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Fragmentation of Ice Sheets during Impact

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Abstract: The paper deals with a computational approach for modelling the fragmentation of ice sheets during their impact with stationary structures. The modelling takes into consideration the intact continuum behaviour of the ice as a rate-sensitive elastoplastic material. During impact, the ice sheet can undergo fragmentation, which is controlled by a brittle strength criterion based on the current stress state. The fragmentation allows the generation of discrete elements of the ice sheet, the movements of which are governed by the equations of motion. The contact between individual fragments is governed by a Coulomb criterion. The individual fragments can themselves undergo further fragmentation with a size-dependent brittle strength criterion. The modelling is applied to examine the interaction and fragmentation of an ice sheet of finite dimensions as it impacts a stationary object.

Keywords: Ice-structure interaction, viscoplasticity, fragmentation, dynamics of ice sheets, rate sensitivity, discrete elements.

1 Introduction

The interaction of ice masses and either stationary or moving objects is of critical importance to navigation in ice-infested waters, construction of offshore resource exploration structures in the high arctic, ice booms to contain floating ice cover from entering hydraulic structures and in the study of offshore structures, such as bridge piers and beacons, used in marine transportation. Accounts of advances and historical aspects of the progress of research are documented by Sodhi and Cox (1987), Sinha, Timco and Frederking (1987), Hallam and Sanderson (1987), Sanderson, (1988), Dempsey, Bazant, Rajapakse and Sunder (1993), Selvadurai and Sepehr (1998), Ibrahim, Chaloub and Falzarano (2007) and Konuk, Gürtner and Yu (2009). The incidence of ice-structure interaction is expected to increase in the next decades as global warming can result in year-round navigable passages with offshore ice covers, which were previously accessible only during summer

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months. The interaction between a moving ice sheet and a stationary structure is a complex problem in both constitutive modelling and multi-body dynamics of interacting ice fragments. The interaction process is governed by a number of factors including the velocity of motion of the moving ice sheet, its rate-sensitive constitutive behaviour, ability to fragment, the frictional interaction of fragments and the process of continued fragmentation during the various stages of loading. The paper presents the application of a computational approach to modelling continuum-to-fragmentation during in-plane mechanics of a floating ice sheet. The methodology is demonstrated through application to typical problems involving in-plane interaction of a moving ice sheet and a stationary structure (Figure 1).

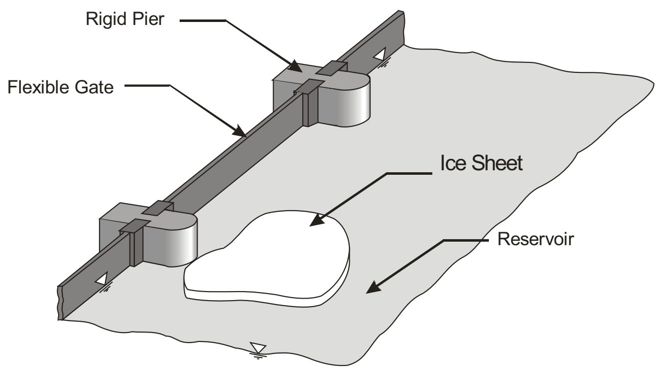

Figure 1: Impact of a floating ice sheet with a stationary structure

2 Constitutive models for fragmentable ice

A typical feature of the dynamic interaction between an ice feature and a stationary rigid structure is the process of fragmentation. Fragment development is the end result of many micromechanical scale- and rate-dependent processes such as stable and unstable micro-crack evolution, damage development, crack coalescence to form macro-crack generation, etc, which are influenced by the morphology of the ice feature (i.e. columnar grained vs. polycrystalline), the temperature and rate effects. The transition from the development of stable micro-crack to macro-crack formation will also depend on a variety of mechanical factors and loading histories, and in situ stress states, which are difficult to accommodate in a realistic fashion. While computational methods based on finite element and boundary element techniques can be effectively utilized to examine the progress of micro-crack extension and coalescence to form macro-cracks and damage, frictional effects and interface plasticity phenomena on crack surfaces (Selvadurai, 2004, 2005; Selvadurai and

Au, 1991; Selvadurai and Mahyari, 1997, 1998; Jing, 2003; Han and Atluri, 2003; Selvadurai and Yu, 2005; Liu, Han, Rajendran and Atluri, 2006; Xu, Dong and Zhang, 2008; Zhou, Li and Yu, 2008), these procedures cannot conveniently handle the presence of multiple cracks and the resulting generation of intact fragments. Furthermore, as intact fragments develop, the computational scheme should be able to examine the interaction of multiple but discrete continuum regions. A computational approach that accounts for transition from a continuum to a fragmented state offers the most appropriate method for dealing with the transformation processes. Codes such as NIKE2D (Hallquist, 1979), DYNA 2D and 3D (Goudreau and Hallquist, 1982) and more current versions (LS-DYNA) are considered the earliest of formal development of computational approaches that incorporate continuum-to-fragmentation processes. The application of continuum-to-fragmentation procedures to mechanics of ice-structure interaction is discussed by Hocking, Mustoe and Williams (1985a, b) Williams, Mustoe and Worgan, 1986; and Mustoe, Williams, Hocking and Worgan (1987). These procedures have been extended by Selvadurai and Sepehr (1999) to examine fragment size limiting effects in ice fragmentation processes during dynamic interaction. In this paper we shall examine the class of ice-structure interaction problems involving floating ice sheets and stationary structures, where fragmentation occurs by preserving the in-plane character of the deformations.

2.1 A model for intact ice

In this section we shall briefly review some of the salient features associated with the two-dimensional modelling of continuum-to-fragment transition in rate sensitive geomaterials. Since the constitutive behaviour is assumed to be rate sensitive, we need to consider an incremental formulation, such that the incremental total strain tensor $d\varepsilon_{ij}$ is given by

$$d\varepsilon_{ij} = d\varepsilon_{ij}^{el} + d\varepsilon_{ij}^{vp} \tag{1}$$

where $d\varepsilon_{ij}^{el}$ and $d\varepsilon_{ij}^{vp}$ refer to the incremental components of the elastic and viscoplastic strain tensors respectively. For an isotropic linear elastic geomaterials the constitutive response has the form (Davis and Selvadurai, 1996)

$$d\varepsilon_{ij}^{el} = \frac{d\sigma_{ij}}{2G} + \left(\frac{1}{9K} - \frac{1}{6G}\right) d\sigma_{kk}\delta_{ij} \tag{2}$$

where G and K are elastic constants. To account for viscoplasticity phenomena, we consider the model proposed by Perzyna (1966) (see also Cristescu and Suliciu, 1982; Selvadurai and Au, 1991; Pellet, Hadju, Deleruyelle and Besnus, 2005;

Sterpi and Gioda, 2009; Ghiabi and Selvadurai, 2009) which has the incremental form of the basic constitutive response of the viscoplastic type, where the incremental components of the strain are given by

$$d\varepsilon_{ij}^{vp} = \gamma\langle\Phi(F)\rangle \frac{\partial F}{\partial \sigma_{ij}} \tag{3}$$

where γ is a fluidity parameter,$\langle\rangle$ is the Macaulay symbol, $F(\sigma_{ij})$ is a yield function which, for most brittle geomaterials, can be approximated by a Mohr-Coulomb failure criterion (Davis and Selvadurai, 2002) given by

$$F = \frac{I_1}{3}\sin\varphi + \sqrt{J_2}\left\{\cos\Theta - \frac{\sin\Theta\sin\varphi}{\sqrt{3}}\right\} - c\cos\varphi \tag{4}$$

and c and ϕ are the conventional strength parameters associated with cohesion and the angle of internal friction, respectively. Also,

$$\begin{aligned} &I_1 = \sigma_{kk};\ J_2 = \frac{1}{2}\sigma'_{ij}\sigma'_{ij};\ J_3 = \frac{1}{3}\sigma'_{ij}\sigma'_{jk}\sigma'_{ki} \\ &\Theta = \frac{1}{3}\sin^{-1}\left\{-\left(3\sqrt{3}J_3\right)/\left(2J_2^{3/2}\right)\right\}; \quad -\pi/6 \le \Theta \le \pi/6 \end{aligned} \tag{5}$$

and σ'_{ij} is the stress deviator tensor. The flow function in (3) needs to be determined by appeal to experimental results. Several plausible and computationally tractable flow functions have been proposed in the literature; e.g.

$$\langle\Phi(F)\rangle = \begin{cases} \exp\left(\frac{M(F-F_0)}{F_0} - 1\right) \\ \left(\frac{F-F_0}{F_0}\right)^N \end{cases} \tag{6}$$

where M and N are constants and F_0 is a uniaxial failure stress.

2.2 A model for fragmented ice

The preceding developments focus on the mechanics of ice prior to fragmentation. We now focus on criteria that can be used to establish fragmentation of ice upon attainment of either a specified tensile/compressive stress or a specified level of strain energy. The latter condition is perhaps more intuitively acceptable, but suffers from the limitation that the orientation of the fragment separation plane cannot be specified. With the former approach, the orientation of a potential plane of fragmentation can be determined by considering the stress state at a point. For example, for fragment initiation in compression, the criterion can be written in terms of the principal stresses in the form

$$\sigma_1 \geq \sigma_c + \sigma_3 \tan^2(45^0 + \phi/2) \tag{7}$$

where σ_c is the unconfined strength in compression and σ_1 and σ_3 are, respectively, the maximum and minimum principal stresses. The unconfined compressive strength can be related to the shear strength parameters c and ϕ according to

$$\sigma_c = 2c[\sqrt{(\tan^2\phi + 1)} + \tan\phi] \tag{8}$$

For the compression failure mode, there are two possible conjugate orientations of fragmentation inclined at equal angles $(\pi/2 - \phi/2)$ to the directions on either side of it. In two dimensions, these are defined by

$$\theta = \tan^{-1}\{(\sigma_1 - \sigma_{xx})/\sigma_{xy}\} \pm (90 - \varphi)/2 \tag{9}$$

where θ is the angle between the fragmentation plane and the positive global axis, σ_{xx} is the x-component of the stress tensor and σ_{xy} is the shear stress. The criterion for fragmentation in tension takes the simpler form

$$\sigma_3 \geq \sigma_T \tag{10}$$

where (Greek sigma subscript T) is the tensile strength and the orientation is perpendicular to the axis of the tensile principal stress σ_3.

Since both viscoplastic flow and fragmentation are described by appeal to the same basic failure criterion, additional constraints need to be prescribed to identify the conditions under which each process will occur. In the computational developments, it is assumed that the intact ice will experience brittle fragmentation only in situations when either a single principal stress or both principal stresses are in the tensile mode. In contrast, viscoplastic flow occurs when both principal stress components are compressive. Also, if viscoplastic flow occurs first, there is provision for subsequent fragmentation development in tension. This subsequent fragmentation will be governed by prescribed post peak strength characteristics, which can also include softening.

2.3 Size dependency in the fragmentation process

Size dependency in strength is a typical feature in brittle geomaterials, which have an inherent structure consisting of grains in a fabric arrangement. The random occurrence of defects, such as micro-cracks and other inhomogeneities generally contribute to size dependency in the fragmentation strength. A review of experimental data indicating size dependency in brittle geomaterials is given by Selvadurai and Sepehr (1999). From the point of view of computational modelling of fragmentation, the introduction of size dependency is a desirable concept. The progressive increase in the fragmentation process will continually decrease the fragment size and, in the process, lead to an increase in the number of generated fragments. The non-linear processes associated with such unrestricted fragment size development makes the computational procedure unmanageable. This deficiency is remedied by adopting a fragmentation tensile strength that increases with a decrease in the fragment size. Both laboratory and field experiments performed on ice point to variations of the type shown in Figure 2. These results are incorporated in the computational modelling.

2.4 Inter-fragment interaction

Fragment interaction response upon attainment of fragmentation is an important consideration in the study of the fragmentation process. These responses are best described by appeal to an interface response identified at a local contact plane between two interacting surfaces (e.g. Selvadurai and Boulon, 1995; Nguyen and Selvadurai 1998; Willner, 2003; Song, McFarland, Bergman and Vakakis, 2005; Ozaki, Hashiguchi, Okayasu and Chen, 2007; Zozulya, 2009), which can be defined in relation to the differential displacements at the contacting surfaces: i.e.

$$dF_i = k_{ij}(du_j^{(1)} - du_j^{(2)}) \tag{11}$$

where $i = n, s$; dF_i are the incremental changes in the contact force per unit length between contacting surfaces; $du_i^{(1)}$ and $du_i^{(2)}$ are displacements at the contact plane between the regions $^{(1)}$ and $^{(2)}$; and k_{ij} are the stiffness coefficients defined in the normal $(k_{nn} = k_n)$ and shear $(k_{ss} = k_s)$ directions on the average plane of contact (all other $k_{ij} = 0$). These stiffnesses themselves could be functions of the differential displacements $(du_i^{(1)} - du_i^{(2)})$. A Coulomb friction model is the simplest of the models that can characterize the contact process. For such a response,

$$\begin{aligned} &k_s = k_s^*; \quad |d\tau_s| < c_f + \mu\sigma_n \\ &k_s = 0; \quad |d\tau_s| = c_f + \mu\sigma_n \end{aligned} \tag{12}$$

where τ_s is the shear stress, σ_n is the normal stress at the inter-fragment contact plane, c_f is an interface adhesion and μ_f is the Coulomb friction for the contact plane. For interaction responses in the normal direction, it is possible to assume linear elastic behaviour provided the contact force is compressive. The normal stiffness will vanish when the fragments separate. i.e.

$$\begin{aligned} k_n &= k_n^*; \quad d\sigma_n \leq 0 \\ k_n &= k_s = 0; \quad d\sigma_n > 0 \end{aligned} \tag{13}$$

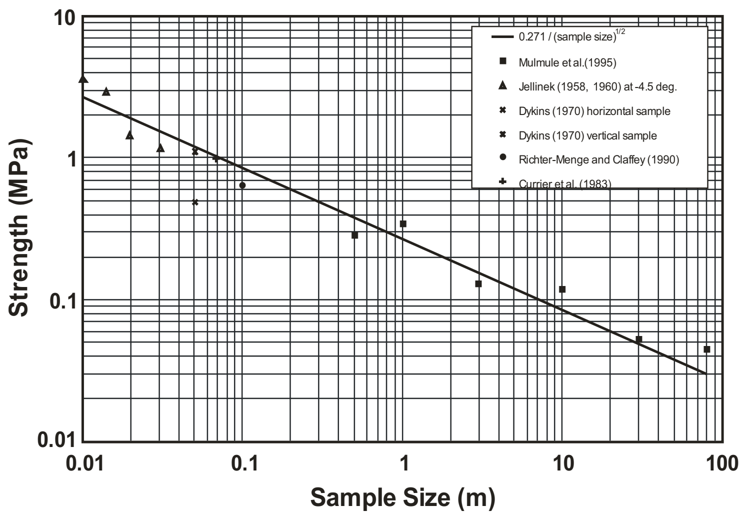

Figure 2: Size-dependent variation of tensile strength of ice [The references cited are given in Selvadurai and Sepehr (1999)]

3 Computational modelling

Approaches to the computational modelling of dynamic problems that include viscoplasticity and other rate-independent non-linear phenomena are described by a number of authors and the reader is referred to Zienkiewicz and Cormeau (1974), Owen and Hinton (1980), Simo and Hughes (1998) and Selvadurai and Sepehr (1999) for further details.

3.1 Computational procedure for viscoplasticity

The viscoplastic strain increment at the time interval $\Delta t_n = (t_{n+1} - t_n)$ can be obtained via a scheme given by

$$(\Delta \boldsymbol{\varepsilon}^{vp})_n = \Delta t_n \left\{ (1-\Omega)(\dot{\boldsymbol{\varepsilon}}^{vp})_n + \Omega (\dot{\boldsymbol{\varepsilon}}^{vp})_{n+1} \right\} \tag{14}$$

where Ω can take the values $0, 1$ and $1/2$ depending on either the fully explicit, fully implicit or the Crank-Nicolson schemes, respectively. The incremental stress at the iteration n can be given in the form

$$(\Delta\boldsymbol{\sigma})_n = [\mathbf{D}^{-1} + \Omega(\Delta t_n)\mathbf{H}_n]^{-1}\{[\mathbf{B}]_n(\Delta\mathbf{d})_n - (\dot{\boldsymbol{\varepsilon}}^{vp})_n(\Delta t_n)\} \tag{15}$$

where

$$\begin{aligned}(\Delta\boldsymbol{\varepsilon})_n &= [\mathbf{B}]_n(\Delta\mathbf{d})_n\\ (\Delta\boldsymbol{\varepsilon}^{vp}) &= (\dot{\boldsymbol{\varepsilon}}^{vp})_n(\Delta t_n) - \{\Omega(\Delta t_n)(\mathbf{H})_n\}(\Delta\boldsymbol{\sigma})_n\\ (\mathbf{H})_n &= (\partial\dot{\boldsymbol{\varepsilon}}^{vp}/\partial\boldsymbol{\sigma})_n\end{aligned} \tag{16}$$

and $\mathbf{D}$ is the elasticity matrix given by

$$\dot{\boldsymbol{\sigma}} = \mathbf{D}\dot{\boldsymbol{\varepsilon}}^e \tag{17}$$

During any time increment (Δt_n), the incremental form of the equation of equilibrium takes the form

$$\int_V [\mathbf{B}]_n^T(\Delta\boldsymbol{\sigma})_n + (\Delta\mathbf{f})_n = 0 \tag{18}$$

where $(\Delta\mathbf{f})_n$ is the vector of applied incremental forces during the time increment (Δt_n). The incremental displacement occurring at this particular time interval can also be written as

$$(\Delta\mathbf{d})_n = [\mathbf{K}_T]_n^{-1}\int_V [\mathbf{B}]_n^T[\mathbf{D}^{-1} + (\mathbf{C})_n]^{-1}(\dot{\boldsymbol{\varepsilon}}^{vp})_n(\Delta t_n)dV + (\Delta\mathbf{f})_n \tag{19}$$

and

$$[\mathbf{K}_T]_n = \int_V [\mathbf{B}]_n^T[\mathbf{D}^{-1} + \Omega(\Delta t)_n(\mathbf{H})_n]^{-1}[\mathbf{B}]_n dV \tag{20}$$

The displacement increments, when back-substituted into (15), give the stress increment

$$(\Delta\boldsymbol{\sigma})_n = D[\mathbf{B}(\Delta\mathbf{d})_n - (\dot{\boldsymbol{\varepsilon}}^{vp})_n(\Delta t)_n] \tag{21}$$

with

$$([\boldsymbol{\sigma}\,;\,\mathbf{d}\,;\,\boldsymbol{\varepsilon}^{vp}])_{n+1} = ([\boldsymbol{\sigma}\,;\,\mathbf{d}\,;\,\boldsymbol{\varepsilon}^{vp}])_n + ([\Delta\boldsymbol{\sigma}\,;\,\Delta\mathbf{d}\,;\,\Delta\boldsymbol{\varepsilon}^{vp}])_n \tag{22}$$

The evaluated stress increment is based on a linearized version of the equilibrium equations in an integral form. The total stresses given by $\boldsymbol{\sigma}$ will not exactly satisfy the complete equations of equilibrium. The incorporation of an out-of-balance residual force Ψ in each cycle will minimize the error, i.e.

$$(\Psi)_{n+1} = \int_V [\mathbf{B}]^T_{n+1} (\boldsymbol{\sigma})_{n+1} dV + (\mathbf{f})_{n+1} \neq 0 \tag{23}$$

and this residual force will be added to each applied force increment at the subsequent step. The time integration is unconditionally stable if $\Omega \geq 1/2$; i.e. the procedure is numerically stable but does not ensure accuracy of solution. Consequently, even for values of $\Omega \geq 1/2$, limits must be imposed on the selection of the time step to achieve a valid result. For viscoplasticity problems, which are based on an associated flow rule, $(Q = F)$, a linear flow function of the form $\Phi(F) = F$, and where F is described by the Mohr-Coulomb failure criterion, a recommended limit for the time increment as specified by Zienkiewicz and Cormeau (1974) that takes the form

$$\Delta t \leq \frac{4(1-\nu)(1-2\nu)F_0}{\gamma(1-2\nu+\sin^2\varphi)E} \tag{24}$$

where F_0 is the equivalent uniaxial yield stress $(c\cos\varphi)$ and E is Young's modulus. The change in the displacement associated with the viscoplastic strain is given by

$$\Delta d = \mathbf{K}^{-1}\left\{\int_V (\mathbf{B}^T \mathbf{D} d\boldsymbol{\varepsilon}^{vp} dV + \Delta\mathbf{f}\right\} \tag{25}$$

where $\Delta\mathbf{f}$ denotes the change in load during the time interval Δt. The resulting stress change is

$$\Delta\boldsymbol{\sigma} = \mathbf{D}[\mathbf{B}\Delta\mathbf{d} - (\Delta t)\, d\boldsymbol{\varepsilon}^{vp}] \tag{26}$$

The updated stress matrix after the time increment Δt is

$$\boldsymbol{\sigma}^{(1)} = \boldsymbol{\sigma}^{(*)} + \Delta\boldsymbol{\sigma} \tag{27}$$

where $\boldsymbol{\sigma}^{(*)}$ is the initial stress matrix. After n time steps, the stress state is given by

$$\boldsymbol{\sigma}^{(n+1)} = \boldsymbol{\sigma}^{(n)} + \Delta\boldsymbol{\sigma}^{(n)} \tag{28}$$

3.2 Computational procedure for discrete elements

The discrete element approach, which includes a transformation of an initially continuum region to a fragmented state, involves a sophisticated computational scheme that has many facets. Two aspects of the computational scheme merit further discussion purely because these procedures have a bearing on the accuracy and efficiency of the solution algorithm. The principal computational aspects associated with the discrete element modelling of a fragmentable viscoplastic material basically involves two components these are

(i) the procedures used to examine the non-linear material phenomena such as a viscoplasticity and non-linear inter-fragment interaction and

(ii) the procedures used in the solution of the dynamic equations of motion associated with the entire system of interacting fragments, intact continuum regions and structural components.

The computational aspects of the discrete element procedure, which accommodate viscoplasticity effects, fragmentation and inter-fragment contact generation-contact loss-a Signorini-type interpenetration constraint in is described in detail in the articles by Hocking, Mustoe and Williams (1985a, b) and Selvadurai and Sepehr (1999). The matrix equation governing the dynamic process can be expressed in the conventional form:

$$[\mathbf{M}]\left\{\frac{d^2}{dt^2}\{\mathbf{u}\}\right\}+[\mathbf{C}]\left\{\frac{d}{dt}\{\mathbf{u}\}\right\}+[\mathbf{K}]\{\mathbf{u}\}=\{\mathbf{f}\} \tag{29}$$

where $\{\mathbf{u}\}$ is the displacement vector, $\{\mathbf{f}\}$ is the force vector, $[\mathbf{M}]$, $[\mathbf{C}]$ and $[\mathbf{K}]$ are, respectively, mass, damping and stiffness matrices, and

$$\begin{aligned}&\left\{\frac{d}{dt}[\mathbf{u}]\right\}_{n+1/2}=\left\{\frac{d}{dt}[\mathbf{u}]\right\}_{n-1/2}+\left\{\frac{d^2}{dt^2}[\mathbf{u}]\right\}_{n}(\Delta t)\\&\{[\mathbf{u}]\}_{n+1/2}=\{[\mathbf{u}]\}_{n}+\left\{\frac{d}{dt}[\mathbf{u}]\right\}_{n+1/2}(\Delta t)\end{aligned} \tag{30}$$

where Δt is the time increment and the subscripts denote the time step number.

The stability of the computational scheme is controlled by the time increment in the integration scheme. This time increment has to satisfy criteria applicable to the solution of viscoplastic problems as well as the time increment criteria applicable to the integration of the dynamic equations (29). For viscoplasticity problems, the recommended limit for the time increment is given by (24). For the integration of the equations of dynamic equilibrium, the stability condition for the time increment Δt, which employs an explicit-explicit, partitioning (which is strictly applicable only to linear systems) takes the form

$$\Delta t \le 2[(1+\tilde{D}^2)^{1/2} - \tilde{D}]/\varpi_{\max} \tag{31}$$

where $\varpi_{\max}$ is the maximum frequency of the combined system involving both rigid body motion and deformability of the system, and $\tilde{D}$ is the fraction of critical damping at $\varpi_{\max}$. Other criteria have been proposed in the literature on computational mechanics (Owen and Hinton, 1980): e.g.

$$\Delta t \le 2/\varpi_{\max}; \quad \Delta t \le \beta l \left[\frac{\rho\,(1+\nu)\,(1-2\nu)}{E\,(1-\nu)}\right]^{1/2} \tag{32}$$

where β is a coefficient that depends on the element type and l is the smallest length between any two nodes. As is evident, if extensive fragmentation takes place without a limit on the smallest fragment size, the time increment has to be reduced accordingly to preserve stability of the computational scheme. This makes the computational procedure computing intensive. The computational code used in the studies was DEC-ICE, which was modified to accommodate the additional criteria related to limits on fragment generation and the dual possibility of either generation of viscoplastic flow and /or fragment evolution.

In addition to the computational modelling of fragmentation and the dynamic interaction between fragments, in the case of floating ice features, the hydrodynamic interaction between the fluid and the dynamically moving floating ice fragments should also be taken into consideration in correctly defining the interaction process. A complete hydrodynamic modelling of a moving ice feature is not attempted in this computational approach. The influence of hydrodynamic resistance is modelled by a drag force acting on an equivalent circular body moving on the water surface.

4 Interaction between an ice sheet and a stationary rigid structure

The computational scheme described briefly in the preceding sections is applied to the study of the impact of an ice sheet with a stationary immovable pier. We first consider the problem of the direct impact of an ice sheet of thickness of 1m and a circular plan form of diameter 20m (Figure 3). These dimensions also will prevent out of plane buckling and fragmentation that is usually associated with relatively thin ice floes, where the forces are generated through accumulation of ice fragments as opposed to in plane failure processes. The ice sheet moves with a steady velocity of 0.2 m/sec.

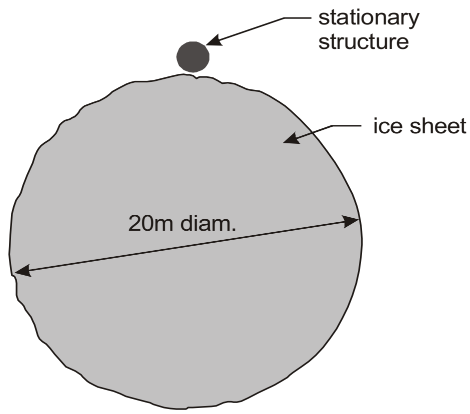

Figure 3: Impact of a moving ice sheet with a stationary rigid pier

The properties of the ice sheet are as follows:

Ice Sheet (*intact*):

$$\begin{cases} E = 3.5\text{ GPa}; \quad \nu = 0.35 \\ c = 1.5\text{ MPa}; \quad \varphi = 30^0; \quad \sigma_T = 0.5\text{ MPa} \\ \gamma = 1.0\times 10^{-2}\,\text{sec}^{-1}; \quad N = 1 \end{cases}$$

Ice Sheet (*failed*):

$$\begin{cases} E = 3.5\text{ GPa}; \quad \nu = 0.35 \\ c_{res} = 3.5\text{ Pa}; \quad \varphi_{res} = 3^0; \quad \sigma_{T\,res} = 0.5\text{ Pa} \\ \gamma = 1.0\times 10^{-2}\,\text{sec}^{-1}; \quad N = 1 \end{cases}$$

Fragmented Ice:

$$\begin{cases} E = 3.5\text{ GPa}; \quad \nu = 0.35 \\ c = 1.5\text{ MPa}; \quad \varphi = 30^0; \quad \sigma_T = (0.271/\sqrt{L})\text{ MPa} \\ \text{where } L \text{ is the fragment size} \end{cases}$$

Ice Fragments:

$$\begin{cases} k_n = 1.0\text{ GPa}; \quad k_s = 1.0\text{ GPa} \\ c = 0; \quad \varphi = 30^0 \end{cases}$$

The failed ice is assigned nominal values of cohesion, internal friction and tensile strength, which are denoted by the residual values (subscript $_{res}$). The ice sheet moves with a velocity of 0.2 m/sec and impacts a stationary rigid circular obstacle at an obliquity of 45^0 to the axis of the obstacle. The computational approach is applied to determine the average normal contact stresses that are generated at the contact zone between the indenting ice feature and the stationary rigid pier. Figure 4 illustrates the time history of the development of contact stresses and the corresponding development of fragmentation. As is evident, the peak contact stress develops at the intial impact and the stress decreases as fragmentation occurs. Figure 5 illustrates the time history of the contact stress development during oblique impact of the ice sheet (velocity of 0.2 m/sec and in plan, inclined at 45^0 to the reference coordinate system). Also in this case the peak contact stress occurs during the initial impact and the magnitude is considerably reduced due to the obliquity.

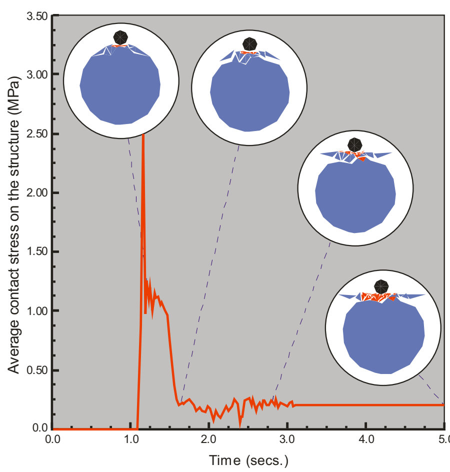

Figure 4: Time-dependent evolution of interactive contact stresses between the impacting ice-sheet and the stationary rigid pier - collinear impact.

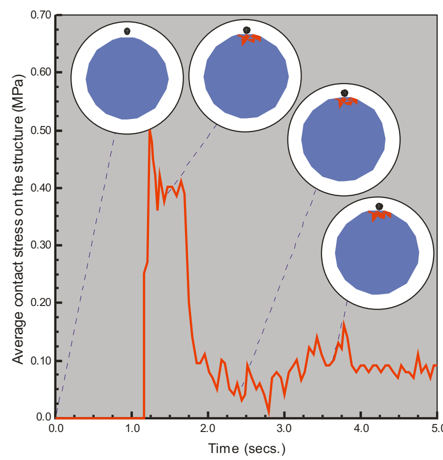

Figure 5: Time-dependent evolution of interactive contact stresses betwe

We next consider the problem of the oblique impact of a moving ice sheet of irregular plan shape with a stationary rigid pier. The plan area of the ice sheet is equivalent to a 10m diameter circular region of thickness 1m. The initial finite element discretization of the ice sheet is shown in Figure 6. A relatively coarse distribution of triangular elements is considered sufficient for the purpose of the computational modelling. The constitutive parameters for the ice sheet are identical to those used previously. The oblique impact occurs with a velocity of 0.2 m/sec. Figure 7 illustrates the time history of the interactive contact stress and the subsequent generation of fragmentation. The failure illustrated is not intended to be characteristic of the most probable modes of failure of the ice sheet; the mode of failure is governed by a number of factors including the strength characteristics of the ice, the obliquity of impact and the approach velocity. The results nonetheless illustrate plausible fragmentation processes in impacting ice features. It is also noted thatin the case of isolated ice features, the peak stress occurs prior to initiation of fragmentation.

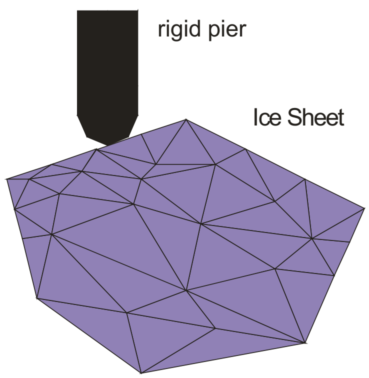

Figure 6: The finite element discretization used in the modelling of the intact ice sheet

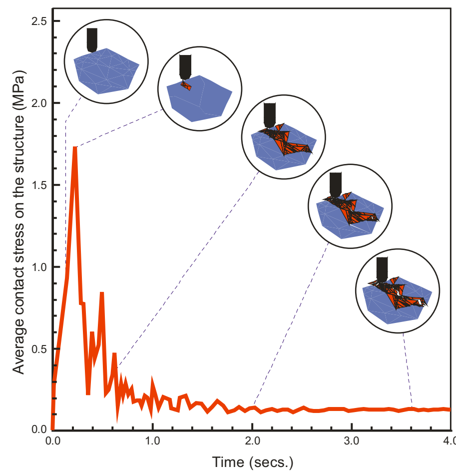

Figure 7: The oblique impact of an ice sheet with a stationary rigid structure.

5 Concluding remarks

The paper presents a computational approach for examining the dynamic interaction between a fragmentable ice sheet and a stationary rigid object. The modelling takes into consideration the fragmentation of an initially intact ice sheet that displays rate-sensitive constitutive properties. The fragments themselves can undergo frictional unilateral contact interactions that can initiate further break-up. To limit the uncontrolled generation of fragments, the strength characteristics governing fragmentation are assumed to increase with decreasing fragment size. The computational results indicate trends that are observed in dynamic interaction problems involving ice sheets and stationary structures. The fragment shape development is controlled by the shape of elements used in the initial discretization. This can result largely in quadrilateral and triangular ice fragments with an elongated geometry;

this satisfies the criteria controlling fragment generation mainly through splitting induced by the action of compressive inter-element contact forces. The analysis can be extended to include other forms of fragmentation that can include bending failure but excludes unattainable fragment geometries (Figure 7).

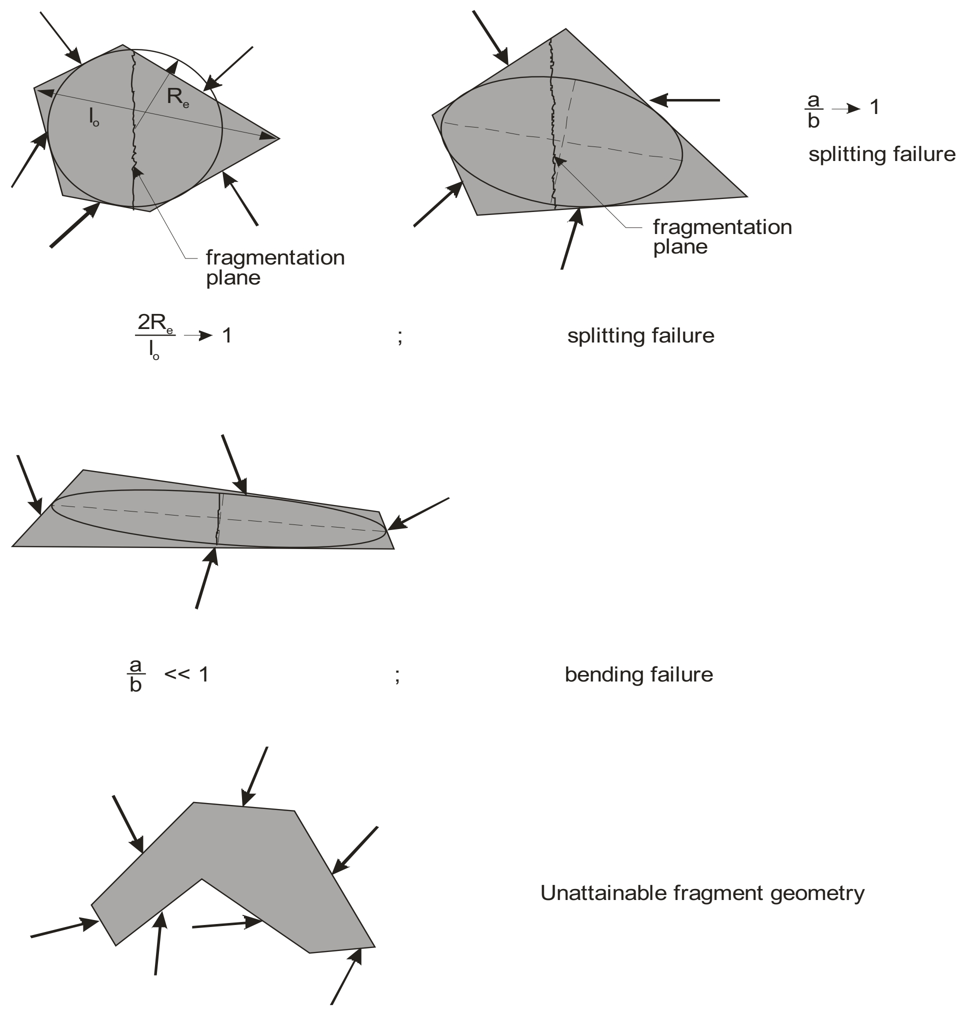

Figure 8: Fragment shapes

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