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Technical Note

Torsion of a layered composite strip

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This paper examines the problem of the torsional loading of a composite strip composed of eleven layers of unidirectionally reinforced elements that are fully bonded. The shear moduli are obtained from the experimental torsional stiffnesses (T/ϕ) and data reduction techniques based on the orthotropic rectangular strip torsion model by Lekhnitskii. The stacking of laminae is also used to construct a computational model of the strip.

1. Introduction

The main objective of this paper is to examine the predictive capabilities of a computational approach for examining the torsional behaviour of a composite strip consisting of an assembly of bonded unidirectionally reinforced laminae. The computational approach uses the transverse isotropic effective elasticity properties of the unidirectionally reinforced composite determined from both computational modelling that incorporates the micromechanical arrangement of the fibre [1] and the use of effective estimates for the elasticity properties of a unidirectionally reinforced solid [2]. The results of the computational modelling of the composite strip are compared with results obtained from torsion experiments conducted on strip specimens obtained from a carbon fibre reinforced composite plate.

When a homogenous isotropic material is subjected to torsion, the state of stress is that of pure shear [3,4]. When heterogeneous materials are subjected to torsion, the state of stress at the microscale can be three-dimensional [5,6]. When the material properties are regularly ordered, such as in a composite laminate, the elastic behaviour can be modelled by appeal to a theory of orthotropic elasticity. A solution for the torsion of an orthotropic prismatic solid was presented by Lekhnitskii [5]. Fig. 1 shows an arrangement where a sample of a composite strip with a rectangular cross-section is subjected to a torque T, which induces a relative twist of ϕ radians over the sample length L. The cross-sectional dimensions of the sample are: a = width, and b = thickness. The torque versus angle of twist (T/ϕ) can be expressed as

$$\frac{T}{\phi} = G_{xy}\frac{ab^3}{L}\beta \qquad (1)$$

where β is given by

$$\beta = \frac{32a^2G_{yz}}{\pi^4 b^2 G_{xy}}\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n^4}\left[1-\frac{2a}{n\pi b}\sqrt{\frac{G_{yz}}{G_{xy}}}\tanh\left(\frac{n\pi b}{2a}\sqrt{\frac{G_{xy}}{G_{yz}}}\right)\right] \qquad (2)$$

and G_{xy} and G_{yz} are in-plane and out-of-plane shear moduli, respectively.

2. Strip lay-up

Two pairs of carbon fibre reinforced polymer (CFRP) strips, SC-0°, SC-90° (Fig. 1), with the same cross-sectional dimensions but with fibre orientations normal to each other were cut from the same plate. The tested specimens measured 200 mm × 25.4 mm × 2.4 mm. Scanning electron microscope investigations were made on samples measuring 25.0 mm × 5.0 mm × 2.4 mm to identify the fibre arrangement. Fig. 2 shows the scanned results for the physical arrangement of fibres in the SC-0° strip. The strips consisted of 11 orthogonally oriented layers. SC-0° and SC-90° had a lay-up of [$(90°/0°)^2$, 90°, 0°, 90°, $(90°/0°)^2$] and [$(0°/90°)^2$, 0°, 90°, 0°, $(0°/90°)^2$] relative to their longitudinal direction, respectively. The diameter of a typical fibre was 8 μm. The image processing toolbox in the MATLAB™ software was used with changes in the area and orientation of a Representative Area Element (RAE) of the fibre arrangement to estimate the representative fibre area fraction in a single layer. The results indicated that the representative fibre volume fraction was approximately 61% for a large square area with an area greater than 40 times the cross-sectional area of a single fibre. In a companion study [1], the effective transversely isotropic properties of a unidirectionally fibre reinforced composite were evaluated using a computational simulation that took into account the spatial arrangements of fibre configurations. It was shown that the effective transversely isotropic elastic properties of the unidirectionally

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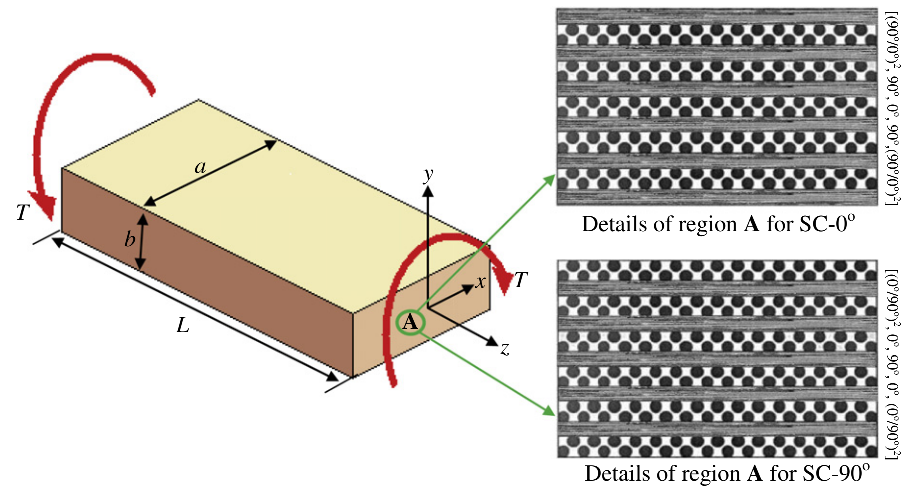

Fig. 1. Coordinate and dimensions of rectangular bar for Lekhnitskii model and details of strip lay-up at region **A**.

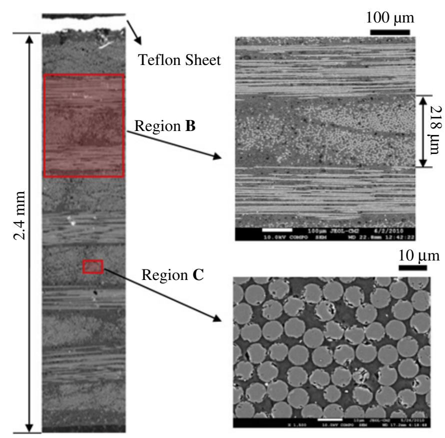

Fig. 2. Fibre arrangement as observed from a scanning electron microscope.

reinforced composite determined from experimental results coupled with computational simulations were in close agreement with the results based on the theoretical relationships proposed by Hashin and Rosen [2]. Predictions for the transversely isotropic elastic constants of a single layer obtained from the RAE method [1], that considers irregular fibre arrangements, were as follows: $E_{11} = 146.26$ GPa, $\nu_{12} = 0.23$, $E_{22} = 12.11$, $\nu_{23} = 0.29$, $G_{23} = 4.85$ where the fibres are aligned in the *L*-direction. This paper presents an extension of the work to allow the identification of the in-plane and out-of-plane shear moduli G_{xy} and G_{yz} of a laminated composite, using the results of torsion tests.

3. The torsion test, computations and results

The torsion testing apparatus used in the research is shown in Fig. 3. One specimen from each test set (SC-0°, SC-90°) was tested up to failure (Fig. 4). The ultimate torque capacity of the SC-0° and SC-90° samples was 149.1 N.m and 168.1 N.m, respectively. Both specimens showed brittle failure with the failure plane aligned roughly normal to the axis of the specimen. The second sample in each test pair was tested 3 times under quasi-static loading and unloading up to a rotation of 0.524 radians over the 200 mm length. The torque–rotation results were linear in the range of the applied torque (Fig. 4). Computational modelling of the strips with their laminated arrangements was also performed using the general-purpose finite element code ABAQUS™. Transversely isotropic stiffness coefficients [7] were determined using the five mechanical properties obtained using the RAE approach [1] and each layer was modelled as a homogenous transversely isotropic elastic material. Perfect bonding was assumed to exist between the separate layers. The computational modelling was

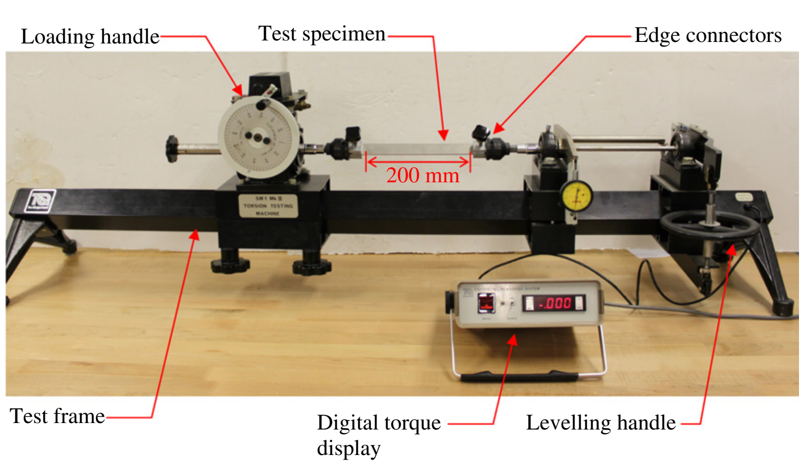

Fig. 3. Static torsion test setup.

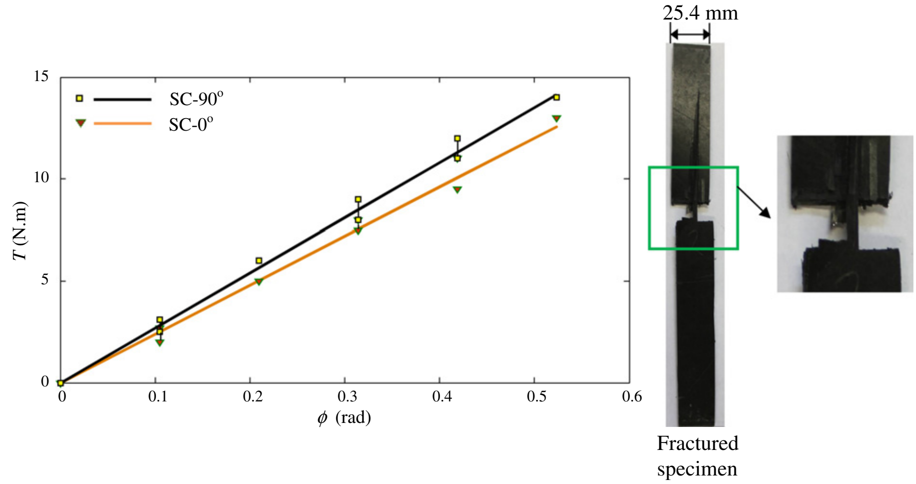

Fig. 4. Torque versus rotation curves and a typical fractured CFRP specimen.

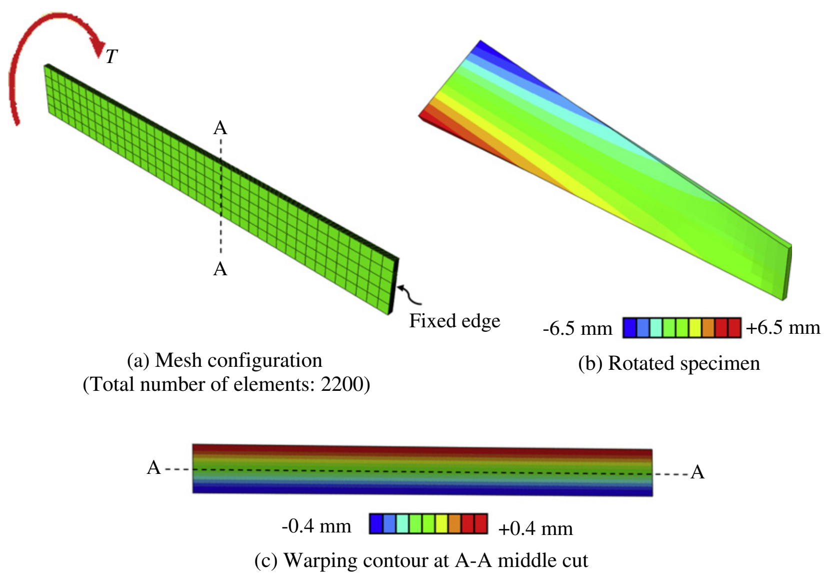

Fig. 5. Mesh configuration and deflection contours of SC–90° specimen due to T = 10.0 N .m.

performed using a standard 8-node linear brick element available in the element library of ABAQUS™. The mesh configuration and deformation contour for SC-90° is presented in Fig. 5. In order to use Lekhnitskii's model, the constants K_1 and K_2 were defined as the slope of the torque-rotation relationship in the linear region, i.e.

$$K_i = \frac{T_i}{\phi_i}, \quad i = 1(\mathrm{SC}-90^\circ),\ 2(\mathrm{SC}-0^\circ) \tag{3}$$

Then, Eq. (1) can be rewritten as:

$$\frac{K_1}{K_2} = \chi \frac{\sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n^4}\left[1 - \frac{2a}{n\pi b}\sqrt{\chi}\tanh\left(\frac{n\pi b}{2a}\sqrt{\frac{1}{\chi}}\right)\right]}{\sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n^4}\left[1 - \frac{2a}{n\pi b}\sqrt{\frac{1}{\chi}}\tanh\left(\frac{n\pi b}{2a}\sqrt{\chi}\right)\right]} \tag{4}$$

where

$$\chi = \frac{G_{yz}}{G_{xy}} \tag{5}$$

is the only unknown. By solving for χ, the in-plane and out-of-plane shear moduli can be expressed as:

$$G_{yz} = \frac{\pi^4 K_1 L}{32a^3 b \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n^4}\left[1 - \frac{2a}{n\pi b}\sqrt{\chi}\tanh\left(\frac{n\pi b}{2a}\sqrt{\frac{1}{\chi}}\right)\right]} \tag{6}$$

$$G_{xy} = \frac{G_{yz}}{\chi} \tag{7}$$

Results obtained from the experiments and computations were as follows: $(G_{xy})_{\mathrm{Experimental}}$ = 6.45 GPa, $(G_{xy})_{\mathrm{Computational}}$ = 6.69 GPa, $(G_{yz})_{\mathrm{Experimental}}$ = 5.92 GPa, $(G_{yz})_{\mathrm{Computational}}$ = 6.17 GPa, which are in close agreement. It should be mentioned that it is expected that when the number of layers is increased, the results for the in-plane and out-of-plane shear moduli will converge to a unique value similar to that of an isotropic strip.

4. Concluding remarks

The mechanical behaviour of a rectangular CFRP laminated composite strip, with two orthogonal fibre orientations and subjected to torsion, was examined using both experimental results and computational simulations. The in-plane and out-of-plane shear moduli were estimated using Lekhnitskii's theory. Consistent results were obtained from both the experiments and finite element simulations based on the effective material properties derived from RAE estimations. A limitation of the RAE method is the need to perform optical experiments to determine the fibre arrangement prior to developing the RAE element. The RAE approach, however, captures the geometric features of the fibre configuration, which is absent in effective property estimates proposed in the literature. The investigation, however, establishes the validity of theories available in the literature for estimating the effective elasticity parameters of fibre reinforced composites based on theoretical relationships that consider representative cell arrangements and variational procedures.

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References

[1] Selvadurai APS, Nikopour H. Transverse elasticity of a unidirectionally reinforced composite with an irregular fibre arrangement: experiments, theory and computations. Compos Struct 2012;94:1973–81.
[2] Hashin Z, Rosen BW. The elastic moduli of fibre-reinforced materials. J Appl Mech 1964;31:223–32.
[3] Timoshenko SP, Goodier JN. Theory of elasticity. New York (USA): McGraw-Hill; 1970.
[4] Selvadurai APS. Partial differential equations in mechanics. The biharmonic equation Poisson's equation, vol. 2. Berlin (Germany_: Springer-Verlag; 2000.
[5] Lekhnitskii SG. Theory of elasticity of an anisotropic body. San Francisco, USA: Holden-Day; 1963.
[6] Davalos JF, Qiao P, Wang J, Salim HA, Schlussel J. Shear moduli of structural composites from torsion tests. Compos Mater 2002;36:1151–73.
[7] Green AE, Zerna W. Theoretical elasticity. Oxford (UK): Clarendon Press; 1968.