Historical Notes: A Momentary Lapse of Concentration by the Genius?

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Sketches and manuscripts written by Sir Isaac Newton during the time of completion of the Principia were exhibited at several locations. Among the Principia papers are sketches drawn by Newton, clearly indicative of a multi-faceted, multi-tasking intellect. One sketch contains two arithmetical calculations written down side by side. A detailed look at these calculations indicates that one of these could be in error. The purpose of this note is to use digital imaging techniques to assess the accuracy of Newton's calculations.

Sir Isaac Newton's genius needs no explanation. What the world would be like without his contributions to mathematics, mechanics and science in general is difficult to comprehend. The reverence with which Isaac Newton is held in scientific circles in the world is legendary.[1] A recent volume by Feingold [1] (see also [2]) presents an authoritative account of the events that serve as the backdrop for Newton's discoveries and his many conflicts, disagreements and public disputes with scientists of the day including Robert Hooke and Gottfried Wilhelm Leibnitz. The frontispiece for this volume contains a 'Newton Doodle', typical of a scientist of that period, that is indicative of Newton's ability to multi-task. The exact explanations for the sketches could be an interesting exercise by itself. Our attention, however, is drawn to an arithmetical calculation that appears on the upper right-hand corner of the frontispiece (Figure 1).

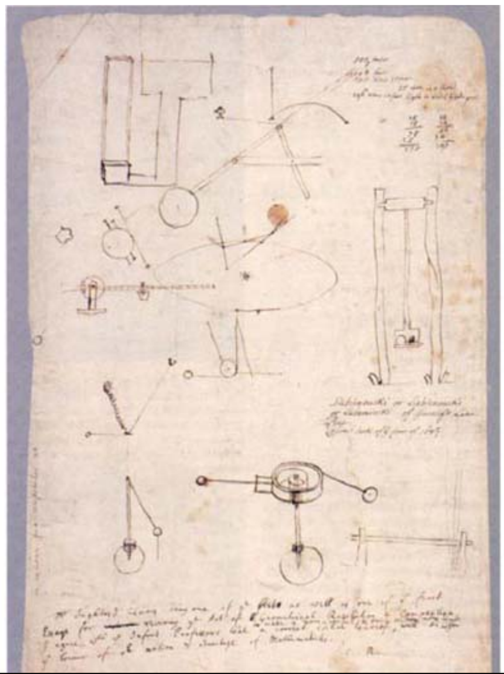
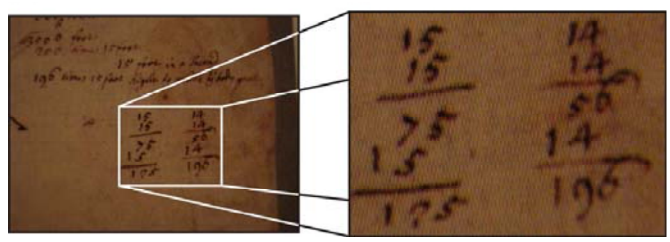


Figure 1: Sketches of an apparatus, sundry objects and arithmetical calculations drawn by Newton [3] (see also [1]).

A detail of the arithmetical calculations is shown in Figure 2(i). There are two calculations; clearly the one on the right involving 14^2 is correct. The second calculation for 15^2 is given as what would appear to be 125 instead of 225. (Even the 2 is written very casually, bearing a resemblance to the number 7. However, if one compares the other 7 in the calculation, there is a clear difference with the middle number in the final result, where the front of the number is curved downward, as would be normal when writing the numeral 2.) The question is whether the numerals can be further examined to conclusively prove whether the first number in the sum corresponds to either 1 or 2?

The approach adopted here is to examine, through digital image analysis, the other handwritten versions of 1 in the detail of the calculation. If one excludes the number 1 in question, shown at the bottom left of the first calculation, there are still seven other number 1s in both calculations. The digital image analysis of these seven images are given in Figure 3. Digital image analysis was performed in Matlab [4], using the following techniques: first, the calculation was magnified to the extent seen in Figure 2(ii), composed of $386,400$ pixels in the RGB additive colour spectrum model; this was then converted to a singular intensity represented as a double precision floating-point machine number [5]. Upon further examination of Figure 2(ii), we notice that the texture of the paper has added noise to the image which needs to be filtered out. A simple disk filter, with a 10 pixel radius, was applied to reduce the sharpness of the image caused by the grainy paper used in the late 17th century. Next, an adaptive threshold filter [6] was employed. Traditional threshold filters examine the intensity value of each pixel and will reassign a binary value (i.e. black or white) with respect to its position relative to the chosen threshold; this is applied over the entire global domain. An adaptive threshold filter is similar, but represents the final image using binary coding. However, this method is employed to accommodate for changing lighting conditions. Using local thresholding (rather than global), the assumption that illumination over the smaller regions is more uniform is generally held. In the case of our photographic image the adaptive threshold technique has been used since the picture was not professionally taken, thus lighting itself is an issue. Figure 3 shows a representation of the image after all steps of the filtering and thresholding were completed.



(i) The detail of the upper right-hand corner of Figure 1. (2048×1536 pixels)

(ii) Magnified image of the calculation. (736×525 pixels)

Figure 2

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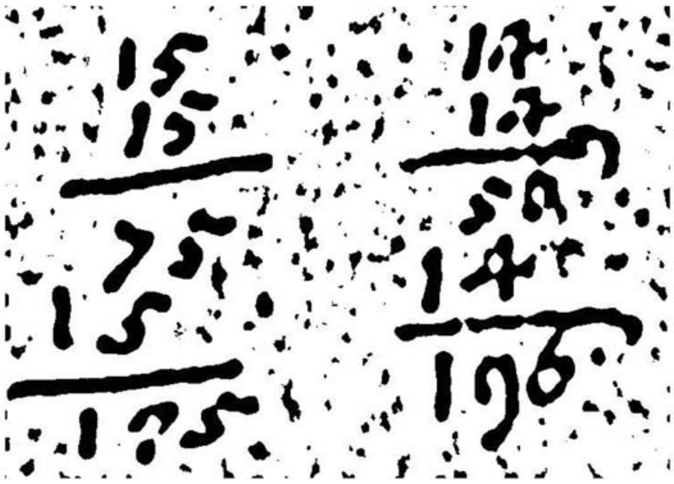

Figure 3: Magnified image enhanced using various filtering and thresholding techniques.

Finally, the comparison of individual numbers in the enhanced image seen in Figure 3 was developed. Each numeral 1 was cropped to an appropriate size containing n rows by m columns of pixels. A number of 'zero-value' pixels were added to the individual 1s ensuring that they all contained the same number of rows and columns as the largest 1 in the set of cropped images. The following equation was used to analyse the correlation coefficient between the images being compared:

$$r_{A-B} = \frac{\sum_n \sum_m \left(A_{mn} - \bar{A}\right)\left(B_{mn} - \bar{B}\right)}{\sqrt{\left(\sum_n \sum_m \left(A_{mn} - \bar{A}\right)^2\right)\left(\sum_n \sum_m \left(B_{mn} - \bar{B}\right)^2\right)}}$$

where A_{nm} and B_{nm} are the pixel value (0 or 1) at the nth row and mth column location in the first and second image, respectively, and $\bar{A}$ and $\bar{B}$ are the mean pixel values over the entire image of A and B, respectively.

Using these seven images, it is possible to examine the correlation between the various numeral 1s exhibited in Newton's handwriting (Figure 4). The distinct correlation between the 1 in question and the seven other 1s is apparent. Whether the images are equivalent can be assessed either by visual comparison or through more sophisticated correlation techniques commonly used in digital image processing. From the results shown in Figure 4, it appears that the first digit in the result for the left-hand-side multiplication corresponds to 1 rather than 2. As a consequence, Newton's result for the multiplication appears to be a slip up. The error by itself is perhaps insignificant, but what went on in the mind of the genius when he wrote down the calculation would be a much more interesting quest! Sir Isaac Newton's prowess in performing arithmetical calculations is not to be underestimated. During the course of his researches into binomial expansions, he calculated several logarithms up to fifty-five places [7]. ❏

Notes

1. The late Sir James Lighthill FRS, like Sir Isaac Newton, was the holder of the Lucasian Chair of Mathematics at Cambridge from 1969 to 1979 and was regarded as one of the foremost authorities in the world on mathematical fluid dynamics and aero-acoustics. Both during his undergraduate days at Cambridge and during his school days at Winchester College, there existed a friendly rivalry between Sir James and Freeman Dyson, who went on to become Professor of Theoretical Physics at Princeton University. As is well known, the friendly rivalry continued and Sir James is known to have commented on the fact that the chair he held had as a predecessor, Isaac Newton, whereas the predecessor of the chair held by Freeman Dyson was only Albert Einstein.

References

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4. Matlab version 7.9a. (2009) The MathWorks Inc., Natick, Massachusetts.
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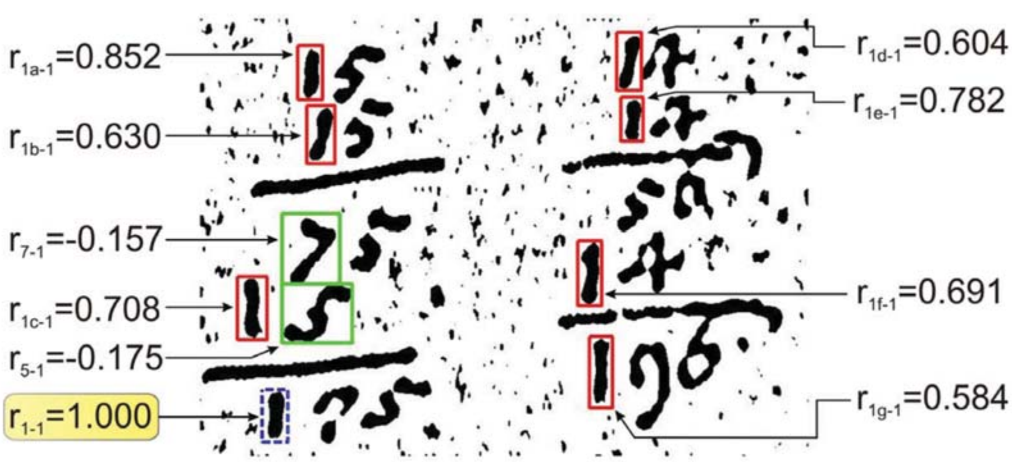

Figure 4: Correlation coefficients for the seven correct numeral 1s (red) with respect to the 1 in question (blue). The digits 7 and 5 (green) were also compared to the 1 in question and demonstrate low correlation values.