A geophysical application of an elastostatic contact problem

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Abstract

This paper develops an analytical solution to the problem of the interaction between an elastic sealing geologic layer that is embedded within elastically dissimilar geologic media of semi-infinite extent. The axisymmetric mechanics of the elastic sealing layer during the application of pressurization of circular regions located in both halfspace regions is evaluated using an interaction analysis based on the classical theory of elasticity. In particular, the flexural behavior of the elastic sealing layer is modeled as an elastic plate that satisfies the Germain–Poisson–Kirchhoff thin plate theory. Integral expressions are developed for the deflections and stresses in the sealing layer, which are essential for establishing its sealing integrity. Specific numerical results are presented for the case where a single halfspace region is subjected to pressurization over a circular area of finite thickness.

Keywords

Geomechanics, elastic interaction, contact mechanics for a plate, thin plate theory, integral transform solutions, geologic sequestration, distributed centers of dilatation

I. Introduction

In recent years concerted attention has been given to the geologic sequestration of anthropogenic carbon dioxide in supercritical form as a possible abiotic strategy for mitigating the impact of greenhouse gases [1-5]. The objective of such an endeavor is to use the pore space of sparsely fractured storage rocks for the hydrodynamic trapping of the injected supercritical fluids. The hydrodynamic trapping mechanism should lead to the development of a stable plume at the base of an elastic sealing layer, which develops as the fluids in the storage rock are displaced by both buoyancy and the pressures exerted during injection of the supercritical fluids. The requirement for sparse fracturing of any storage horizon or target rock is dictated by its suitability for trapping for long periods in excess of several thousand years. A heavily fractured target rock is most likely to be confined by sealing rock exhibiting a similar fracture pattern, making the site unsuitable for storage of carbon dioxide in any form. Furthermore, a fractured storage formation presents a significant challenge: it would require extensive temporary and permanent sealing operations to ensure that the pore space in the intact units is fully utilized to provide the volume necessary for trapping large quantities of carbon dioxide in a supercritical form. A comprehensive study of geologic sequestration should also address both the geochemical and geomechanical aspects that arise during the injection of supercritical carbon dioxide, particularly into saline geologic formations, which can include full thermo-hydro-mechanical-chemical couplings. The research in this area has largely focused on either the consideration of fluid dynamics or geochemical influences of the carbon dioxide injection process

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Figure 1. The idealized configuration of an embedded elastic sealing layer.

[6–9], and the subject matter is being extended in a limited sense to include hydro-geomechanics aspects of the sequestration process [10–15].

The objective of this paper is to demonstrate that analytical solutions that are based on the classical theory of elasticity continue to provide valuable results that can serve as important benchmarking solutions; these can also be used to effectively assess the role of parameter variability on results of interest to sequestration strategy. In this paper we examine the interaction between an elastic sealing layer and dissimilar elastic geologic media when such interaction is induced by the action of pressures that are imposed by the injection of fluids. The problem is examined within the classical theory of isotropic elasticity, which has been a mainstay in the study of problems in geomechanics and geosciences [16,17] and continues to provide the basic solutions that can be applied to develop important preliminary estimates relevant to engineering applications.

2. Mechanics of the elastic sealing layer during pressurization

The paper focuses on a specific problem related to the interaction between a flexible elastic sealing layer, which is embedded between dissimilar elastic regions of semi-infinite extent. The mechanical behavior of the sealing layer is modeled by a thin pate that satisfies the Germain–Poisson–Kirchhoff thin plate theory [18–20]. The interaction between the elastic halfspace regions and the embedded elastic plate is induced by pressurization of circular regions of differing radii that are located at finite distances from the boundary of the elastic sealing layer. In order to develop an analytical solution of the interaction problem, it is assumed that the elastic layer is oriented horizontally such that the flexural interaction can be treated as an axisymmetric problem in classical elasticity theory. The paper develops generalized results for the flexural response of the pressurized regions of differing intensity and dimensions located in halfspace regions with dissimilar properties (Figure 1). Typical numerical results are presented to demonstrate how the pressurization in one halfspace region influences the deflections and flexural moments in the sealing elastic layer.

The choice of elasticity as a first approximation is a pragmatic one, which renders the problem tractable for most competent geomaterials. The elastic model is entirely relevant [17] for the range of stresses for which the embedded layer is expected to maintain its sealing integrity without the development of cracks and other defects. The problem can be examined by recourse to a more complicated computational multi-physics model of the

interaction between the various geologic horizons, which could include poroelasto-plasticity [21] of the geologic layers and the consideration of regions of pressurization that have a moving pressure front associated with hydrodynamic displacement of immiscible fluids. This degree of refinement in the modeling is not warranted in the present circumstances, where the intention is to develop a convenient analytical result that can be used to estimate material properties and variability in the geometric parameters in the problem; these can include the thickness of the elastic sealing layer, the radius of the pressurized zones and their proximity to the sealing layer. The problem assumes that the sealing layer can be modeled as a thin plate that exhibits flexural behavior governed by a Germain–Poisson–Kirchhoff thin plate theory. The justification of the thin plate approximation for the elastic sealing layer will be governed largely by its thickness in relation to the dimension (radius) of the pressurized region. The classical plate theory has been successfully applied to model the flexure of the earth's lithospheric crust supported on a dense fluid-like asthenosphere or mantle [22–27]. Alternative models of the elastic sealing layer can include plate models, which account for both flexural and shear deformation behavior [28–30]. The sealing layer is assumed to be contained between the geologic media A($r \in (0, \infty)$; $z \in (0, -\infty)$) and B($r \in (0, \infty)$; $z \in (0, \infty)$), which are modeled as isotropic elastic halfspace regions; the analysis is restricted to axisymmetric conditions. The deflection of the elastic sealing layer w(r) is governed by the plate bending equation

$$D\tilde{\nabla}^2\tilde{\nabla}^2 w(r) - q^{\mathrm{A}}(r) + q^{\mathrm{B}}(r) = 0$$
⁽¹⁾

where

$$\tilde{\nabla}^2 = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}$$
(2)

is the axisymmetric form of Laplace's operator in two dimensions, $D(=G_ch^3/6(1-\nu_c))$ is the flexural rigidity of the elastic sealing layer, h is its thickness, G_c and v_c are, respectively, its linear elastic shear modulus and Poisson's ratio and $q^{(A)}$ and $q^{(B)}$ are the contact stresses on the faces of the layer in contact with the respective halfspace regions (Figure 1). The interaction between the embedded sealing layer and the adjacent geologic media is induced by pressurized circular regions of radii R_A and R_B, located in the halfspace regions and at distances $l_{\rm A}$ and $l_{\rm B}$, respectively, from the faces of the plate. The pressurization regions are represented by distributions of centers of dilation of intensities p_A and p_B applied over thicknesses t_A and t_B , respectively. The centers of dilation in elastic media are based on the concepts originally proposed by Dougall [31] and Love [32] (see also [33, 34]) and subsequently applied by several investigators to examine fluid extraction problems related to fluid-saturated media [35–40]. The conventional solutions for pressure reduction in the halfspace regions due to fluid extraction are applicable to situations where the surface of a halfspace region is traction free; these results are inapplicable to the situation being modeled here due to the presence of the bonded elastic sealing layer. Also, the development of the fundamental problem related to a center of dilatation assumes that the elastic medium has a Poisson's ratio $\nu < 0.5$. The injection pressures can be visualized as those applied over and above any hydrostatic stresses that can be present in the halfspace regions. Finally, the elastic sealing layer is assumed to be have two extreme types of interface conditions that can vary from either (i) *perfect bonded contact* with the elastic halfspace regions, which, by virtue of the plate flexure model for the elastic sealing layer, induces an inextensibility constraint at the surfaces in contact with the halfspace regions or (ii) has a frictionless contact, which imposes a shear traction-free requirement at the surfaces of the halfspace regions in contact with the elastic sealing layer. The kinematic constraints that should be satisfied simultaneously at the contact between the sealing layer and the halfspace regions are

$$w(r) = u_z^{(A)q_A}(r,0) + u_z^{(A)p_A}(r,0) = u_z^{(B)q_B}(r,0) + u_z^{(B)p_B}(r,0)$$
(3)

$$u_r^{(A)q_A}(r,0) = u_r^{(A)p_A}(r,0) = u_r^{(B)q_B}(r,0) = u_r^{(B)p_B}(r,0) = 0$$
(4)

where $u_r(r, z)$ and $u_z(r, z)$ are, respectively, the radial and axial components of the displacement vector referred to as the cylindrical polar coordinate system (r, θ, z) ; $u_r^{(A)p_A}(r, 0)$, $u_r^{(A)q_A}(r, 0)$ and $u_z^{(A)p_A}(r, 0)$, $u_z^{(A)q_A}(r, 0)$ are, respectively, the radial and axial displacements at the boundary of halfspace region A in contact with the elastic sealing layer induced by the pressurized regions $p_A(r)$ and the contact stresses $q_A(r)$; similar definitions are implied by $u_r^{(B)p_B}(r, 0)$, $u_r^{(B)q_B}(r, 0)$ and $u_z^{(B)p_B}(r, 0)$, $u_z^{(B)q_B}(r, 0)$ for the radial and axial displacements at the boundary of halfspace region B in contact with the elastic sealing layer. In the case where the interfaces in contact with the elastic sealing layer and the halfspace regions are frictionless, the interface condition (3) remains unchanged but the displacement boundary condition (4) is replaced by

$$\sigma_{rz}^{(A)q_A}(r,0) = \sigma_{rz}^{(A)p_A}(r,0) = \sigma_{rz}^{(B)q_B}(r,0) = \sigma_{rz}^{(B)p_B}(r,0) = 0$$
(5)

3. Analysis of interaction between the halfspace regions and the elastic layer

The mathematical analysis of the axisymmetric elastostatic interaction between the elastic plate and the adjoining geologic media induced by the pressurization can be examined most conveniently using a procedure based on Hankel integral transforms [20, 41]. We define the zeroth-order Hankel transform of the function F(r) by $\overline{F}(\xi)$, where

$$\bar{F}(\xi) = \int_0^\infty r F(r) J_0(\xi r) dr$$
(6)

The inverse of (6) follows from self-reciprocity properties of Hankel transforms and $J_0(\xi r)$ is the zeroth-order Bessel function of the first kind. Considering the surface deflection of halfspace region A under the action of the pressurized region of radius R_A and thickness t_A located at an axial distance of l_A from the surface of the halfspace, it can be shown that

$$u_{z}^{(A)p_{A}}(r,0) = \frac{[\beta_{A}]\alpha_{A}(1-2\nu_{A})p_{A}R_{A}t_{A}}{G_{A}}\int_{0}^{\infty} e^{-\xi l_{A}}J_{1}(\xi R_{A})J_{0}(\xi r)\,d\xi$$
(7)

where

$$\beta_{\rm A} = 1$$
 for frictionless contact (8)

$$\beta_{\rm A} = \frac{1}{2(1 - \nu_{\rm A})}$$
 for bonded contact (9)

In (7), α_A is the Biot coefficient [42–44] for the storage region defined by

$$\alpha_{\rm A} = 1 - \frac{K_{\rm A}}{K_{\rm gA}} \tag{10}$$

where K_A is the bulk modulus for the grain fabric (i.e. drained state of the porous medium) and K_{gA} is the bulk modulus for the solid grain material in region A. Considering the deformations of halfspace region A subject to either the kinematic constraint (4) or the traction constraint (5), and the action of the compressive contact stresses $q^{(A)}(r)$ and the circular pressurized region $p^{(A)}$, it can be shown that

$$\bar{u}_{z}^{(A)} = -\frac{[\kappa_{A}]}{\xi G_{A}} \bar{q}^{A}(\xi)$$
(11)

where

 $\kappa_{\rm A} = (1 - \nu_{\rm A})$ for frictionless contact (12)

$$\kappa_{\rm A} = \frac{(3 - 4\nu_{\rm A})}{4(1 - \nu_{\rm A})} \text{ for bonded contact}$$
(13)

Considering the transformed equivalent of the first part of the kinematic constraint (3), it can be shown that

$$\bar{w}(\xi) = \bar{u}_z^{(A)q_A}(\xi) + \bar{u}_z^{(A)p_A}(\xi) = -\frac{[\kappa_A]}{G_A\xi} [\bar{q}^{(A)}(\xi) - \bar{S}^{(A)}(\xi)]$$
(14)

where

$$\bar{S}^{(A)}(\xi) = \frac{p_A R_A t_A \alpha_A (1 - 2\nu_A) [\beta_A]}{[\kappa_A]} \left(\frac{e^{-\xi l_A}}{\xi} J_1(\xi R_A)\right)$$
(15)

Similarly, considering the interactive processes between the sealing layer and halfspace region B, which is subject to either the kinematic constraint (4) or the traction constraint (5), it can be shown that

$$\bar{w}(\xi) = \bar{u}_z^{(B)q_B} + \bar{u}_z^{(B)p_B} = \frac{[\kappa_B]}{G_B \xi} [\bar{q}^{(B)}(\xi) - \bar{S}^{(B)}(\xi)]$$
(16)

where

$$\bar{S}^{(B)}(\xi) = \frac{p_{B}R_{B} t_{B}\alpha_{B}(1-2\nu_{B})[\beta_{B}]}{[\kappa_{B}]} \left(\frac{e^{-\xi l_{B}}}{\xi} J_{1}(\xi R_{B})\right)$$
(17)

In (17), $p_{\rm B}$ refers to the intensity of the centers of pressure applied to region B over a circular region of radius $R_{\rm B}$, thickness $t_{\rm B}$ and located at a distance $l_{\rm B}$ from the sealing layer, and

$$\alpha_{\rm B} = 1 - \frac{K_{\rm B}}{K_{g\rm B}} \tag{18}$$

where $K_{\rm B}$ is the bulk modulus for the grain fabric (i.e. drained state of the porous medium) and $K_{g\rm B}$ is the bulk modulus for the solid grain material in region B.

Operating on the ordinary differential equation (1) governing the flexural behavior of the elastic layer with the zeroth-order Hankel transform (6), we have

$$D\xi^{4}\bar{w}(\xi) - \bar{q}^{(A)}(\xi) + \bar{q}^{(B)}(\xi) = 0$$
(19)

Using (14) and (16) in (19), we can eliminate the transformed expressions for the contact stresses $\bar{q}^{(A)}(\xi)$ and $\bar{q}^{(B)}(\xi)$ to obtain an expression for the transformed equivalent of the mid-plane deflection of the elastic sealing layer; inversion of the Hankel transform gives the following result for the elastic sealing layer deflection w(r):

$$w(r) = \frac{at}{h(G_{\rm A}[\kappa_{\rm B}] + G_{\rm B}[\kappa_{\rm A}])} \int_0^\infty \frac{[F_{\rm A}(\xi) - F_{\rm B}(\xi)]}{[1 + \Phi\lambda^3]} J_0(\lambda r/h) \, d\lambda \tag{20}$$

where

$$F_i(\xi) = p_i R_i t_i \alpha_i (1 - 2\nu_i) [\beta_i] [\kappa_i] e^{-\lambda(t_i/h)} J_1(\lambda R_i/h) \quad ; \quad i = A, B$$
(21)

$$\Phi = \frac{G_{c} [\kappa_{A}][\kappa_{B}]}{6(1 - \nu_{c})(G_{A}[\kappa_{B}] + G_{B}[\kappa_{A}])}$$
(22)

The generalized elasticity solution for deflection of the elastic sealing layer and the adjacent geologic halfspaces during the axisymmetric pressurization of both halfspace regions by circular distributions of centers of dilations is given by the expression (20). The generalized nature of the solution stems from the fact that several types of interface conditions between the elastic sealing layer and the halfspace regions can be accommodated by a suitable selection of the constraints implied by $[\kappa_A]$ and $[\beta_A]$. Integral expressions for the contact stresses and flexural stresses in the elastic sealing layer can be determined by using (12) in the expressions for the flexural moments and shear force distributions:

$$M_{rr}(r) = -D\left[\frac{d^2w(r)}{dr^2} + \frac{v_c}{r}\frac{dw(r)}{dr}\right]$$
(23)

$$M_{\theta\theta}(r) = -D\left[\frac{1}{r}\frac{dw(r)}{dr} + v_{\rm c}\frac{d^2w(r)}{dr^2}\right]$$
(24)

$$V_r(r) = -D\frac{d}{dr} \left[\frac{d^2 w(r)}{dr^2} + \frac{1}{r} \frac{dw(r)}{dr} \right]$$
(25)

These results can be used to estimate the stresses in the elastic sealing layer due to the pressurization of the separate halfspace regions. It should be borne in mind that the geostatic stresses should be added to these pressurization-induced stresses to determine the final stress state. The contact stresses $q^{(A)}(r)$ and $q^{(B)}(r)$ can similarly be determined using expression (20) for w(r) in (11) and its counterpart for domain B.

4. Numerical results

Since the result for the deflection of the elastic sealing layer can be obtained in a compact integral form, it is possible to obtain specific results applicable to an elastic sealing layer and halfspace combinations, where the halfspace regions can be identified as either the domain into which fluids are injected or an overburden region

that contributes to the structural integrity of the elastic sealing layer. The infinite integral (20) can be computed using a suitable quadrature to account for the oscillatory but decaying nature of the integrand that results from the product of two Bessel functions. For the purposes of presentation of numerical results, we consider the particular situation where the pressurized region A is identified as a storage domain (subscript 's') and the halfspace region B is the overburden (subscript 'o') and not subjected to fluid pressure. Region A is pressurized by a circular distribution of centers of dilatation with pressure p_0 , radius a, thickness t and located at a distance lfrom the elastic sealing layer. Also, for the purposes of illustration we assume the kinematic constraints defined by (9) and (13). While several combinations of interface conditions are accommodated by the fundamental result (20), for the purposes of illustration, we shall restrict attention to the kinematic interface conditions defined by (4). For this particular case, the integral result for w(r) given by (20) reduces to the simplified form

$$w(r) = \frac{\Omega at}{h} \int_0^\infty \frac{e^{-(\lambda l/h)}}{[1 + \Phi\lambda^3]} J_1(\lambda a/h) J_0(\lambda r/h) d\lambda$$
(26)

where

$$\Omega = \frac{\alpha_{\rm s}(1-\nu_{\rm s})(1-2\nu_{\rm s})(3-4\nu_{\rm s})p_0}{2[G_{\rm s}(1-\nu_{\rm s})(3-4\nu_{\rm o})+G_{\rm o}(1-\nu_{\rm o})(3-4\nu_{\rm s})]}$$
(27)

$$\Phi = \frac{G_{\rm c}(3-4\nu_{\rm s})(3-4\nu_{\rm o})}{24(1-\nu_{\rm c})[G_{\rm s}(1-\nu_{\rm s})(3-4\nu_{\rm o})+G_{\rm o}(1-\nu_{\rm o})(3-4\nu_{\rm s})]}$$
(28)

The result (26) contains two non-dimensional parameters Ω and Φ , which can be estimated by assigning typical relative values for the elasticity, geometrical and loading parameters in the system. The quantity $(\Omega h/at)$ can be used to make w(r) non-dimensional; the influence of Φ and the geometric parameters l/h, a/h and r/h can be assigned typical values. For example, consider an injection zone of thickness h = 50 m that is loaded by a pressurized circular region of radius a = 500 m and thickness t = 10 m, located in the storage rock region at a depth of l = 250 m from the interface between the elastic sealing layer and the storage domain. These dimensions can be varied to estimate the influence of either the position of the injection zone or its dimensions on both the magnitude and distribution of the sealing layer deflections. The numerical integration of expression (25) was performed using Mathematica[®], taking into consideration a suitable number of intervals to ensure convergence of the result. Figures 2-6 illustrate the distribution of the non-dimensional uplift deflection of the mid-plane of the sealing layer due to injection pressures of constant intensity over a thickness (t) and radius (a), but located at differing depths (l) in relation to the thickness of the sealing layer. In order to perform the numerical evaluations, we set $\Phi = 2$, which approximately corresponds to $(G_c/(G_s + G_o)) \approx 13.5$ for $\nu_{\rm c} = \nu_{\rm s} = \nu_{\rm o} = 1/4$. The numerical results clearly indicate the influence of the proximity of the pressurized region on both the magnitude of the maximum displacement of the sealing layer and on the curvature of the deflected shape, which will influence the development of stresses in the layer. As the location of the pressurized zone in the storage region increases, the corresponding maximum displacements are reduced and abrupt changes to the deflected shape of the elastic sealing layer are also reduced.

Similarly, Figures 7–11 illustrate the non-dimensional distributions of the radial moment with the radial distance. These represent the flexural moment due to the application of the circular distribution of centers of pressure. As the relative flexibility of the sealing elastic layer decreases, the flexural moments tend to be localized in the vicinity of the boundary of the pressurized region. For relatively stiff elastic sealing layers, the location of maximum radial flexural moment in the elastic sealing layer shifts towards the axis of symmetry. The radial flexural stresses can be added to the geostatic stress states to determine the combined stress state. In the developments presented here, the overburden region is assumed to be of semi-infinite extent. In the estimation of the combined stress state, the overburden can be assumed to be of finite depth.

The results presented for the circular pressurization problem can be extended to include other areal distributions of pressure within the injection zone. Considering result (26), we can develop a solution to the problem of a center of pressure of magnitude p_0 acting at a distance l within a halfspace region and distributed over a volume v^{*} in the form

$$w(r) = \frac{\Omega v^*}{2h^2} \int_0^\infty \frac{\lambda e^{-\lambda l/h} J_0(\lambda r/h)}{[1 + \Phi\lambda^3]} d\lambda$$
⁽²⁹⁾

Result (29) can be regarded as a fundamental solution for the interaction between the elastic sealing layer and a center of dilatation resembling the pressure exerted by the injected fluids over the elemental volume V^* . Result



Figure 2. Deflection of the elastic sealing layer due to a pressurized region located in the storage geological medium (a/h = 10; $\Phi = 0.1$).



Figure 3. Deflection of the elastic sealing layer due to a pressurized region located in the storage rock (a/h = 10; $\Phi = 2$).



Figure 4. Deflection of the elastic sealing layer due to a pressurized region located in the storage rock (a/h = 10; $\Phi = 10$).



Figure 5. Deflection of the elastic sealing layer due to a pressurized region located in the storage rock (a/h = 10; $\Phi = 40$).



Figure 6. Deflection of the elastic sealing layer due to a pressurized region located in the storage rock (a/h = 10; $\Phi = 100$).



Figure 7. Distribution of radial flexural moment in the elastic sealing layer due to internal pressurization by a circular distribution of centers of pressure in the storage region (a/h = 10; $\Phi = 0.10$).



Figure 8. Distribution of radial flexural moment in the elastic sealing layer due to internal pressurization by a circular distribution of centers of pressure in the storage region (a/h = 10; $\Phi = 2$).



Figure 9. Distribution of radial flexural moment in the elastic sealing layer due to internal pressurization by a circular distribution of centers of pressure in the storage region (a/h = 10; $\Phi = 10$).



Figure 10. Distribution of radial flexural moment in the elastic sealing layer due to internal pressurization by a circular distribution of centers of pressure in the storage region (a/h = 10; $\Phi = 40$).



Figure 11. Distribution of radial flexural moment in the elastic sealing layer due to internal pressurization by a circular distribution of centers of pressure in the storage region (a/h = 10; $\Phi = 100$).

(29) can be integrated to develop solutions for the deflection of the elastic plate, which is induced by other forms of distributed pressures. Consider the internal pressure distribution $p_0 f(x, y, z)$ applied over a volume region V* (Figure 12); the formal result for the deflection of the embedded sealing layer can be written as

$$w(x,y) = \frac{\Omega h}{4} \iiint_{\mathbb{V}^*} f(\xi,\eta,\zeta) \ G(\xi,\eta,\zeta; x,y,z) \, d\Sigma \, d\Lambda \, d\Xi$$
(30)

where

$$G(\xi,\eta,\zeta;x,y,z) = \int_0^\infty \frac{\lambda e^{-\lambda\Xi} J_0 \left(\lambda \sqrt{(X-\Sigma)^2 + (Y-\Gamma)^2}\right) d\lambda}{[1+\Phi\lambda^3]}$$
(31)

and X = x/h, Y = y/h and Z = z/h are the normalized Cartesian coordinates of the *field point* and $\Sigma = \xi/h$, $\Lambda = \eta/h$ and $\Xi = \xi/h$ are the normalized coordinates of the *source point*. As is evident, the formal result (30) can be evaluated only by recourse to numerical procedures.

5. Concluding remarks

The paper develops an analytical model of the elastic interaction between an elastic sealing layer that is embedded between two elastic rock masses of infinite extent, which acts as a medium for the sequestration of carbon dioxide through pressurized injection. The elasticity model is regarded as a first approximation to develop useful calculations for preliminary estimations of the influence of pressurized injection into a storage rock on the mechanics of the sealing layer. The geometry of this layer, its modeling as a thin plate, the modeling of the storage rock and the overburden rock extended to semi-infinite domains and the restriction of the pressurized region to an axisymmetric thin disk-shaped region enables the development of compact analytical solutions. These can significantly enhance preliminary design calculations to assess the impact of the pressurized injection. The elastic modeling of the geological horizons in the storage location is considered to be adequate, since injection pressures are selected to prevent development of fracture and elasto-plastic effects in the largely brittle rocks. The methodology can be extended to include poroelastic effects in the storage formation and to include more complete modeling of the elastic sealing layer as a deformable layer, as well as non-uniform development of injection pressures in the storage unit. These extensions will be necessary if the injection activity is influenced by time-dependent effects of flow and load transfer from the injecting fluids to the porous skeletal framework of the storage formation. Modifications to the modeling approach will also be necessary if hydraulic fracture is initiated during the injection process.



Figure 12. Spatial distribution of centers of pressure within a volume domain.

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Conflict of interest

None declared.

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