Heave of a surficial rock layer due to pressures generated by injected fluids

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[1] This paper examines the problem of heave developed in a surface or caprock layer of a storage formation due to the injection of pressurized fluids at depth. The problem is of interest to the deep geologic disposal of hazardous wastes and the geologic sequestration of carbon dioxide by injection. The paper develops a mathematical model that assumes elastic behavior of the storage medium while the surface rock layer is modeled as a thin plate. Elastic solutions are developed for the axisymmetric flexure deflections of the surface layer during internal pressurization over a flat circular region. The model provides a convenient tool for the preliminary estimation of the influence of injection pressures on the integrity of surface rock layers. **Citation:** Selvadurai, A. P. S. (2009), Heave of a surficial rock layer due to pressures generated by injected fluids, *Geophys. Res. Lett.*, *36*, L14302, doi:10.1029/2009GL038187.

1. Introduction

[2] Deep geologic storage or disposal of hazardous and toxic fluids, including liquefied nuclear waste and biological wastes by injection into storage formations has always been an option for environmental management [*Apps and Tsang*, 1996; *Selvadurai*, 2006]. The geologic sequestration of carbon dioxide in a supercritical form is now viewed as a possible abiotic solution for mitigating the impact of greenhouse gases [*Bachu and Adams*, 2003]. The objective of sequestration is to use the pore space of sparsely fractured storage rocks for the hydrodynamic and chemical trapping of the injected material. For the geologic sequestration of carbon dioxide in supercritical forms, hydrodynamic processes should lead to the development of a stable plume at the base of a caprock formation, which will develop as the ground water in the storage rock is displaced by both buoyancy and the pressures exerted during injection of the supercritical fluids. Fractured target rocks would make the pore space within the intact regions between fractures more easily accessible for trapping CO_2 over long periods, in excess of several thousand years; indeed a heavily fractured storage rock confined by intact caprock layers and an intact overburden would present the ideal geological setting for sequestration of carbon dioxide. An example of such a configuration is the carbon dioxide storage project at In Salah, Algeria [*Rutqvist et al.*, 2008a], where injection takes place in a layer of fractured sandstone. This storage unit is overlain by about 900 meters of competent low permeability caprock and a further 900 meter thick surface layer. The research in the area of geologic sequestration of carbon dioxide has largely focused on either the consideration of fluid dynamics or the geochemical influences of the carbon dioxide injection process [*Bickle et al.*, 2007] and the developments are continually being extended to include hydro-geomechanics aspects of the sequestration process [*Rutqvist et al.*, 2007, 2008b].

[3] This paper examines the problem of the interaction between a surface rock layer and a deep storage formation, when pressurized fluids are injected at a finite depth into the storage formation. It is important to assess the mechanics of the interaction between the surface rock and the storage formation during the injection in order to establish the efficiency of the injection process and to validate the modeling of the geomechanical processes. Furthermore, recent satellite measurements at the In Salah carbon dioxide storage site point to ground surface uplift in the vicinity of the injection wells [*Rutqvist et al.*, 2008a]. An elastostatic model is used to examine the problem; the surface rock layer is represented by a thin elastic plate and the storage formation into which injection takes place is modeled as an elastic halfspace. The pressures applied to the storage formation are represented by a circular disc-shaped region subjected to a distribution of centers of dilatation. The resulting axisymmetric problem in elasticity is examined using integral transform techniques to develop a solution for the heave of the surface rock layer. This problem is motivated by its applicability to geologic sequestration of carbon dioxide and, in particular, to estimating caprock movements during injection. Also, the greatest mechanical impact in a storage location is likely to occur near the ground surface. The existence of the free surface implies that the near surface regions will not be able to sustain large failure stress states because of low in situ stress states. The basic model, however, has wider potential geophysical applications, including the modeling of crustal deformations and ground subsidence during extraction of fluids, crust-lithosphere interaction during magma ascent and in the study of lithospheric dichotomies in planets [*Geertsma*, 1973; *Kruse et al.*, 1997; *Audet and Mareschal*, 2004; *Watters and McGovern*, 2006; *Nishimura*, 2006; *Lisowski*, 2006].

2. Modeling the Mechanics of a Crustal Rock Layer During Pressurization

[4] The present paper focuses on a specific problem related to the interaction between a surface caprock layer and a pressurized injection zone located within a target storage unit (Figure 1). The dimensions of the crustal rock in relation to the dimensions of the storage unit and the dimensions of the pressurization zone are assumed to be

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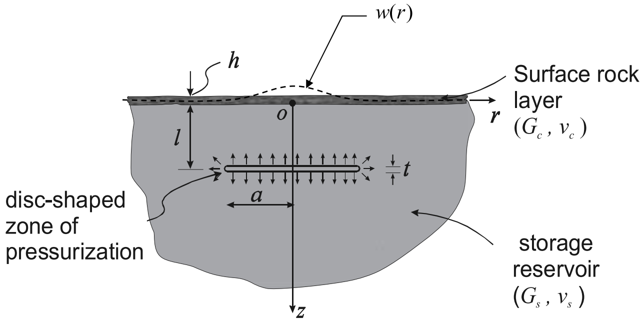

Figure 1. The geometry of the surface rock layer and the internally pressurized storage region.

such that the crustal layer can be adequately modeled as a thin elastic plate. The elastic plate idealization of surface rock layers has found useful applications in several seminal geophysical studies [*Walcott*, 1970; *Cathles*, 1975; *Turcotte and Schubert*, 2002; *Jaeger et al.*, 2007] and geomechanical interactions [*Selvadurai*, 1979, 2007]. The crustal layer of finite thickness rests on a storage unit that is assumed to be elastic and of semi-infinite extent. The storage region is assumed to be fluid-saturated but the analysis is restricted to an elastic model of the domain where the pressurization region is modeled as a distribution of centers of dilatation; these can be represented in terms of the magnitude of the injection pressures and the poroelastic properties of the porous skeleton. The interface between the surficial rock layer and the storage region can exhibit either complete bonding or complete frictionless contact, with Coulomb friction effects occupying an intermediate position. In view of the analytical nature of the solutions developed, the modeling presented accounts for the extreme cases of frictionless or fully bonded contact. The interaction between the surface rock layer and the underlying storage formation is caused by injection pressures of a constant magnitude applied over a circular disk-shaped region located at a finite depth below the base of the surface rock layer (Figure 1). The modeling of the problem focuses on the elastostatic analysis of the interaction between the intact storage unit, the caprock and the overlying rock. The choice of elasticity as a first approximation is a pragmatic one, which not only renders the problem tractable but for most competent rocks, the elastic model is entirely relevant [*Davis and Selvadurai*, 1996; *Selvadurai*, 2007; *Jaeger et al.*, 2007]. The problem can also be examined by recourse to a more complicated computational multi-physics model of the interaction between the various geologic horizons, which could include poroelasto-plasticity of the geologic layers and the consideration of regions of pressurization that have a moving pressure front associated with hydrodynamic displacement of immiscible fluids. This degree of model refinement is not warranted in the present circumstance, where the intention is to develop a straightforward analytical result that can be used to provide convenient analytical estimates for examining the sensitivity of the material and geometric parameters of the problem, which can include the thickness of the caprock, the radius of the pressurized zone and its proximity to the surface rock layer.

[5] The problem assumes that the surface rock layer can be modeled as a thin plate that exhibits flexural behavior governed by a Germain-Poisson-Kirchhoff thin plate theory [*Selvadurai*, 2000]. The justification of the thin plate approximation for the surface layer is governed largely by its thickness in relation to the dimension (radius) of the pressurized region a. The plate rests in contact with the storage unit (superscript 's'), which is modeled as an isotropic elastic halfspace region and the modeling is restricted to axisymmetric conditions, although more general states of deformation can be examined. Since the surface plate is not subjected to external loads, its deflection $w(r)$ is governed by the plate bending equation

$$D\tilde{\nabla}^2\tilde{\nabla}^2 w(r) + q^{(s)}(r) = 0 \tag{1}$$

where

$$\tilde{\nabla}^2 = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} \tag{2}$$

is the axisymmetric form of Laplace's operator in two-dimensions, $D(= G_c h^3/6(1 - \nu_c))$ is the flexural rigidity of the plate, h is the thickness of the surface rock layer, G_c and ν_c are, respectively, the linear elastic shear modulus and Poisson's ratio for the surface rock and $q^{(s)}(r)$ is the contact stress at the interface between the surface rock layer and the storage region. The interaction between the surface rock layer and the storage region is induced by the increase in pressure (of magnitude p_0) in a disc-shaped region of radius a and thickness t located at a distance l from the interface. The injection pressure can be visualized as over and above the hydrostatic stresses that may be present in the storage unit. The elastic effects of the pressurization due to a disc-shaped region of thickness t located within a traction free halfspace region were presented by several authors including *Geertsma* [1973], *Segall et al.* [1994], and *Jaeger et al.* [2007]. These results are, however, inapplicable to the situation being modeled here since the surface rock layer exerts an interactive constraint due to its stiffness. Finally, the surface rock layer is assumed to be either in bonded contact with the underlying halfspace region, or in frictionless bi-lateral contact. The latter assumption assumes that there can be relative slip between the surface rock and the underlying halfspace but there is continuity of normal tractions and normal displacements at the interface between the surface rock and the underlying halfspace during pressurization. The assumption is plausible only when contact stresses at the interface due to gravity effects are sufficient to prevent loss of contact. For fully bonded contact between the surface rock and the halfspace storage unit, the relevant interface conditions are

$$w(r) = u_z^{(s)}(r, 0) + u_z^{(s)} p_0(r, 0) \tag{3}$$

$$u_r^{(s)}(r, 0) = u_r^{(s)p_0}(r, 0) = 0 \tag{4}$$

where $u_r(r, z)$ and $u_z(r, z)$ are, respectively, the radial and axial components of the displacement vector referred to the cylindrical polar coordinate system (r, θ, z), and $u_r^{(s)p_0}(r, 0)$ and $u_z^{(s)p_0}(r, 0)$ are the displacements at the surface of the storage region in contact with the surface rock layer.

For frictionless contact between the surface rock layer and the halfspace storage unit, the relevant interface conditions are

$$w(r) = u_z^{(s)}(r,0) + u_z^{(s)p_0}(r,0) \tag{5}$$

$$\sigma_{rz}^{(s)}(r,0) = \sigma_{rz}^{(s)p_0}(r,0) = 0 \tag{6}$$

3. Analysis of Interaction Between the Surface Rock Layer and the Geologic Medium

[6] The mathematical analysis of the axisymmetric elastostatic interaction between the surface rock layer and the storage region induced by pressurization can be examined most conveniently using a procedure based on Hankel integral transforms [*Sneddon*, 1972; *Selvadurai*, 2000]. We define the zeroth-order Hankel transform of the function $F(r)$ by $\bar{F}(\xi)$, where

$$\bar{F}(\xi) = \int_0^\infty r\,F(r)\,J_0(\xi r)\,dr \tag{7}$$

and $J_0(\xi r)$ is the zeroth-order Bessel function of the first kind. The inverse of (7) follows from self-reciprocity properties of Hankel transforms. Considering the surface deflection of the storage unit under the action of the pressurized region, it can be shown that the general result is

$$u_z^{(s)p_0}(r,0) = -\frac{[\beta]\,\alpha_s(1-2\nu_s)p_0\,a\,t}{G_s}\int_0^\infty e^{-\xi l}J_1(\xi a)\,J_0(\xi r)\,d\xi \tag{8}$$

where

$$\begin{aligned} &\beta = 1; \quad \text{for frictionless contact} \\ &\beta = \frac{1}{2(1-\nu_s)}; \quad \text{for bonded contact} \end{aligned} \tag{9}$$

In (8), α_s is the Biot coefficient for the storage region defined by

$$\alpha_s = 1 - \frac{K}{K_g} \tag{10}$$

where K is the bulk modulus for the grain fabric (i.e. drained state of the porous medium) and K_g is the bulk modulus for the solid grain material itself and ν_s and G_s are, respectively, Poisson's ratio and shear modulus of the storage unit. Considering the deformations of the storage halfspace region subject to either the kinematic constraint (4) or the traction constraint (6) and the action of the compressive contact stresses $q^{(s)}(r)$ and the circular pressurized region, it can be shown that

$$\bar{u}_z^q(\xi) = \frac{[\kappa]\,\bar{q}^s(\xi)}{\xi\,G_s} \tag{11}$$

where

$$\begin{aligned} &\kappa = (1-\nu_s): \quad \text{for frictionless contact} \\ &\kappa = \frac{(3-4\nu_s)}{4(1-\nu_s)}; \quad \text{for bonded contact} \end{aligned} \tag{12}$$

Considering the kinematic constraints (3) and (5) we obtain

$$\bar{w}(\xi) = \bar{u}_z^{(s)}(\xi) + \bar{u}_z^{(s)p_0}(\xi) = \frac{[\kappa]}{G_s\xi}\left[\bar{q}^{(s)}(\xi) - \bar{S}(\xi)\right] \tag{13}$$

where

$$\bar{S}(\xi) = \frac{p_0 a\,t\alpha_s(1-2\nu_s)[\beta]}{[\kappa]}\,e^{-\xi l}J_1(\xi a) \tag{14}$$

and the values applicable to $[\beta]$ and $[\kappa]$ for the respective cases involving bonded and frictionless bi-lateral contact at the interface are given by (9) and (12).

[7] Operating on the differential equation (1) governing the flexural behavior of the surface rock layer with the zeroth-order Hankel transform (7), and eliminating $\bar{q}^{(s)}(\xi)$ between the resulting equation and (13), yields a result for $\bar{w}(\xi)$; inversion of the Hankel transform gives the result for the deflection $w(r)$. We can write the results applicable to either the frictionless or the fully bonded interfaces in the general form (note that there is no summation on repeated index i)

$$w(r) = \frac{\Omega_i a t}{h}\int_0^\infty \frac{e^{-(\lambda l/h)}}{\left[1+\Phi_i\lambda^3\right]}J_1(\lambda a/h)\,J_0(\lambda r/a)\,d\lambda \quad ; \quad (i = f \text{ or } b) \tag{15}$$

where

$$\begin{aligned} &\text{(i) for the } \textit{frictionless} \text{ interface:} \\ &\Omega_f = \frac{\alpha_s(1-2\nu_s)p_0}{G_s}; \quad \Phi_f = \frac{(1-\nu_s)}{6(1-\nu_c)}\left(\frac{G_c}{G_s}\right) \\ &\text{(ii) for the } \textit{bonded} \text{ interface:} \\ &\Omega_b = \frac{\alpha_s(1-2\nu_s)p_0}{2(1-\nu_s)G_s}; \quad \Phi_b = \frac{(3-4\nu_s)}{24(1-\nu_s)(1-\nu_c)}\left(\frac{G_c}{G_s}\right) \end{aligned} \tag{16}$$

[8] The formal elasticity solution for the interaction between the surface rock layer and the underlying geologic medium during pressurization is given by the expression (15) for the deflection of the caprock layer. Integral expressions for the contact stress and flexural stresses in the caprock can be determined by using (15) in the expressions for the flexural moments and shear force distributions in the plate-like surface layer; e.g. the radial flexural moment, $M_{rr}(r)$, in the layer is given by

$$M_{rr}(r) = \frac{\Omega_i G_c t\,h^2}{6a(1-\nu_c)}\int_0^\infty \frac{e^{-(\lambda l/h)}J_1\left(\frac{\lambda a}{h}\right)}{\left[1+\Phi_i\lambda^3\right]} \cdot \left(\lambda^2 J_0\left(\frac{\lambda r}{a}\right) - \frac{\lambda a}{r}(1-\nu_c)J_1\left(\frac{\lambda r}{a}\right)\right)d\lambda \tag{17}$$

(with $i = f$ or b). These results can be used to estimate the stresses in the caprock due to the pressurization. The geostatic stresses should be added to these pressurization-

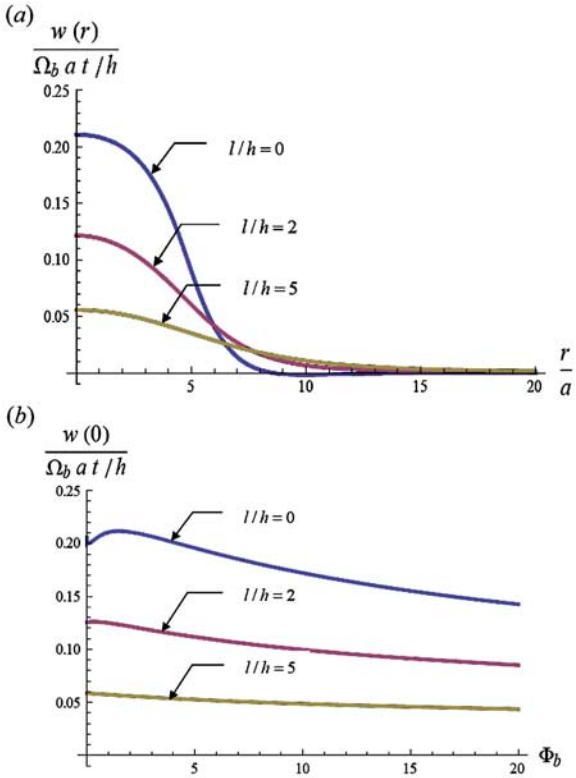

Figure 2. Influence of the relative depth of embedment of the pressurized injection zone on the deflections of the surface rock layer. (a) The variation of displacement with distance. (b) The variation of maximum deflection with the relative stiffness parameter.

induced stresses to determine the final stress state in the surface rock layer. The contact pressure $q^{(s)}(r)$ can similarly be determined using the expression (12) for $w(r)$ in (13).

4. Numerical Results

[9] Since the result for the deflection of the surface rock layer can be obtained in a compact integral form, it is possible to obtain specific results applicable to a target rock-caprock system once the specific values for the elasticity properties, the dimensions of the layer, the dimensions of the pressurized region and its depth of location are specified. The infinite integral (15) can be computed using a suitable quadrature to account for the oscillatory, but decaying, nature of the integrand that results from the product of the two Bessel functions. The integral result for $w(r)$ given by (15) contains two non-dimensional parameters Ω and Φ; these can be estimated by assigning typical relative values for the elasticity, geometrical and loading parameters in the system. The quantity $(\Omega h/at)$ can be used to non-dimensionalize $w(r)$, and the influence of Φ and the geometric parameters l/h, a/h and r/h can be assigned typical values. As an example, we consider a surface rock layer of thickness h = 200 m loaded by a pressurized circular region of radius a = 1000 m and thickness t = 10 m, located in the storage rock region at depth ratios ranging from l/h = 1, 2 and 5, with l measured in relation to the interface between the surface layer and the storage region (Figure 1). These dimensions can be varied to estimate the influence of either the position of the injection zone or its dimensions on both the magnitude and distribution of the caprock deflections. The numerical integration of expression (15) was performed using MATHEMATICA®, taking into consideration a suitable number of intervals to ensure convergence of the result. We present results for the case of the fully bonded interface where $a/h = 5$ and the parameter $\Phi_b = 2$, which approximately corresponds to $(G_c/G_s) \approx 13.5$ and $\nu_c = \nu_s \approx 1/4$. Figure 2a illustrates the distribution of the non-dimensional uplift deflection of the mid plane of the surface rock layer due to injection pressures of constant intensity applied over a thickness (t) and radius (a) but located at different depths (l) in relation to the thickness of the caprock. The numerical results clearly indicate the influence of the proximity of the pressurized region on both the magnitude of the maximum caprock displacement and the curvature of the deflected shape, which will influence the development of stresses in the caprock. As the depth of location of the pressurized zone in the storage region increases in relation to the thickness of the surface layer, the corresponding maximum displacements are reduced and abrupt changes to the deflected shape of the caprock layer are also reduced. Figure 2b presents results for the maximum heave of the surface layer as a function of the relative stiffness parameter $\Phi_b \in (0, 20)$. The reduction in the maximum displacement is most noticeable when the pressurized region is located at the interface between the surface layer and the underlying geologic medium. The influence of the depth of embedment of the pressurized region on the distribution of radial flexural moments in the surface layer is illustrated in Figure 3.

5. Concluding Remarks

[10] The paper presents an elementary model of the interaction between a surface rock layer and a deep rock mass that is pressurized during the injection of storage fluids. The elasticity model of the interaction is facilitated by modeling the surface rock layer as an elastic plate and the storage unit as an elastic halfspace. Solutions are presented for interface conditions that account for either fully bonded or frictionless bi-lateral contact at the interface. The effects of pressurization are accommodated through the use of fundamental elasticity solutions associated with centers of pressure. The analysis provides a convenient tool that can be used to estimate both heave of the surface rock

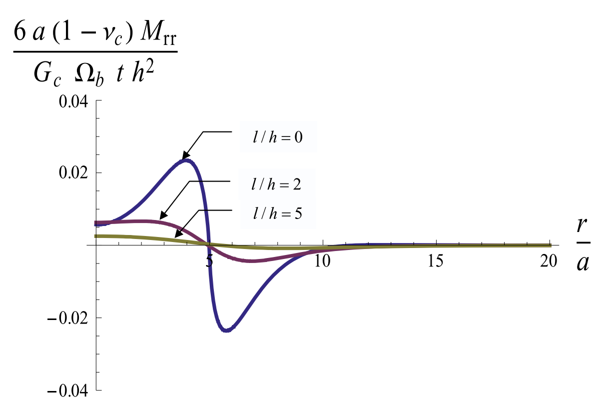

Figure 3. Influence of the relative depth of the flexural moments in the surface rock layer.

layer and the stresses generated due to a disc-shaped region of pressurization. The basic results can be easily extended to include a moderately thick surface layer that accounts for its shear deformations and for situations where injection occurs along a finite region of a borehole. The advantage of the elementary result developed in the paper is that it is amenable to convenient numerical evaluation that can be used more readily in sensitivity analysis of the effects of pressurized injection, which are influenced by a range of geometric and material parameters.

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