Sneddon[22] subsequently presented a simpler result which was based on the reduction of the dual system (28) to an integral equation of the Abel type. This solution gives

$$L(\xi)=\frac{2}{\pi a}\int_0^a\left\{\frac{1}{t}\frac{\mathrm{d}}{\mathrm{d}t}\int_0^t\frac{r^2f(r)\,\mathrm{d}r}{\sqrt{(t^2-r^2)}}\right\}\sin(\xi t/a)\,\mathrm{d}t \tag{29}$$

where

$$f(r)=-\frac{2G\Omega a^4 r}{(3-4\nu)} \tag{30}$$

From (29) and (30) we obtain

$$D(\xi)=-\frac{8G\Omega a^5}{\pi\xi^5(3-4\nu)}\{\sin\xi-\xi\cos\xi\} \tag{31}$$

Again, formal integral expressions for the displacement and stress fields in the halfspace region $z\geqslant 0$ can be determined by making use of the relevant general results. In order to evaluate the moment–rotation characteristics for the embedded circular foundation we evaluate the normal stress at the bonded interface $\sigma_{zz}(r,\theta,0^+)$. We observe that from the antisymmetry of the deformation $\sigma_{zz}(r,\theta,0^+)=-\sigma_{zz}(r,\theta,0^-)$. The integral expression for $\sigma_{zz}(r,\theta,0^+)$ is

$$\sigma_{zz}(r,\theta,0^+)=-\frac{16G\Omega(1-\nu)\cos\theta}{\pi(3-4\nu)}\int_0^\infty\left[\frac{\sin\xi}{\xi}-\cos\xi\right]J_1(\xi r/a)\,\mathrm{d}\xi \tag{32}$$

Evaluating (32) we have

$$\begin{aligned}\sigma_{zz}(r,\theta,0^+)&=-\frac{16G\Omega(1-\nu)r\cos\theta}{\pi(3-4\nu)\sqrt{(a^2-r^2)}}; && 0<r<a\\ \sigma_{zz}(r,\theta,0^+)&=0; && a<r<\infty\end{aligned} \tag{33}$$

The moment exerted by the elastic medium on the embedded disc foundation is

$$M_0=\int_0^a\int_{\pi}^{\pi}[\sigma_{zz}(r,\theta,0^+)-\sigma_{zz}(r,\theta,0^-)]r^2\cos\theta\,\mathrm{d}r\,\mathrm{d}\theta \tag{34}$$

Evaluating (34) we have

$$M_0=-\frac{64G\Omega(1-\nu)a^3}{3(3-4\nu)} \tag{35}$$

It is of interest to note that when the elastic medium is incompressible, the rotational stiffness $(M_0/G\Omega a^3=32/3)$ of the disc foundation embedded in the infinite medium is twice that obtained for the rotational stiffness of a disc foundation bonded to an incompressible elastic halfspace. (This latter result is given by Florence.[23]) It would appear that the similarity argument proposed earlier for the axisymmetrically loaded embedded circular foundation can also be extended to the asymmetrically loaded case.

The results developed in this paper, which are of primary interest in geotechnical engineering, may be summarized in the following manner. The displacement of the embedded rigid circular foundation subjected to a concentrated load P with eccentricity e is given by

$$\frac{u_z(r,\theta,0)}{\{P(3-4\nu)/32Ga(1-\nu)\}}=\left\{1+\frac{3}{2}\left(\frac{e}{a}\right)\left(\frac{r}{a}\right)\cos\theta\right\} \tag{36}$$

The normal contact stress on the interface $z = 0^+$ is given by

$$\frac{\sigma_{zz}(r, \theta, 0^+)}{p_0 a^2} = \frac{\{1 + 3(e/a)(r/a)\cos\theta\}}{4\sqrt{[1-(r/a)^2]}} \tag{37}$$

where $p_0 = P/\pi a^2$ is an average stress. Also from the asymmetry of the problem $\sigma_{zz}(r, \theta, 0^+) = -\sigma_{zz}(r, \theta, 0^-)$.

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